

## Lecture 10.

### Mie theory and remote sensing applications of direct solar radiation

1. Mie Theory revisited
2. What can we infer from Mie Theory?
3. Remote sensing applications based on measurements of direct solar radiation

#### Required reading:

S: 5.6, 5.7; P: 12.3

#### Additional reading

L: 5

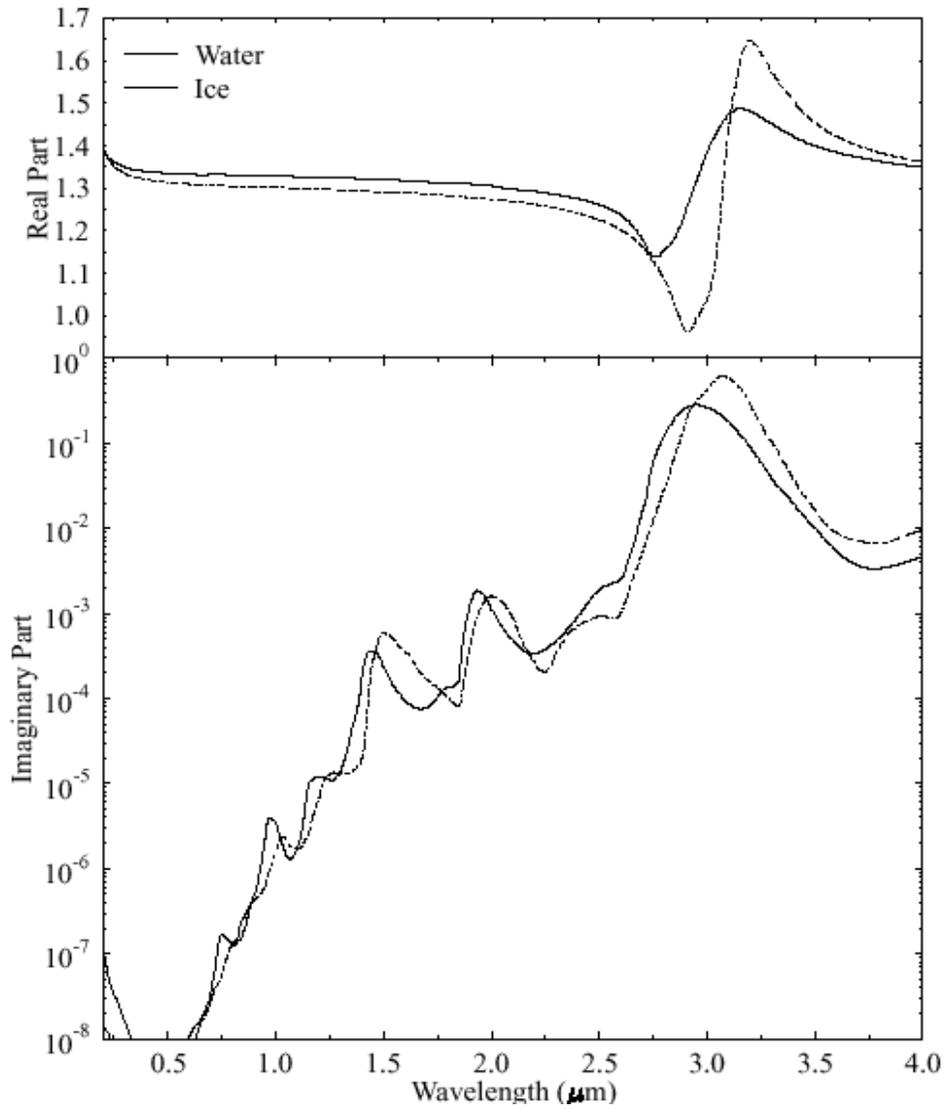
### 1. Mie theory revisited

Refractive indices database: <https://refractiveindex.info/>

Prahl (Online) Mie scattering calculator: [https://omlc.org/calc/mie\\_calc.html](https://omlc.org/calc/mie_calc.html)

