

Lecture 9

Scattering and absorption by aerosol and cloud particles: Mie theory

1. Properties of aerosol and clouds.
2. Scattering and absorption by aerosol and cloud particles: Intro to Mie theory

Required reading:

S: 1.6; 5.1-5.4, 5.6, 5.7

Additional reading

Liou:

Petty: 4, 7, 12

Advanced reading

Bohren, C. F., and D. R. Huffman, *Absorption and scattering of light by small particles*. John Wiley&Sons, New York, pp. 531, 1983.

1. Properties of aerosol and clouds.

The interaction of particulates (aerosols, cloud drops, ice crystals, rain drops, etc.) with electromagnetic radiation is controlled by the particle amount, size, composition (refractive index), and shape.

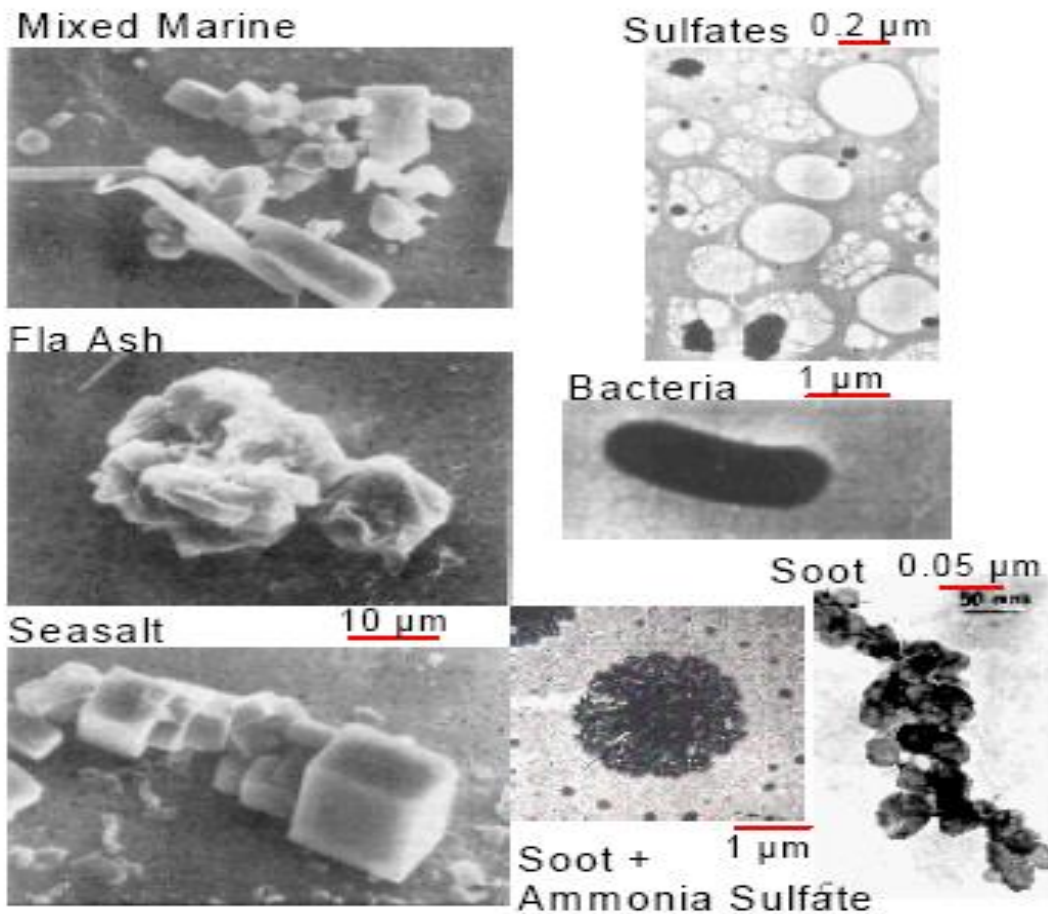
- Atmospheric aerosols are solid and/or liquid (or mixed phase) particles suspended in the air with diameters between about 0.002 μm to about 50-100 μm .

Chemical composition:

Individual chemical species: sulfate (SO_4^{2-}), nitrate (NO_3^-), soot (elemental carbon or black carbon), sea-salt (NaCl); minerals (e.g., quartz, SiO_4 , clays, feldspar, etc.)

