Lecture 11

Multiple scattering as a source of radiation

1. Direct and diffuse radiation
2. Radiative transfer equation
3. Multiple scattering

Required Reading:

S: 6.1, 6.3, 6.4, Appendix 1

Suggested Reading:

Petty: 11, Into to Chapter 13, 13.1

1. Direct and diffuse radiation

The solar radiation field is traditionally considered as a sum of two distinctly different components: **direct** and **diffuse:** $I = I_{dir} + I_{dif}$

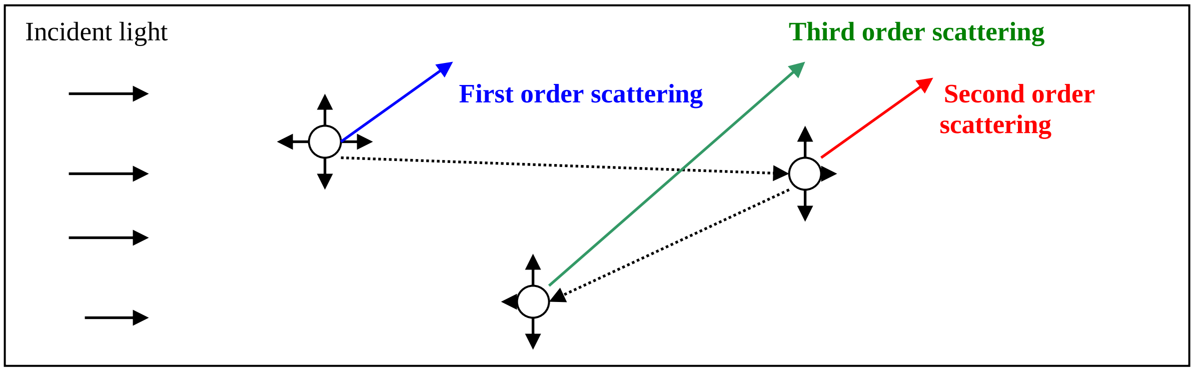
Direct radiation is a part of the radiation field that has survived the extinction passing through a layer with optical depth $\boldsymbol{\tau}$ and it obeys the Beer-Bouguer-Lambert law :

$$I_{dir}^{\downarrow} = I_0 \exp(-\tau / \mu_0)$$

where I_0 is the incident intensity at a given wavelength at the top of a layer and μ_0 is a cosine of the incident zenith angle θ_0 ($\mu_0 = \cos(\theta_0)$).

Diffuse radiation

Diffuse radiation arises from the light that undergoes one scattering event (**single scattering**) or many (**multiple scattering**).



For single scattering

$$\delta I_\lambda(\vec{\Omega}) = k_{s,\lambda} ds \frac{P_\lambda(\vec{\Omega},\vec{\Omega}')}{4\pi} I_\lambda(\vec{\Omega}')\delta\Omega' \qquad [11.1]$$

where $I_\lambda(\vec{\Omega}')$ is the incident intensity in the direction defined by a solid angle $\vec{\Omega}'(\mu',\varphi')$.

For **multiple scattering,** integrating over all directions:

$$dI_\lambda(\vec{\Omega}) = k_{s,\lambda} ds \int_{4\pi} \frac{P_\lambda(\vec{\Omega},\vec{\Omega}')}{4\pi} I_\lambda(\vec{\Omega}') d\Omega' \qquad [11.2]$$

NOTE: The above equation shows that the phase function redirects the incident intensity from the direction $\vec{\Omega}'(\mu',\varphi')$ to the direction $\vec{\Omega}(\mu,\varphi)$, and the integral accounts for all possible scattering events within the 4π solid angle.

According to the Beer-Bouguer-Lambert law, scattering radiance from path ***ds*** can be expressed as $dI_\lambda = k_{e,\lambda} J_\lambda ds$ thus (from Eq.[11.2]) **the scattering source function** is

$$J_\lambda(\vec{\Omega}) = \frac{\omega_{0,\lambda}}{4\pi} \int_{4\pi} I_\lambda(\vec{\Omega}') P_\lambda(\vec{\Omega},\vec{\Omega}') d\Omega' \qquad [11.3]$$

where $\omega_{0,\lambda} = k_{s,\lambda} / k_{e,\lambda}$ is the single scattering albedo.