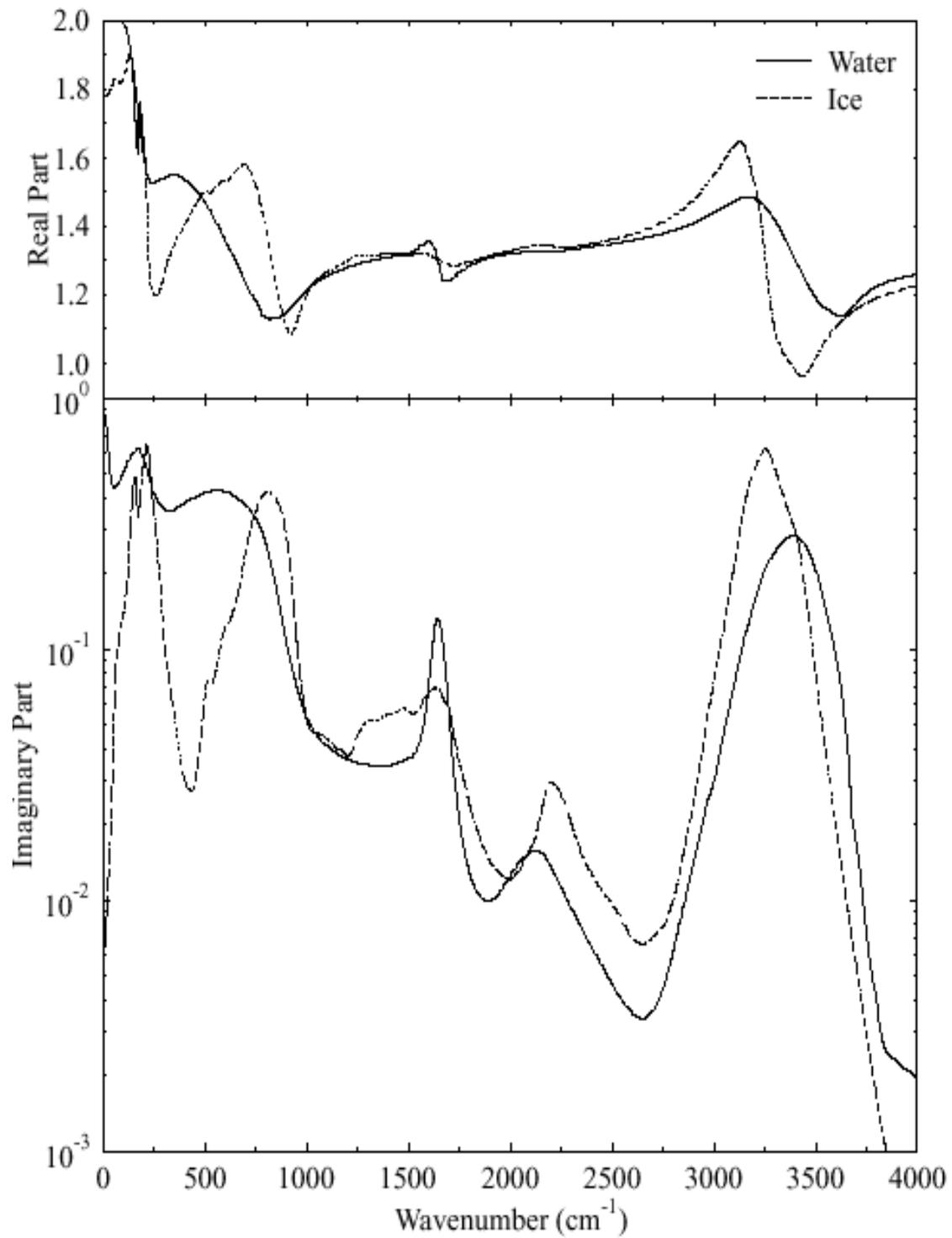
#### ➤ Refractive indices of water, ice, and aerosol species.

**Refractive index (or optical constants),  $m = n - i\kappa$** , is the material properties of dielectric that determines its radiative properties. In general, each material has its own spectral refractive index. The imaginary part  $\kappa$  of the refractive index determines the absorption of the wave as it propagates through the medium; the real part  $n$  of the refractive index gives the phase velocity of propagation.