Multi-component (MC) aerosols: complex make-up of many chemical species (called internally mixed particles)

Shape:



Spheres: all aqueous aerosol particles (e.g., sulfates, nitrates, etc.)

Complex shapes: dust, soot (i.e., solid particles)

“Classical” representation of particle size spectrum:

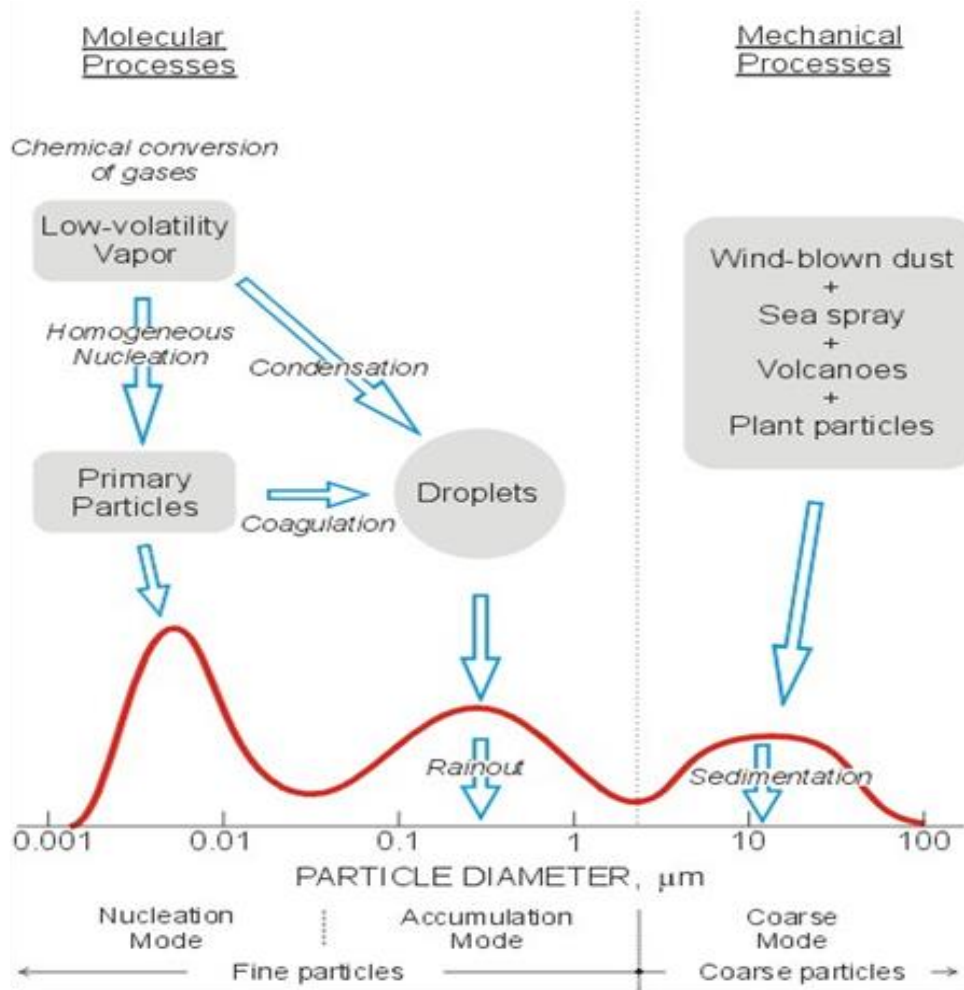


Figure 9.1 Idealized schematic of the size distribution of atmospheric aerosol particle (from Whitby and Cantrell, 1976).

NOTE: **fine mode** ($d < 2.5 \mu\text{m}$) and **coarse mode** ($d > 2.5 \mu\text{m}$); fine mode is divided on the **nuclei mode** (about $0.005 \mu\text{m} < d < 0.1 \mu\text{m}$) and **accumulation mode** ($0.1 \mu\text{m} < d < 2.5 \mu\text{m}$).

