Lecture 3 | August 26, 2019

Properties of electromagnetic radiation. Polarization. Stokes' parameters.

1. Concepts of extinction (scattering + absorption) and emission.

2. Polarization. Stokes' parameters.

Required reading:

S: 2.3-2.4

Suggested reading:

Petty: 2.3

1. Concepts of extinction (scattering + absorption) and emission.

Electromagnetic radiation in the atmosphere interacts with gases, aerosol particles, and cloud particles.

- **Extinction** and **emission** are two main types of the interactions between an electromagnetic radiation field and a medium (e.g., the atmosphere).

General definition:

Extinction is a process that decreases the radiative **intensity**, while **emission** increases it.

NOTE: "same name": **extinction = attenuation**

Radiation is **emitted** by **all** bodies that have a temperature above absolute zero (^{O} K) (often referred to as **thermal emission**).

- **Extinction** is due to **absorption** and **scattering**.

Absorption is a process that removes the radiative energy from an electromagnetic field and transfers it to other forms of energy (conservation of energy law states that energy cannot be created or destroyed, but it can be converted or transferred from one form to another)

Scattering is a process that **does not** remove energy from the radiation field, but may redirect it.

NOTE: Scattering can be thought of as **absorption** of radiative energy followed by **re-emission** back to the electromagnetic field with negligible conversion of energy. Thus, scattering can remove radiative energy of a light beam traveling in one direction, but can be a "source" of radiative energy for the light beams traveling in other directions.

- Elastic scattering is the case when the scattered radiation has the same frequency as that of the incident field. Inelastic (Raman) scattering results in scattered light with a frequency different from that of the incident light.

2. Polarization. Stokes parameters.

Electromagnetic radiation travels as **transverse** waves, i.e., waves that vibrate in a direction perpendicular to their direction of propagation. **Polarization** is a phenomenon peculiar to **transverse** waves.

NOTE: Unlike electromagnetic (transverse) waves, sound is a **longitudinal** wave that travels through media by alternatively forcing the molecules of the medium closer together, then spreading them apart.

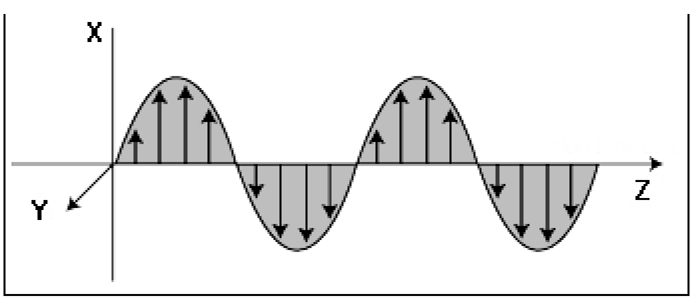
Polarization is the distribution of the electric field in the plane normal to the propagation direction.

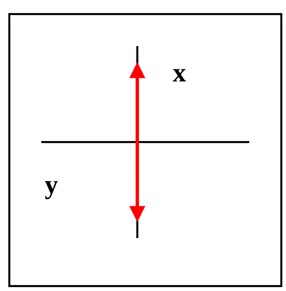
Unpolarized radiation (or randomly polarized) is an electromagnetic wave in which the orientation of the electrical vector changes randomly.

If there is a definite relation of phases between different scatterers => radiation is called **coherent.** If there is no relations in phase shift => light is called **incoherent**

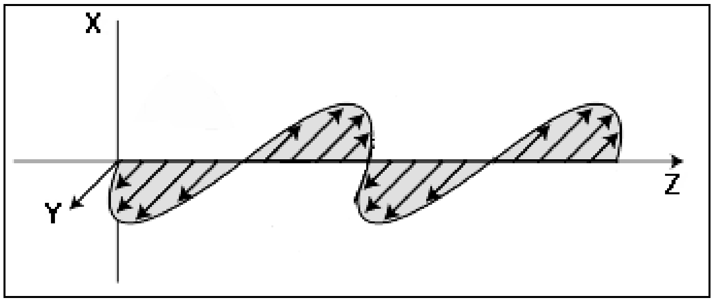
- **Natural light is incoherent.**
- **Natural light is unpolarized.**

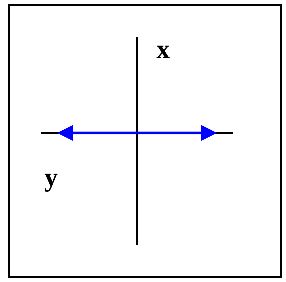
Vertically polarized wave is an EM wave in which electric field lies only in the x-z plane.





Horizontally polarized wave is one for which the electric field lies only in the y-z plane.





- Horizontal and vertical polarizations are an example of **linear polarization**.