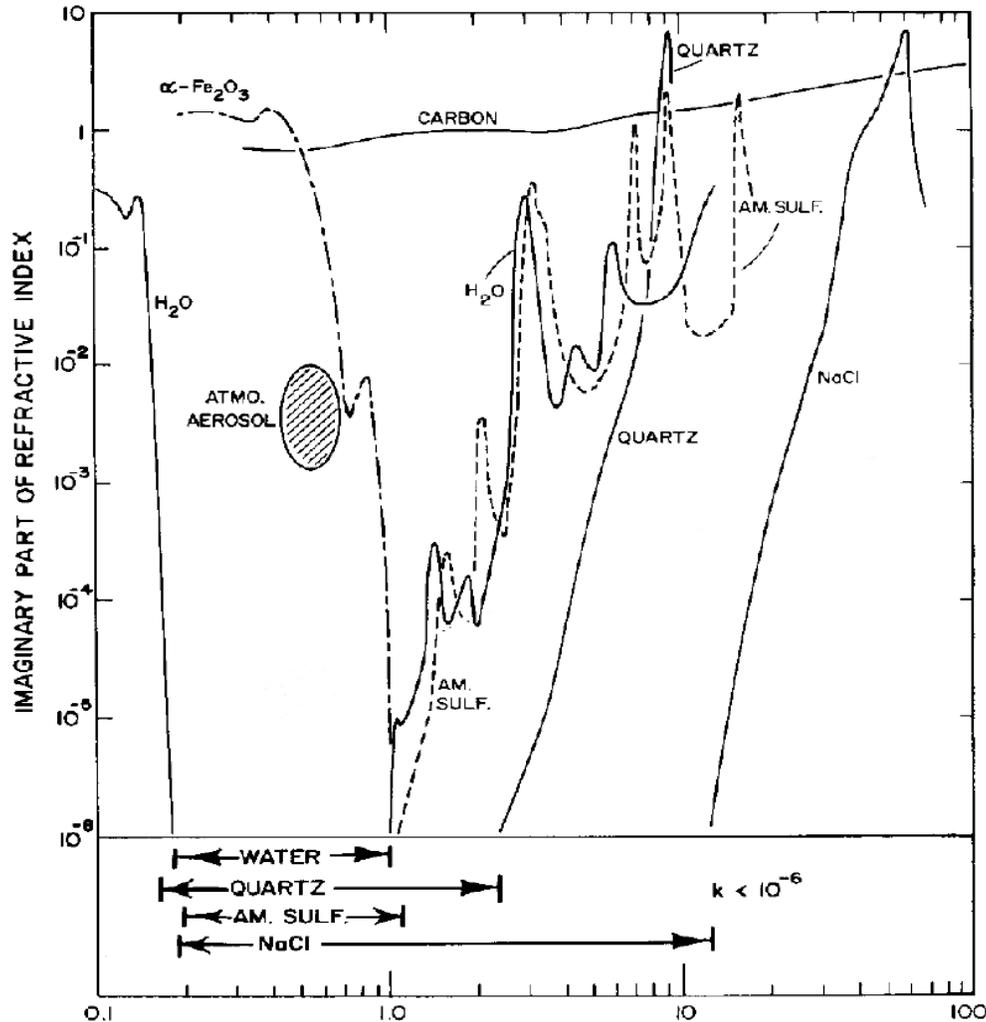


**Figure 10.1** The refractive index of water and ice in the visible and near-IR.

**NOTE:** water has low imaginary part in the visible => negligible absorption by water drops in the visible



**Figure 10.2** The refractive index of water and ice in the IR.



**Figure 10.3** “Classical plot” of the imaginary part of the refractive indexes of some aerosol materials as a function of wavelength (in  $\mu\text{m}$ ) (Bohren and Huffman, Fig.4.16).

**NOTE:** Main absorbing species in the solar spectrum are black carbon (soot) and hematite (dust), but in the thermal IR various species have high imaginary parts of the refractive index.

**NOTE:** Aerosol particles often consist of several chemical species (called the **internal mixture**). There are several approaches (called **mixing rules**) to calculate an **effective refractive index**  $m_e$  of the internally mixed particles by using the refractive indices of the individual species. For instance, **Volume weighted mixing**:

$$m_e = \sum_j m_j f_j \quad [10.1]$$

where  $m_j$  is the refractive index of the  $j$ -species and  $f_j$  is its volume fraction.

## 2. Scattering and absorption by aerosol and cloud particles: Mie theory.

**Mie theory** describes the scattering and absorption of electromagnetic radiation by spherical particles through solving the Maxwell equations.

**NOTE:** Mie theory is also called **Lorenz-Mie theory** or **Lorenz-Mie-Debye theory**.

===== **Mie theory outline** (FROM PREVIOUS LECTURE)=====

### General outline of Mie theory:

#### *Key Assumptions:*

- i) Particle is a **sphere**;
- ii) Particle is **homogeneous** (therefore it is characterized by a **single refractive index**  $m=n - i\kappa$  at a given wavelength);

**NOTE:** Mie theory requires the relative refractive index that is the refractive index of a particle divided by the refractive index of a medium. For air  $m$  is about 1, so one needs to know the refractive index of the particle (i.e., refractive index of the material of which the particle is composed).

**NOTE:** If a particle has complex chemical composition such as some atmospheric aerosols, the effective refractive index must be calculated at a given wavelength.

- Mie theory calculates the scattered electromagnetic field at all points within the particle (called internal field) and at all points of the homogeneous medium in which the particle is embedded. For all practical applications in the atmosphere, light scattering observations are carried out in the **far-field zone** (i.e., at the large distances from a particle).