The particle size distribution of aerosols are often approximated by a sum of three log-normal functions as

$$N(r) = \sum_i \frac{N_i}{\sqrt{2\pi} \ln(\sigma_i) r} \exp\left(-\frac{\ln(r/r_{0,i})^2}{2 \ln(\sigma_i)^2}\right) \quad [9.1]$$

where $N(r)$ is the particle number concentration, N_i is the total particle number concentration of i -th size mode with its median radius $r_{0,i}$ and geometric standard deviation σ_i .

Property of the log-normal function: the k -moment is

$$\int r^k N(r) dr = N_0 r_0^k \exp(k^2 (\ln \sigma)^2 / 2) \quad [9.2]$$

NOTE: Eq [9.2] helps to quickly calculate the mass, volume, and surface area size distribution from the particle number size distribution.

➤ **Clouds:**

- ✓ Cloud droplet sizes vary from a few micrometers to 100 micrometers with average diameter in the 10 to 20 μm range.
- ✓ Cloud droplet concentration varies from about 10 cm^{-3} to 1000 cm^{-3} with an average droplet concentration of a few hundred cm^{-3} .
- ✓ The liquid water content of typical clouds, often abbreviated LWC, varies from approximately 0.05 to 3 $\text{g}(\text{water}) \text{ m}^{-3}$, with most of the observed values in the 0.1 to 0.3 $\text{g}(\text{water}) \text{ m}^{-3}$ region.

Cloud droplets size distribution is often approximated by a **modified gamma distribution**

$$N(r) = \frac{N_0}{\Gamma(\alpha) r_n} \left(\frac{r}{r_n} \right)^{\alpha-1} \exp(-r / r_n) \quad [9.3]$$

where N_0 is the total number of droplets (cm^{-3}); r_n in the radius that characterizes the distribution ; α in the variance of the distribution, and Γ is the gamma function.

Table 9.1 Characteristics of representative size distributions of some clouds (for $\alpha = 2$)

Cloud type	N_0 (cm^{-3})	r_m (μm)	r_{max} (μm)	r_e (μm)	LWC (g m^{-3})
Stratus: over ocean over land	50 300-400	10 6	15 15	17 10	0.1-0.5 0.1-0.5
Fair weather cumulus	300-400	4	15	6.7	0.3
Maritime cumulus	50	15	20	25	0.5
Cumulonimbus	70	20	100	33	2.5
Altostratus	200-400	5	15	8	0.6

NOTE: For many practical applications, the optical properties of water clouds are parameterized as a function of the **effective radius** and **liquid water content (LWC)**.

The **effective radius** is defined as

$$r_e = \frac{\int \pi r^3 N(r) dr}{\int \pi r^2 N(r) dr} \quad [9.4]$$

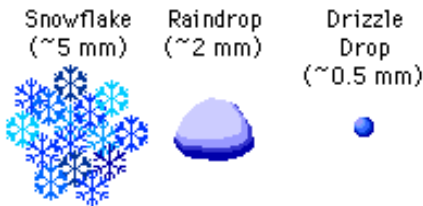
where $N(r)$ is the droplet size distribution (e.g., in units $\text{m}^{-3}\mu\text{m}^{-1}$).

NOTE: Mean radius: $r_m = (\alpha + 1) r_n$ Effective radius: $r_e = (\alpha + 3) r_n$

The **liquid water content (LWC)** is defined as

$$LWC = \rho_w V = \frac{4}{3} \rho_w \int \pi r^3 N(r) dr \quad [9.5]$$

➤ **Raindrops**



Nonspherical particles: shape depends on size of a rain drop.

Raindrop **size distribution** is often represented by the **Marshall-Palmer distribution**:

$$N(r) = N_0 \exp(-2\Lambda r) \quad [9.6]$$

where $N_0 = 8 \times 10^3 \text{ m}^{-3} \text{ mm}^{-1}$, but, in general, N_0 depends on the rain type;

Λ is related to the rainfall rate, Rr (in mm/hour) as $\Lambda = 4.1 Rr^{-0.21} \text{ mm}^{-1}$

➤ **Ice crystals:**

- ✓ Sizes range from $\sim 5\text{-}10 \mu\text{m}$ to a few mm
- ✓ Exhibit a large variety of shapes (called habits): for instance, plates - nearly flat hexagon; columns - elongated, flat bottoms; needles - elongated, pointed bottoms; dendrites - elongated arms (six), snowflake shape.