The scattering source function:

1) has units of intensity

2) plays the role of the Planck function in thermal radiative transfer but the scattering source function is more complex

3) depends on the radiation intensity in the incident direction, $I_\lambda(\vec{\Omega}')$; fraction of radiation which is scattered, $\omega_{0,\lambda}$; and fraction scattered into the new direction $\frac{P(\vec{\Omega},\vec{\Omega}')}{4\pi} d\Omega'$

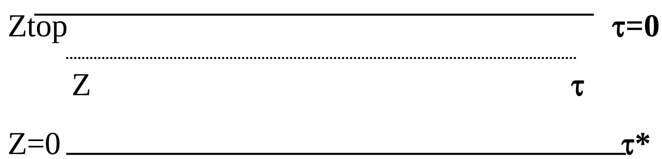
2. Radiative transfer equation

The monochromatic radiative transfer equation for a plane-parallel atmosphere:

Expresses the net change in intensity due to extinction and scattering along path ***dz:***

dI = dI (extinction) + dI(scattering)

A plane-parallel atmosphere



Using ***ds = dz/cos(θ)***, the **radiative transfer equation** can be written as

$$\cos(\theta)\frac{dI_\lambda(z,\theta,\varphi)}{k_{e,\lambda}dz} = -I_\lambda(z,\theta,\varphi) + J_\lambda(z,\theta,\varphi) \qquad [11.4]$$

Introducing the optical depth measured from the outer boundary downward as

$$\tau_\lambda(z,0) = \int_0^z k_{e,\lambda}(z)dz \qquad [11.5]$$

and using $d\tau_\lambda = -k_{e,\lambda}(z)dz$ and ***μ = cos(θ),*** we have

$$\mu\frac{dI_\lambda(\tau;\mu;\varphi)}{d\tau} = I_\lambda(\tau;\mu,\varphi) - J_\lambda(\tau,\mu,\varphi) \qquad [11.6]$$

NOTE: Eq.[11.6] is called the Schwarzchild's equation and and the differential form of the radiative transfer equation for the plane-parallel atmosphere.

Upward (or upwelling) intensity $I_\lambda^\uparrow$ is for $1 \geq \mu \geq 0$ (or $0 \leq \theta \leq \pi/2$);

Downward (or downwelling) intensity $I_\lambda^\downarrow$ is for $-1 \le \mu \le 0$ (or $\pi/2 \le \theta \le \pi$)

The **radiative transfer equation** [11.6] can be written for **upward and downward intensities:**

$$\mu \frac{dI_\lambda^\uparrow(\tau,\mu,\varphi)}{d\tau} = I_\lambda^\uparrow(\tau,\mu,\varphi) - J_\lambda^\uparrow(\tau,\mu,\varphi) \qquad [11.7a]$$

$$-\mu \frac{dI_\lambda^\downarrow(\tau,-\mu,\varphi)}{d\tau} = I_\lambda^\downarrow(\tau,-\mu,\varphi) - J_\lambda^\downarrow(\tau,-\mu,\varphi) \qquad [11.7b]$$

The solution of Eq.[11.7a] gives the upward intensity in the plane-parallel atmosphere:

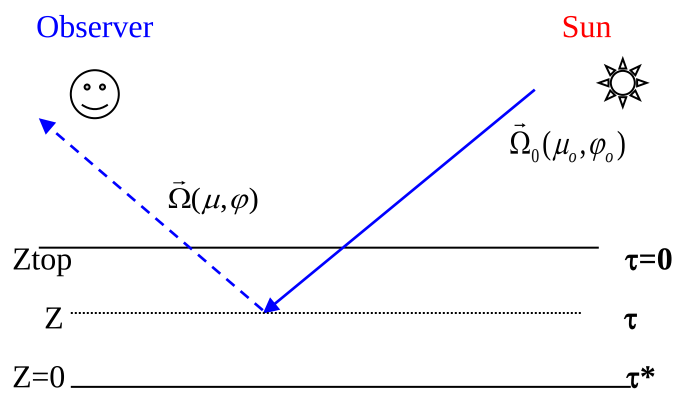
$$I_\lambda^\uparrow(\tau,\mu,\varphi) = I_\lambda^\uparrow(\tau^*,\mu,\varphi)\exp(-\frac{\tau^*-\tau}{\mu}) + \frac{1}{\mu}\int_\tau^{\tau^*}\exp(-\frac{\tau'-\tau}{\mu})\ J_\lambda^\uparrow(\tau',\mu,\varphi)d\tau' \qquad [11.8a]$$