In the far-field zone (i.e., at the large distances  $R$  from a sphere), the solution of the wave equation can be obtained as

$$\begin{bmatrix} E_l^s \\ E_r^s \end{bmatrix} = \frac{\exp(-ikR + ikz)}{ikR} \begin{bmatrix} S_2 & S_3 \\ S_4 & S_1 \end{bmatrix} \begin{bmatrix} E_l^i \\ E_r^i \end{bmatrix} \quad [9.8]$$

here  $k = 2\pi/\lambda$ ,  $E_l^i$  and  $E_r^i$  are the parallel and perpendicular components of incident electrical field, and  $E_l^s$  and  $E_r^s$  are the parallel and perpendicular components of scattered electrical field,

$\begin{bmatrix} S_2(\Theta) & S_3(\Theta) \\ S_4(\Theta) & S_1(\Theta) \end{bmatrix}$  is the **amplitude scattering matrix** (unitless)

**For spheres:**  $S_3(\Theta) = S_4(\Theta) = \mathbf{0}$ , and thus Eq.[6.1] gives

$$\begin{bmatrix} E_l^s \\ E_r^s \end{bmatrix} = \frac{\exp(-ikR + ikz)}{ikR} \begin{bmatrix} S_2(\Theta) & 0 \\ 0 & S_1(\Theta) \end{bmatrix} \begin{bmatrix} E_l^i \\ E_r^i \end{bmatrix} \quad [9.9]$$

**Eq. [9.9] is a fundamental equation of scattered radiation by a sphere including polarization.**

**Mie theory solution for the scattering amplitudes:**

$$S_1(\Theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} [a_n \pi_n(\cos \Theta) + b_n \tau_n(\cos \Theta)] \quad [9.10]$$

$$S_2(\Theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} [b_n \pi_n(\cos \Theta) + a_n \tau_n(\cos \Theta)] \quad [9.11]$$

where  $\pi_n$  and  $\tau_n$  are **Mie angular functions**

$$\pi_n(\cos \Theta) = \frac{1}{\sin(\Theta)} P_n^1(\cos \Theta) \quad [9.12]$$

$$\tau_n(\cos \Theta) = \frac{d}{d\Theta} P_n^1(\cos \Theta) \quad [9.13]$$

where  $P_n^1$  are the associated Legendre polynomials.

Mie theory also gives the **scattering phase matrix**  $P(\Theta)$  that relates the Stokes parameters  $\{I_0, Q_0, U_0 \text{ and } V_0\}$  of incident radiation field and the Stokes parameters  $\{I, Q, U \text{ and } V\}$  of scattered radiation:

-----END of Mie theory outline-----

## 2. What can we infer from Mie theory

- **Efficiencies (or efficiency factors)** for extinction, scattering and absorption are defined as

$$\boxed{Q_e = \frac{\sigma_e}{\pi r^2} \quad Q_s = \frac{\sigma_s}{\pi r^2} \quad Q_a = \frac{\sigma_a}{\pi r^2}} \quad [10.2]$$

where  $\pi r^2$  is the area of a article with radius  $r$ .

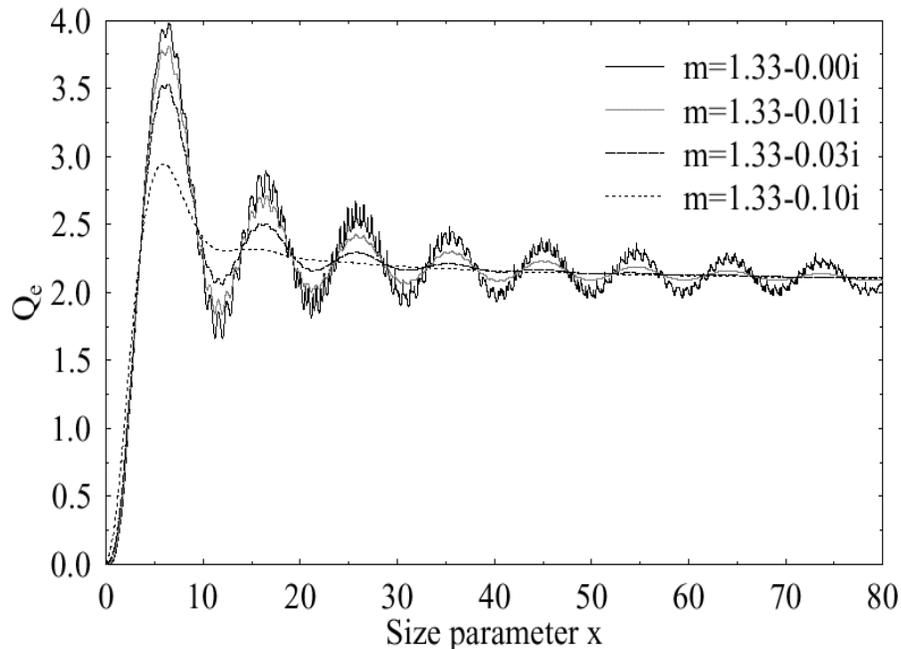
Mie theory gives the solution for  $Q_e$ ,  $Q_s$  and  $Q_a$  in terms coefficient  $a_n$  and  $b_n$  (i.e., coefficients in the expressions for the scattering amplitudes  $S_1(0^0)$  and  $S_2(0^0)$ ).