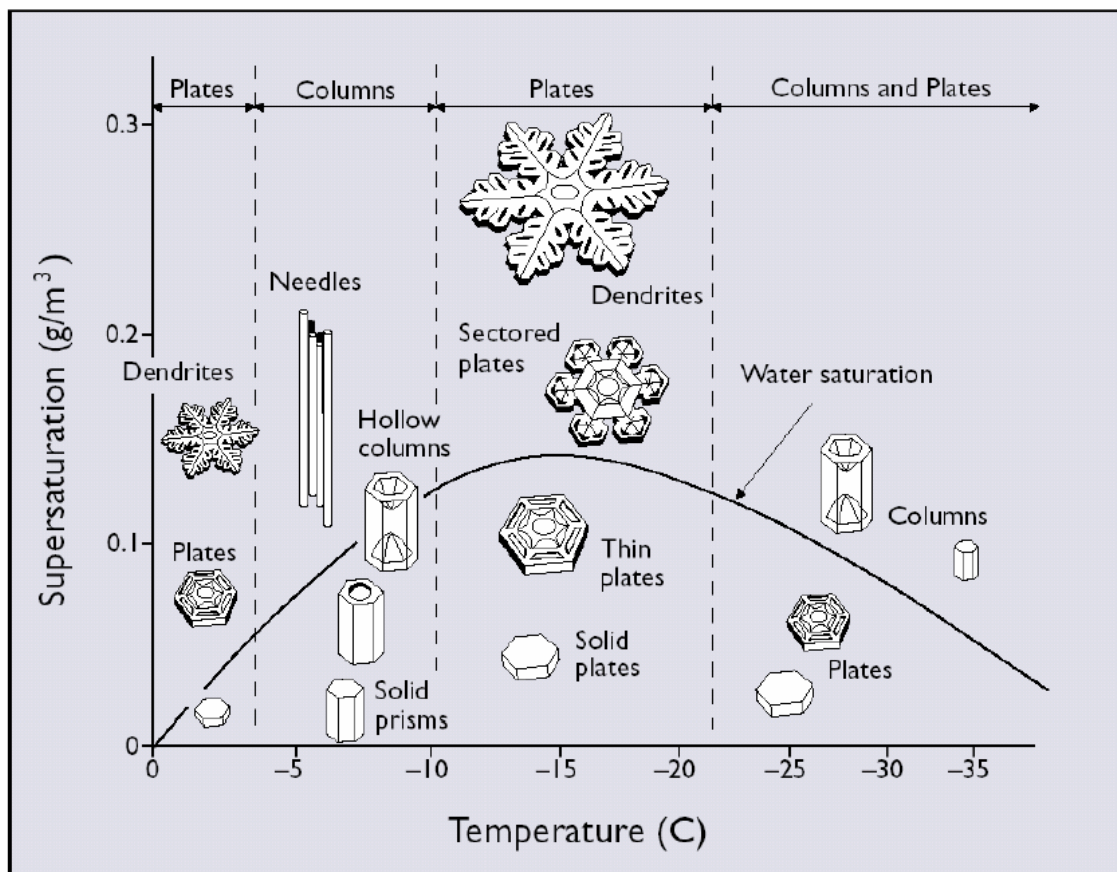


Figure 9.2 Ice crystal's shape as a function of supersaturation and temperature – a “classical” view

➤ Refractive indices of water, ice, and aerosol species.

Refractive index (or optical constants), $m=n - ix$, is the material properties of dielectric that determines its radiative properties. In general, each material has its own spectral refractive index. The imaginary part x of the refractive index determines the absorption of the wave as it propagates through the medium; the real part n of the refractive index gives the phase velocity of propagation.