The solution of Eq.[11.7b] gives the downward intensity in the plane-parallel atmosphere:

$$I_\lambda^\downarrow(\tau,-\mu,\varphi) = I_\lambda^\downarrow(0,-\mu,\varphi)\exp(-\frac{\tau}{\mu}) + \frac{1}{\mu}\int_0^{\tau}\exp(-\frac{\tau-\tau'}{\mu})\ J_\lambda^\downarrow(\tau',-\mu,\varphi)d\tau' \qquad [11.8b]$$

3. Multiple scattering

➢ **First-order scattering:**



Direct solar radiation reaching the altitude z is

$$F^{\downarrow}(z) = F_0 \exp(-k_e(z_t - z)/\mu_0) = F_o \exp(-\tau/\mu_0)$$

where $\boldsymbol{F_0}$ is the solar constant at the top of the atmosphere.

Scattering of the direct beam is the source of diffuse radiation (see Eq.[11.1])

$$J(\tau,\mu,\varphi) = \frac{\omega_0}{4\pi} F_0 \exp(-\tau/\mu_0) P(\mu,\varphi,\mu_0,\varphi_0) \qquad [11.9]$$

Assuming <u>no surface reflection (dark surface),</u> the upwelling intensity at the level Z (or τ) can be found from Eq.[11.8a] as

$$I^{\uparrow}(\tau,\mu,\varphi) = \int_{\tau}^{\tau^*} J(\tau',\mu,\varphi) \exp[-(\tau'-\tau)/\mu] d\tau'/\mu \qquad [11.10]$$

Substituting in the source function

$$I^{\uparrow}(\tau,\mu,\varphi) = \frac{\omega_0}{4\pi} F_0 P(\Theta) \int_{\tau}^{\tau^*} \exp[-(\tau'-\tau)/\mu - \tau'/\mu_0] d\tau'/\mu \qquad [11.11]$$

An observer (i.e., a satellite sensor) at Ztop (or τ= 0) measures

$$I^{\uparrow}(0,\mu,\varphi) = \frac{\omega_0}{4\pi} F_0 P(\Theta) \frac{\mu_0}{\mu+\mu_0} [1 - \exp(-(\frac{1}{\mu_o} + \frac{1}{\mu})\tau^*)] \qquad [11.12]$$

If **τ^* <1 (called the single scattering approximation), Eq.[11.12] simplifies to**

$$\boxed{I^{\uparrow}(0,\mu,\varphi) = \frac{\omega_0}{4\pi} F_0 P(\Theta) \frac{\tau^*}{\mu}} \qquad [11.13]$$

Radiative transfer equation with multiple scattering:

In the general case of multiple scattering

$$J(\tau,\mu,\varphi) = \frac{\omega_0}{4\pi} \int_0^{2\pi} \int_{-1}^{1} I(\tau,\mu',\varphi') P(\mu,\varphi,\mu',\varphi') d\mu' d\varphi' + \\ + \frac{\omega_0}{4\pi} F_0 P(\mu,\varphi,\mu_0,\varphi_0) \exp(-\tau/\mu_0) \qquad [11.14]$$

Using the source function for scattering, we can write the **radiative transfer equation for diffuse radiation** as an integro-differential equation:

$$\mu \frac{dI(\tau,\vec{\Omega})}{d\tau} = I(\tau,\vec{\Omega}) - \frac{\varpi_0}{4\pi} \int_{4\pi} I(\tau,\vec{\Omega}) P(\vec{\Omega},\vec{\Omega}') d\Omega' - \frac{\omega_0}{4\pi} F_0 P(\vec{\Omega},\vec{\Omega}_0) \exp(-\tau/\mu_0) \qquad [11.15]$$

NOTE: To solve Eq.[11.15], one needs to know the scattering coefficient $\mathbf{k}_{s,\lambda}$, absorption coefficient $\mathbf{k}_{a,\lambda}$ and scattering phase function ***P*(μ, φ, μ',φ')** as a function of wavelength in each atmospheric layer.

NOTE: Various approximate and "exact" (such as Discrete-ordinate, Adding-doubling, Monte-Carlo, etc.) techniques have been developed to solve the radiative transfer equation for diffuse radiation. Each technique requires a sophisticated numerical code. There are a number of various numerical radiative transfer codes that are openly available to the scientific community.