$$\boxed{Q_e = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1) \operatorname{Re}[a_n + b_n]} \quad [10.3]$$

$$\boxed{Q_s = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1)[|a_n|^2 + |b_n|^2]} \quad [10.4]$$

and the absorption efficiency can be calculated as

$$\boxed{Q_a = Q_e - Q_s} \quad [10.5]$$



**Figure 10.4** Examples of  $Q_e$  calculated with the Mie theory for several refractive indexes.

### Some highlights of Mie theory results:

- Extinction efficiency vs. size parameter  $x$  (assuming NO ABSORPTION):
  - 1) small in Rayleigh limit:  $Q_e \propto x^4$
  - 2) largest  $Q_e$  when particles and wavelength have similar size
  - 3)  $Q_e \rightarrow 2$  in the geometric limit ( $x \rightarrow \infty$ )
  - 4) **Oscillations** (see Fig. 4.7) from interference of transmitted and diffracted waves
- Period in  $x$  of interference oscillations depends on the refractive index.  
**Absorption reduces interference oscillations and kills ripple structure.**
- Scattering and absorption efficiencies vs. size parameter with ABSORPTION:  
As  $x \rightarrow \infty$ :  $Q_s \rightarrow 1$  and , entering rays are absorbed inside particle. Smaller imaginary part of the refractive index requires larger particle to fully absorb internal rays.
- Scattering phase function: forward peak height increases dramatically with  $x$ . For single particles – number of oscillations in  $P(\Theta)$  increases with  $x$ .

**For a single spherical particle, Mie theory gives the extinction, scattering and absorption cross-sections (and efficiency factors), the scattering amplitudes and phase matrix.**

### *Integration over the particle size distribution:*

If particles have the size distribution  $N(r)$ , the volume extinction, scattering and absorption coefficients (in units  $\text{LENGTH}^{-1}$ ) are determined as

$$k_e = \int_{r_1}^{r_2} \sigma_e(r) N(r) dr = \int_{r_1}^{r_2} \pi r^2 Q_e N(r) dr \quad [10.6]$$

$$k_s = \int_{r_1}^{r_2} \sigma_s(r) N(r) dr = \int_{r_1}^{r_2} \pi r^2 Q_s N(r) dr \quad [10.7]$$

$$k_a = \int_{r_1}^{r_2} \sigma_a(r) N(r) dr = \int_{r_1}^{r_2} \pi r^2 Q_a N(r) dr \quad [10.8]$$

The optical depth of an aerosol layer (between  $S_1$  and  $S_2$ ) is

$$\tau_{\lambda}(s_1, s_2) = \int_{s_1}^{s_2} k_{e,\lambda} ds = \int_{s_1}^{s_2} M k_{m,e,\lambda} ds \quad [10.9]$$

where  $k_{m,e,\lambda}$  is the mass extinction coefficient and  $M$  is the mass of particles.

**NOTE:** Mass coefficients = volume coefficients/particle mass concentration,  $M$

The **single scattering albedo** gives the percentage of light which will be scattered in a single scattered event and it is defined as

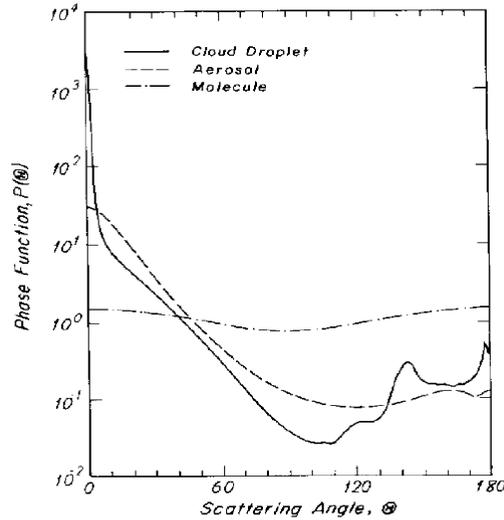
$$\omega_0 = \frac{k_s}{k_e} \quad [10.10]$$