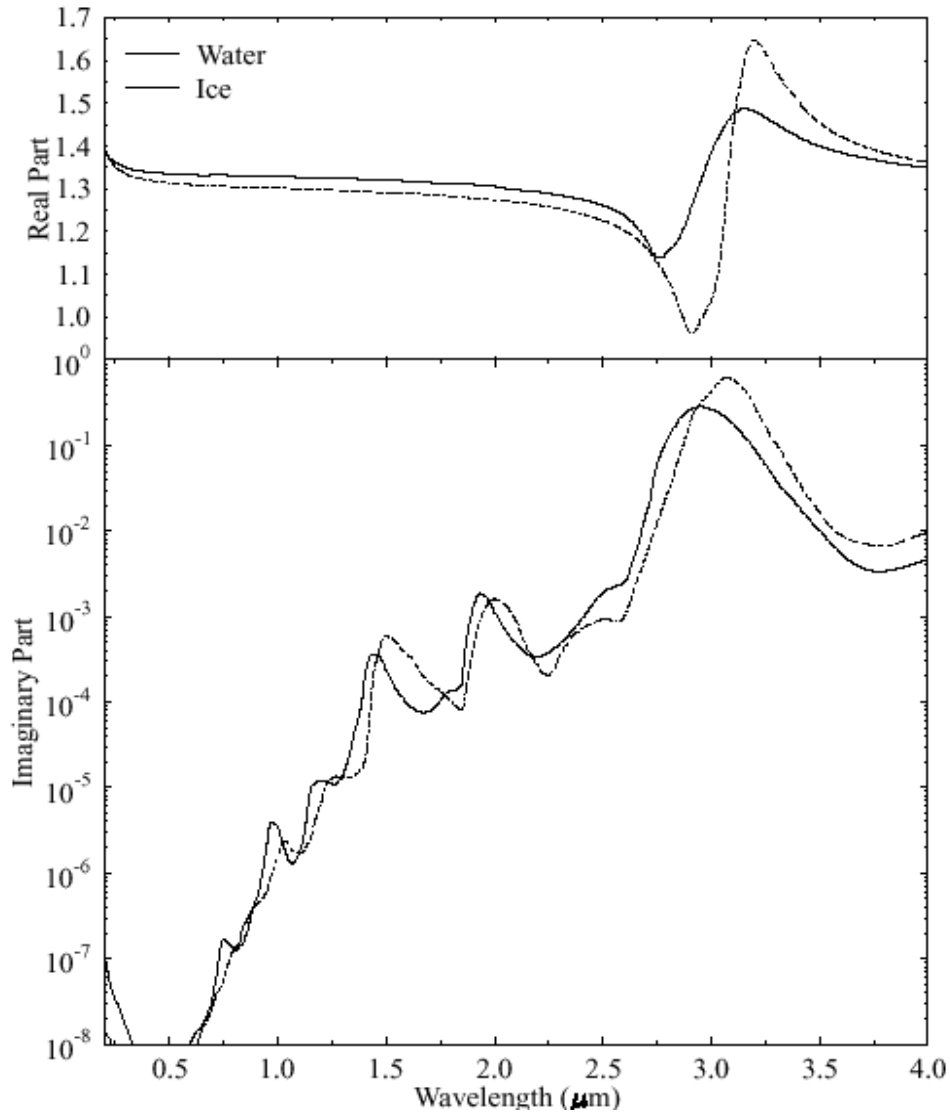


Figure 9.3 The refractive index of water and ice in the visible and near-IR.

NOTE: water has low imaginary part in the visible => negligible absorption by water drops in the visible