Mathematical representation of a plane wave propagating in the direction *z (2D case)* is

$$E = E_0 \cos(kz - \omega t + \varphi_0) \qquad [3.1]$$

where E_0 is the **amplitude;**

k is the propagation (or wave) constant, ***k* = 2π/λ ;**

ω is the circular frequency, **ω** = ***kc* = *2πc/λ*** ; φ_0 is the constant (or initial phase)

$\varphi = (kz - \omega t + \varphi_0)$ is **the phase of the wave.**

Introducing complex variables, Eq.[3.1] can be expressed as

$$E = E_0 \exp(i\varphi) \qquad [3.2]$$

NOTE: we use $\exp(\pm i\varphi) = \cos(\varphi) \pm i \sin(\varphi)$

The electric vector $\vec{E}$ may be decomposed into the parallel E_l and perpendicular E_r components as

$$\vec{E} = E_l \vec{l} + E_r \vec{r}$$

We can express E_l and E_r in the form

$$E_l = E_{l0} \cos(kz - \omega t + \varphi_{l0})$$

$$E_r = E_{r0} \cos(kz - \omega t + \varphi_{r0})$$

Then we have

$$E_l / E_{l0} = \cos(\zeta)\cos(\varphi_{l0}) - \sin(\zeta)\sin(\varphi_{l0})$$

$$E_r / E_{r0} = \cos(\zeta)\cos(\varphi_{r0}) - \sin(\zeta)\sin(\varphi_{r0})$$

where $\zeta = kz - \omega t$.

After some mathematical manipulation, we obtain

$$(E_l / E_{l0})^2 + (E_r / E_{r0})^2 - 2(E_l / E_{l0})(E_r / E_{r0})\cos(\Delta\varphi) = \sin^2(\Delta\varphi) \qquad [3.3]$$

where $\Delta\varphi = \varphi_{lo} - \varphi_{r0}$ called the **phase shift**.

Eq.[3.3] defines an ellipse => **elliptically polarized wave**.

If the phase shift $\Delta\varphi$ **= n π (n=0, +/-1, +/-2,…),** then

$\sin(\Delta\varphi) = 0$ and $\cos(\Delta\varphi) = \pm 1$, and Eq.[3.3] becomes

$$\left(\frac{E_l}{E_{l0}} \pm \frac{E_r}{E_{r0}}\right)^2 = 0 \quad \text{or} \quad E_r = \pm\frac{E_{ro}}{E_{lo}} E_l \qquad [3.4]$$

Eq.[3.4] defines straight lines => **linearly polarized wave**

If the phase shift $\Delta\varphi$ **= n π /2 (n= +/-1, +/-3,…)** and $E_{l0} = E_{r0} = E_0$, then

$\sin(\Delta\varphi) = \pm 1$ and $\cos(\Delta\varphi) = 0$, and Eq.[3.3] becomes

$$E_l^2 + E_r^2 = E_0^{\ 2} \qquad [3.5]$$

Eq.[3.5] defines a circle => **circular polarized wave**

NOTE: The sign of the phase shift gives **handedness**: right-handed and left-handed polarization.

- The state of polarization is completely defined by four parameters: two amplitudes, and the magnitude and the sign of the phase shift (see Eq.[3.3]). Because the phase difference is hard to measure, the alternative description called a **Stokes vector** is often used.

Stokes Vector consists of four parameters **(called Stokes parameters)**:

intensity *I*,

the degree of polarization *Q*,

the plane of polarization *U*,

the ellipticity *V*.

Notation

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} \quad \text{or} \quad \{I, Q, U, V\}$$

- **Stokes parameters** are defined via the intensities which can be measured:

I = total intensity

Q= I_0-I_{90} = differences in intensities between horizontal and vertical linearly polarized components;

U = I_{+45} –I_{-45}= differences in intensities between linearly polarized components oriented at +45^0 and -45^0

V = I_{RHC} –I_{LCH} = differences in intensities between right and left circular polarized components.

- **Stokes parameters** can be expressed via the amplitudes and the phase shift of the parallel and perpendicular components of the electric field vector

$$I = E_{ro}^2 + E_{lo}^2$$

$$Q = E_{ro}^2 - E_{lo}^2 \qquad [3.6]$$

$$U = 2E_{ro}E_{lo}\cos(\Delta\varphi)$$

$$V = 2E_{ro}E_{lo}\sin(\Delta\varphi)$$

***Example*:** Stokes parameters for the vertical polarization:

For this case $E_l = 0$

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} E_{ro}^2 \\ E_{r0}^2 \\ 0 \\ 0 \end{pmatrix} = E_{ro}^2 \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

For **unpolarized** light:

$$Q = U = V = 0 \qquad [3.7]$$

The **degree of polarization *P*** of a light beam is defined as

$$P = (Q^2 + U^2 + V^2)^{1/2} / I \qquad [3.8]$$

The **degree of linear polarization *LP*** of a light beam is defined by neglecting U and V

$$LP = -\frac{Q}{I} \qquad [3.9]$$

NOTE: Measurements of polarization are often used in remote sensing in the solar and microwave regions and in active remote sensing techniques

- Polarization in the microwave – mainly due to reflection from the surface.
- Polarization in the solar – reflection from the surface and scattering by molecules and particulates.
- Active remote sensing (e.g., radar, lidar) commonly uses polarized radiation.