**NOTE:** No absorption (conservative scattering):  $\omega_0 = 1$

No scattering:  $\omega_0 = 0$

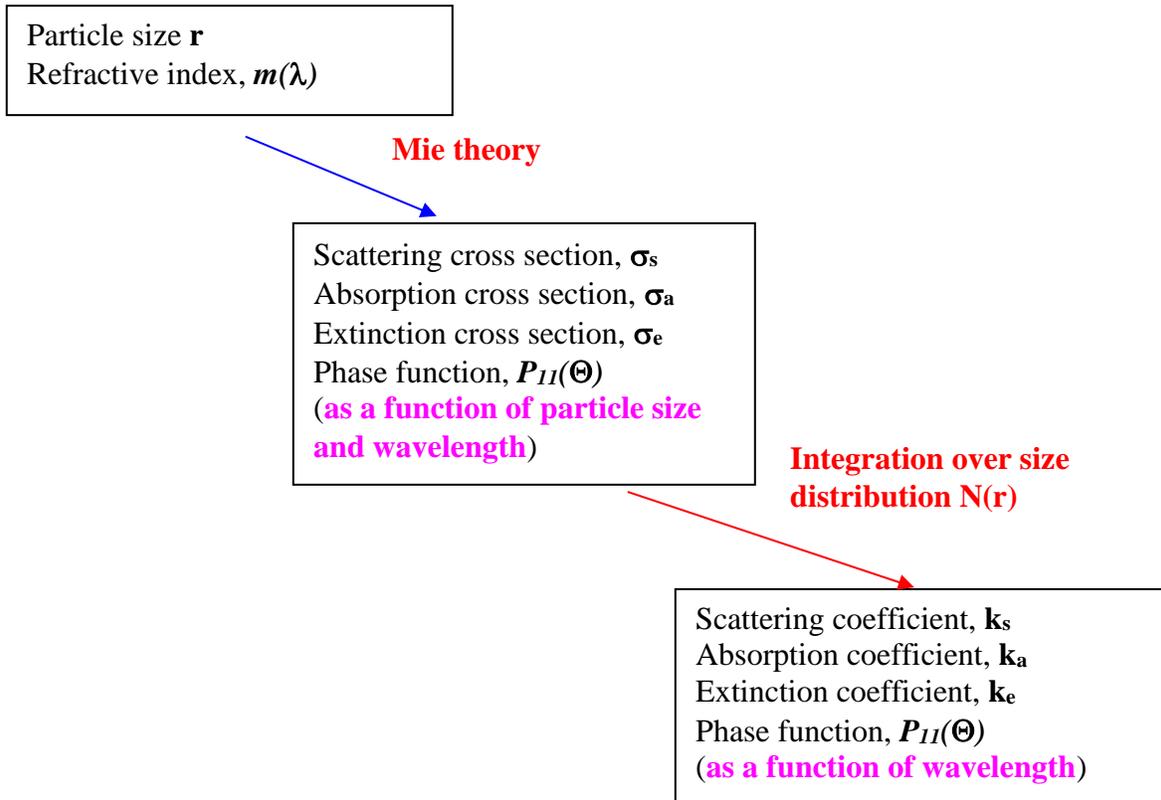
**Scattering phase function** of particles with the size distribution  $N(r)$ :

$$P(\Theta) = \frac{\int_{r_1}^{r_2} P(\Theta, r) \sigma_s N(r) dr}{\int_{r_1}^{r_2} \sigma_s N(r) dr} \quad [10.11]$$



**Figure 10.5** Examples of representative scattering phase functions (at a wavelength of  $0.5 \mu\text{m}$ ) for aerosol and clouds particles. The molecular (Rayleigh) scattering phase function is also shown for comparison.

How to calculate optical characteristics of an ensemble of spherical particles:



- Optical properties of the external mixture (i.e., the mixture of several types of particles)

$$k_e = \sum_i k_e^i \quad k_s = \sum_i k_s^i \quad k_a = \sum_i k_a^i \quad [10.12]$$

where  $k_e^i$ ,  $k_s^i$  and  $k_a^i$  are calculated for each particle type characterized by its particle size distribution  $N_i(r)$  and a refractive index (or effective refractive index)  $m_i$ .

**NOTE:** Do not sum the single scattering albedo and scattering phase functions!!! See below Eq.[10.14-10.15] how to do it.

- How to calculate the effective optical properties of an atmospheric layer consisting of gas and aerosols (and clouds):

In general, an atmospheric layer consists of molecules, aerosols and/or cloud particles.

Thus, one needs to calculate the **effective optical properties** of the layer:

**Effective (also called total) optical depth:**

$$\tau_{\lambda} = \tau_{a,\lambda}^M + \tau_{s,\lambda}^M + \tau_{a,\lambda}^A + \tau_{s,\lambda}^A \quad [10.13]$$

where  $\tau_{a,\lambda}^M$  and  $\tau_{s,\lambda}^M$  are optical depth due to absorption by gases and molecular (Rayleigh) scattering, respectively;

$\tau_{a,\lambda}^A$  and  $\tau_{s,\lambda}^A$  are optical depth due to absorption and scattering by aerosol particles, respectively.