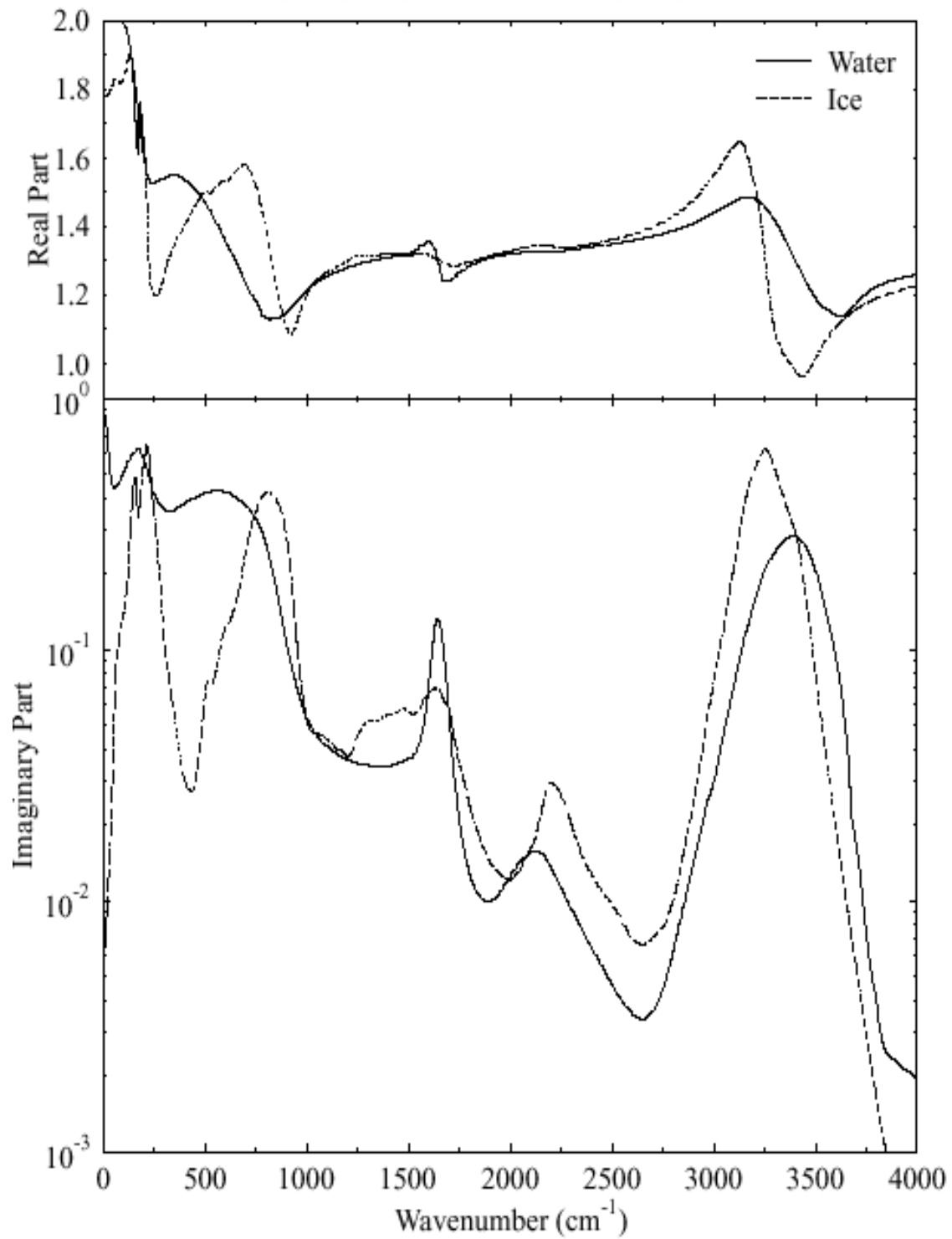


Figure 9.4 The refractive index of water and ice in the IR.