**Effective single scattering albedo:**

$$\omega_{0,\lambda} = \frac{\tau_{s,\lambda}^M + \tau_{s,\lambda}^A}{\tau_{\lambda}} \quad [10.14]$$

**Effective scattering phase function:**

$$P_{\lambda}(\Theta) = \frac{\tau_{s,\lambda}^M P_{\lambda}^M(\Theta) + \tau_{s,\lambda}^A P_{\lambda}^A(\Theta)}{\tau_{s,\lambda}^M + \tau_{s,\lambda}^A} \quad [10.15]$$

**Effective asymmetry parameter:**

$$g_{\lambda} = \frac{\tau_{s,\lambda}^A g_{\lambda}^A}{\tau_{s,\lambda}^M + \tau_{s,\lambda}^A} \quad [10.16]$$

### **3. Remote sensing applications based on measurements of direct solar radiation**

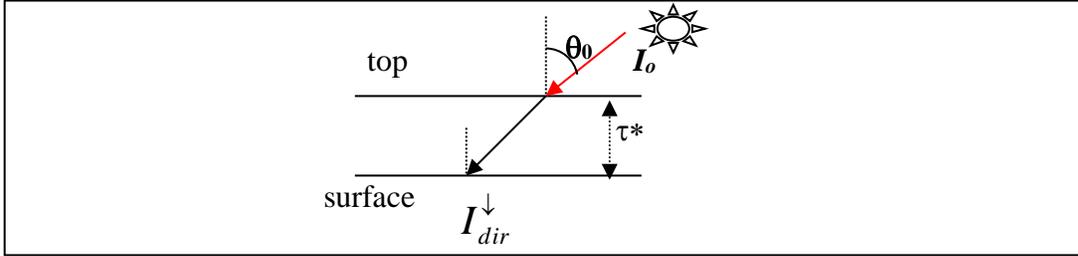
**Direct radiation** is a part of the radiation field that has survived the extinction passing a layer with optical depth  $\tau$  and it obeys the Beer-Bouguer-Lambert law (or Extinction law):

$$I_{dir}^{\downarrow} = I_0 \exp(-\tau / \mu_0) \quad [10.17]$$

where  $I_0$  is the incident intensity at a given wavelength at the top of a layer and  $\mu_0$  is a cosine of the incident zenith angle  $\theta_0$  ( $\mu_0 = \cos(\theta_0)$ ).

**Applying the Extinction law to direct solar radiation:**

$I_0$  is the solar intensity at a given wavelengths at the top of the atmosphere



Thus direct solar radiation reaching the surface is

$$I_{dir}^{\downarrow} = I_0 \exp(-\tau / \mu_0) \quad [10.18]$$

where  $\tau$  is the **optical depth on the entire atmosphere**.

**NOTE:** Optical depth is defined along vertical coordinate (i.e., altitude  $z$ ) in the atmosphere

**Retrieval of aerosol optical depth from ground-based sunphotometer measurement:**

A sunphotometer (a narrow-field of view radiometer that tracks the sun) measures the direct solar radiation attenuated by the atmosphere:

$$F_{dir,\lambda}^{\downarrow} = F_{0,\lambda} \exp(-\tau_{\lambda}^* / \mu_0) \quad [10.19]$$

where  $F_{dir,\lambda}^{\downarrow}$  is the downward direct solar **flux** reaching the surface (i.e., the downward direct solar intensity integrated over the viewing angle of the sunphotometer) and  $\tau_{\lambda}^*$  is the optical depth of the entire atmospheric column, i.e.

$$\tau_{\lambda}^* = \tau_{\lambda}(z_{top}, 0) = \int_0^{z_{top}} k_{e,\lambda}(z) dz \quad [10.20]$$

For cloud-free atmospheric conditions,  $\tau_{\lambda}^*$  is due to attenuation by aerosols, molecular (Rayleigh) scattering and gaseous absorption (e.g.,  $O_3$  and  $NO_2$  depending on  $\lambda$ ). Thus

$$\tau_{\lambda}^* = \tau_{a,\lambda}^M + \tau_{s,\lambda}^M + \tau_{\lambda}^A \quad [10.21]$$

From Eq.[10.19] we have

$$\ln(F_{dir,\lambda}^\downarrow) = \ln(F_{0,\lambda}) - \tau_\lambda^* / \mu_0 \quad [10.22]$$

and

$$\tau_\lambda^* = \mu_0 \{ \ln(F_{0,\lambda}) - \ln(F_{dir,\lambda}^\downarrow) \} \quad [10.23]$$

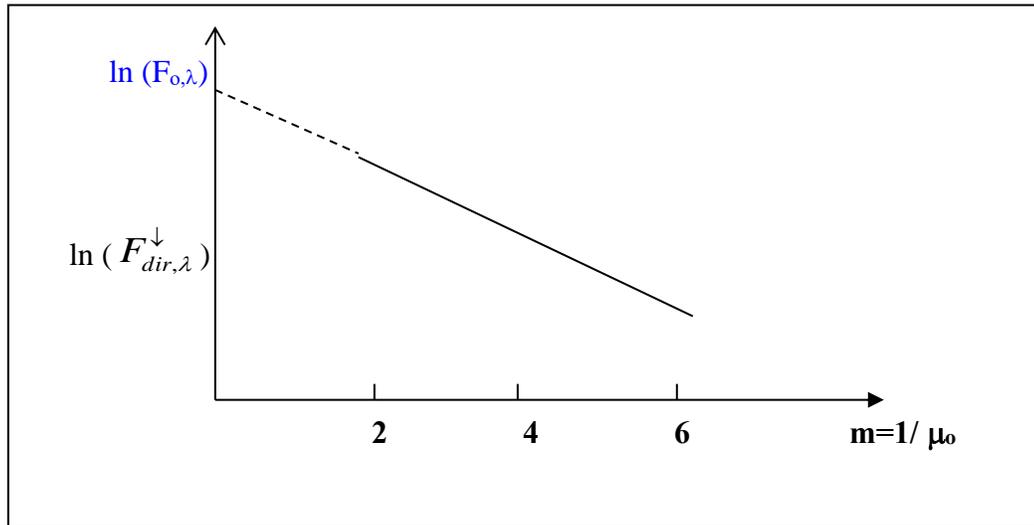
Thus

$$\tau_\lambda^A = \mu_0 \{ \ln(F_{0,\lambda}) - \ln(F_{dir,\lambda}^\downarrow) \} - \{ \tau_{a,\lambda}^M + \tau_{s,\lambda}^M \} \quad [10.24]$$

**NOTE:** To retrieve the aerosol optical depth, one needs to correct for Rayleigh scattering and gaseous absorption.

**Other applications:**

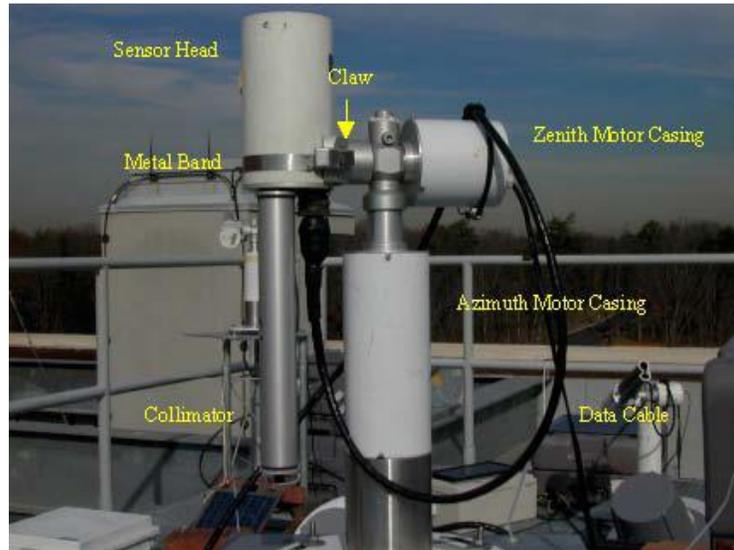
i) If the aerosol optical depth remains constant during the day, Eq.[10.24] enables to measure the solar flux  $F_{0,\lambda}$  by plotting measured  $F_{dir,\lambda}^\downarrow$  vs.  $\mu_0$  (called the Langley plot).



ii) If the aerosol optical depth is known or negligibly small, Eq.[10.24] gives the optical depth due to absorption of gases (used in the retrievals of O<sub>3</sub> and H<sub>2</sub>O column amount).

**Example:** AERONET - <http://aeronet.gsfc.nasa.gov/> - (AErosol RObotic NETwork) program is a federation of ground-based remote sensing aerosol networks established originally by NASA and CNRS and expanded by many international agencies,

universities, etc. The program provides a long-term, continuous and readily accessible public data of aerosol optical properties (and water vapor column amount) for research and validation of satellite aerosol optical depth retrievals.



**Figure 10.6** Cimel Sunphotometer operated by AERONET. Typical spectral bands (channels) of the Cimel sunptometer (central wavelengths): 340 nm, 380 nm, 440nm, 500 nm, 675 nm, 870 nm and 1020nm.

Measurements of aerosol spectral optical depth can be used to compute the **Angstrom parameter** (also called the **Angstrom exponent**) which relates to the aerosol particle size distribution: for measurements of optical depth  $\tau_{\lambda_1}$  and  $\tau_{\lambda_2}$  taken at two different wavelengths  $\lambda_1$  and  $\lambda_2$  respectively, the Angstrom exponent  $\alpha$  is given by

$$\alpha = - \frac{\ln \frac{\tau_{\lambda_1}}{\tau_{\lambda_2}}}{\ln \frac{\lambda_1}{\lambda_2}}$$

or

$$\frac{\tau_{\lambda_1}}{\tau_{\lambda_2}} = \left( \frac{\lambda_1}{\lambda_2} \right)^{-\alpha} \quad [4.68]$$

**NOTE:** For air (pure molecular scattering):  $\alpha = 4$  and for large particles (compared to the wavelength)  $\alpha = 0$