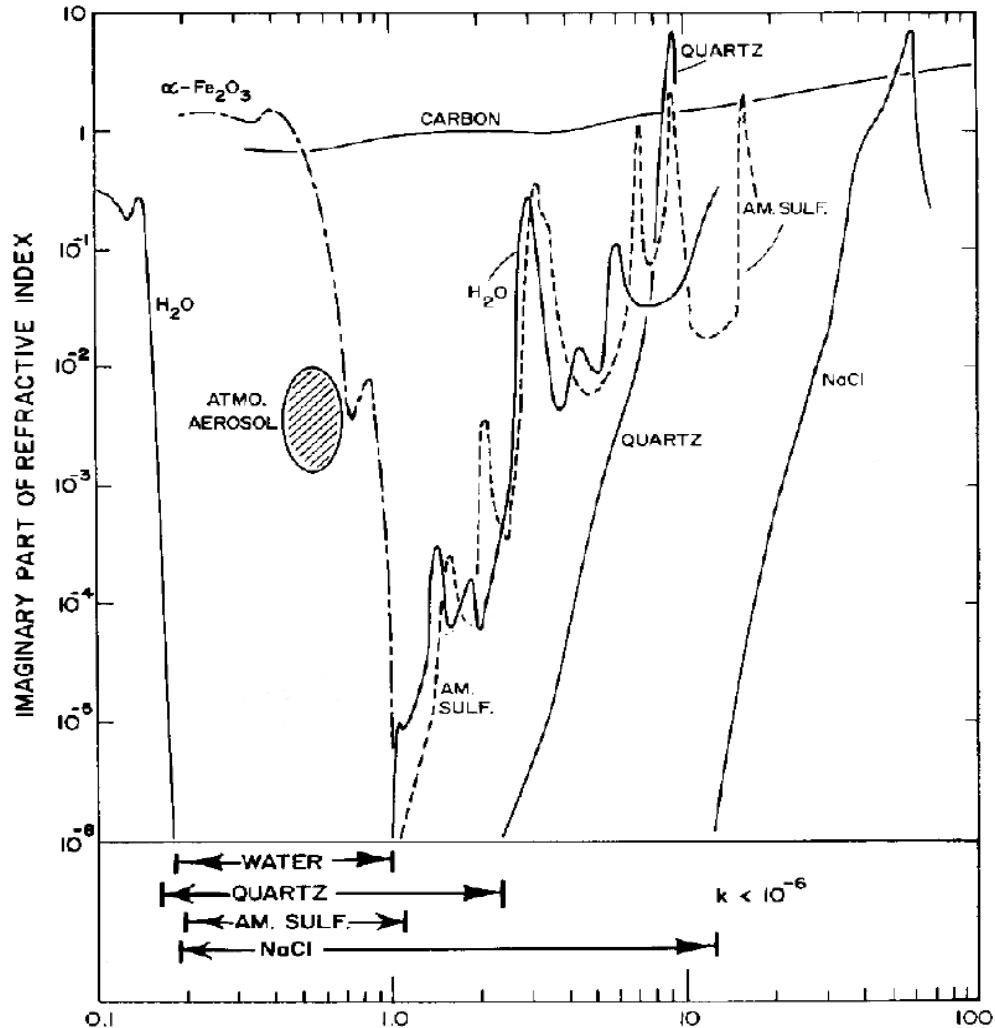


Figure 9.5 “Classical plot” of the imaginary part of the refractive indexes of some aerosol materials as a function of wavelength (in μm) (Bohren and Huffman, Fig.4.16).

NOTE: Main absorbing species in the solar spectrum are black carbon (soot) and hematite (dust), but in the thermal IR various species have high imaginary parts of the refractive index.

NOTE: Aerosol particles often consist of several chemical species (called the **internal mixture**). There are several approaches (called **mixing rules**) to calculate an **effective refractive index** m_e of the internally mixed particles by using the refractive indices of the individual species. For instance, **Volume weighted mixing**:

$$m_e = \sum_j m_j f_j \quad [9.7]$$

where m_j is the refractive index of the j -species and f_j is its volume fraction.

2. Scattering and absorption by aerosol and cloud particles: Mie theory.

Mie theory describes the scattering and absorption of electromagnetic radiation by spherical particles through solving the Maxwell equations.

NOTE: Mie theory is also called **Lorenz-Mie theory** or **Lorenz-Mie-Debye theory**.

===== **Mie theory outline** =====

General outline of Mie theory:

Key Assumptions:

- i) Particle is a **sphere**;
- ii) Particle is **homogeneous** (therefore it is characterized by a **single refractive index** $m=n - ix$ at a given wavelength);

NOTE: Mie theory requires the relative refractive index that is the refractive index of a particle divided by the refractive index of a medium. For air m is about 1, so one needs to know the refractive index of the particle (i.e., refractive index of the material of which the particle is composed).

NOTE: If a particle has complex chemical composition such as some atmospheric aerosols, the effective refractive index must be calculated at a given wavelength.

- Mie theory calculates the scattered electromagnetic field at all points within the particle (called internal field) and at all points of the homogeneous medium in which the particle is embedded. For all practical applications in the atmosphere, light scattering observations are carried out in the **far-field zone** (i.e., at the large distances from a particle).

In the far-field zone (i.e., at the large distances R from a sphere), the solution of the wave equation can be obtained as

$$\begin{bmatrix} E_l^s \\ E_r^s \end{bmatrix} = \frac{\exp(-ikR + ikz)}{ikR} \begin{bmatrix} S_2 & S_3 \\ S_4 & S_1 \end{bmatrix} \begin{bmatrix} E_l^i \\ E_r^i \end{bmatrix} \quad [9.8]$$

here $k = 2\pi/\lambda$, E_l^i and E_r^i are the parallel and perpendicular components of incident electrical field, and E_l^s and E_r^s are the parallel and perpendicular components of scattered electrical field,

$\begin{bmatrix} S_2(\Theta) & S_3(\Theta) \\ S_4(\Theta) & S_1(\Theta) \end{bmatrix}$ is the **amplitude scattering matrix** (unitless)

For spheres: $S_3(\Theta) = S_4(\Theta) = \mathbf{0}$, and thus Eq.[6.1] gives

$$\begin{bmatrix} E_l^s \\ E_r^s \end{bmatrix} = \frac{\exp(-ikR + ikz)}{ikR} \begin{bmatrix} S_2(\Theta) & 0 \\ 0 & S_1(\Theta) \end{bmatrix} \begin{bmatrix} E_l^i \\ E_r^i \end{bmatrix} \quad [9.9]$$

Eq. [4.37] is a fundamental equation of scattered radiation by a sphere including polarization.

Mie theory solution for the scattering amplitudes:

$$S_1(\Theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} [a_n \pi_n(\cos \Theta) + b_n \tau_n(\cos \Theta)] \quad [9.10]$$

$$S_2(\Theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} [b_n \pi_n(\cos \Theta) + a_n \tau_n(\cos \Theta)] \quad [9.11]$$

where π_n and τ_n are **Mie angular functions**

$$\pi_n(\cos \Theta) = \frac{1}{\sin(\Theta)} P_n^1(\cos \Theta) \quad [9.12]$$

$$\tau_n(\cos \Theta) = \frac{d}{d\Theta} P_n^1(\cos \Theta) \quad [9.13]$$

where P_n^1 are the associated Legendre polynomials.

Mie theory also gives the **scattering phase matrix** $P(\Theta)$ that relates the Stokes parameters $\{I_0, Q_0, U_0 \text{ and } V_0\}$ of incident radiation field and the Stokes parameters $\{I, Q, U \text{ and } V\}$ of scattered radiation:

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \frac{\sigma_s}{4\pi r^2} P \begin{bmatrix} I_o \\ Q_o \\ U_o \\ V_o \end{bmatrix} \quad [9.14]$$

where

$$P = \begin{bmatrix} P_{11} & P_{12} & 0 & 0 \\ P_{12} & P_{22} & 0 & 0 \\ 0 & 0 & P_{33} & -P_{34} \\ 0 & 0 & P_{34} & P_{44} \end{bmatrix} \quad [9.15]$$

NOTE: In general, for a particle of any shape, the scattering phase matrix consists of 16 independent elements, but for a sphere this number reduces to four.

For spheres: $P_{22} = P_{11}$ and $P_{44} = P_{33}$

Thus **for spheres**, Eq.[9.14] reduces to

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \frac{\sigma_s}{4\pi r^2} \begin{bmatrix} P_{11} & P_{12} & 0 & 0 \\ P_{12} & P_{11} & 0 & 0 \\ 0 & 0 & P_{33} & -P_{34} \\ 0 & 0 & P_{34} & P_{33} \end{bmatrix} \begin{bmatrix} I_o \\ Q_o \\ U_o \\ V_o \end{bmatrix} \quad [9.16]$$

and each element of the scattering phase matrix is expressed via the scattering amplitudes $S_1(\Theta)$ and $S_2(\Theta)$.

$P_{11}(\Theta) = P(\Theta)$ is the scattering phase function of a particle.

From Mie theory it follows that the **extinction cross-section** of a particle is

$$\sigma_e = \frac{4\pi}{k^2} \text{Re}[S(0^0)] \quad [9.17]$$

But for the forward direction (i.e. $\Theta = 0^0$) from Eqs.[1]-[4], we have

$$S_1(0^0) = S_2(0^0) = \frac{1}{2} \sum_{n=1}^{\infty} (2n+1)(a_n + b_n)$$

-----END of Mie theory outline-----