Lecture 14:
Passive Remote Sensing using emission

October 6, 2019

1. Radiative transfer review
2. Radiative transfer with emission

Required reading S: 7.1, 7.3.1

Additional reading P: 8

1 Radiative transfer review

We began the course by going over the fundamental physics concepts employed in remote sensing of the atmosphere and oceans. We discussed topics like scattering and absorption as they pertain to single particle-radiation interactions.

The theory of radiative transfer (RT) describes the *accumulated* effects of many particle-radiation interactions as the EM radiation is transferred from one volume to another along a given path. The two forms of RT we have discussed thus far consider:

1. effects from extinction only
2. effects from extinction and multiple scattering

1.1 RTE for Exinction Only

Start with the Beer-Lambert Law, which states that the intensity of radiation emerging from the end of a given path length ds is proportional to the incoming intensity.

$$dI_\lambda = -k_{ext} I_\lambda ds \tag{1}$$

NOTE: This equation only accounts for radiation LOST along the specified path (does note include radiation accrued along the path from scattering and emission).

Rearranging we get,

$$\frac{dI_\lambda}{ds} = -k_{ext} I_\lambda \tag{2}$$

(This is the Radiative Transfer Equation (RTE) for EXTINCTION ONLY.)

Rearranging and solving this Ordinary Differential Equation (ODE) by,

$$\int_{I_0}^{I_\lambda} \frac{dI_\lambda^{'}}{I_\lambda^{'}} = \int_{s_1}^{s_2} -k_{ext} ds \tag{3}$$

$$ln(\frac{I_\lambda}{I_0}) = \tau_\lambda \tag{4}$$

where the optical depth is defined as $\tau_\lambda = -\int_{s_1}^{s_2} k_{ext} ds$

Then,

$$I_\lambda = I_0 exp(-\tau_\lambda) \tag{5}$$

which is the solution to the RTE for extinction only, and otherwise known as Beer's Law.

If we want to consider extinction in an atmosphere that is a combination of molecules (gas) and aerosols, then we need to include optical depth terms for both molecules and aerosols (AOD) by,

$$I_\lambda = I_0 exp[-(\tau_\lambda^{molecules} + \tau_\lambda^{aerosols})] \tag{6}$$

1.2 RTE for Extinction and Multiple Scattering

Extinction-only RTE used only single-scattering accumulated events and describes the radiation *removed* along a path.

This method does not include multiple scattering effects seen in the presence of clouds and dense aerosols (high AOD). In these cases, photons can scatter and reappear again in the path along the line-of-sight of the sensor.To account for multiple scattering, we need to introduce a mathematical expression for this reappearance of photons.

Now we must account for both direct and diffuse radiation,

$$I = I_{dir} + I_{dif} \tag{7}$$

Direct radiation obeys the Beer-Lambert Law, so I_{dir} is given by Eqn. 5.

Diffuse radiation arises from the radiation that undergoes one scattering event (single scattering) or many scattering events (multiple scattering).

For single scattering, we have an incremental increase in radiation given by,

$$\delta I_\lambda(\vec{\Omega}) = k_{s,\lambda} ds \frac{P_\lambda(\vec{\Omega}, \vec{\Omega}')}{4\pi} I_\lambda(\vec{\Omega}') \delta\Omega' \tag{8}$$

where $I_\lambda(\vec{\Omega}')$ is the incident intensity (radiance) in the direction defined by a solid angle $\vec{\Omega}'(\mu', \phi')$.

For multiple scattering, we integrate the above equation over all directions to get,

$$dI_\lambda = k_{s,\lambda} ds \int_{4\pi} \frac{P_\lambda(\vec{\Omega}, \vec{\Omega}')}{4\pi} I_\lambda(\vec{\Omega}') d\Omega' \tag{9}$$

NOTE: The above equation shows that the phase function redirects the incident intensity from the direction $\vec{\Omega}'(\mu', \phi')$ to the direction $\vec{\Omega}(\mu, \phi)$.

We can write the Beer-Lambert Law in terms of a new function,

$$dI_\lambda = k_{e,\lambda} J_\lambda ds \tag{10}$$

From Eqn. 9, we define the **scattering source function** as,

$$J_\lambda(\vec{\Omega}) = \frac{\omega_0}{4\pi} \int_{4\pi} P_\lambda(\vec{\Omega}, \vec{\Omega}') I_\lambda(\vec{\Omega}') d\Omega' \tag{11}$$

where $\omega_0 = \frac{k_s}{k_e}$, is the single scattering albedo.

Now our RTE to include extinction and multiple scattering is described as,

$$dI = dI_{ext} + dI_{sca} \tag{12}$$

and is given by,

$$\mu \frac{dI_\lambda(z, \theta, \phi}{d\tau} = I_\lambda(z, \theta, \phi) - J_\lambda(z, \theta, \phi) \tag{13}$$

where the optical depth, τ is measured from the TOA downward toward the surface and used here as, $d\tau_\lambda = -k_{e,\lambda}(z)dz$, and $\mu = cos(\theta)$. This is the form of the RTE for upward intensity. The solution to this equation is given by,

$$I(\tau, \mu, \phi) = I(\tau^*, \mu, \phi) exp[-\frac{(\tau^* - \tau)}{\mu}] + \frac{1}{\mu} \int_\tau^{\tau^*} exp[-\frac{(\tau^* - \tau)}{\mu}] J(\tau', \mu, \phi) d\tau' \tag{14}$$

where λ subscripts have been dropped for ease of writing. The first term gives the radiation removed by extinction and the second term gives the radiation accrued by multiple scattering.

Now, at the top of the atmosphere (TOA), we begin with the downward flux given by,

$$F(z) = F_0 exp[-k_e(z_t - z)/\mu_0] = F_0 exp[-\tau/\mu_0] \tag{15}$$

In first-order or single scattering, the beam of direct radiation scattered is the SOURCE of diffuse radiation,

$$J(\tau, \mu, \phi) = \frac{\omega_0}{4\pi} F_0 exp[-\tau/\mu_0] P(\Theta) \tag{16}$$

Assuming no surface reflection, the upward RTE solution becomes,

$$I(\tau, \mu, \phi) = \int_\tau^{\tau^*} J(\tau', \mu', \phi') exp[-(\tau' - \tau)/\mu] d\tau'/\mu \tag{17}$$

Substituting in the source function gives,

$$I(\tau, \mu, \phi) = \frac{\omega_0}{4\pi} F_0 P(\Theta) \int_\tau^{\tau^*} exp[-((\tau' - \tau)/\mu) - (\tau'/\mu_0)] d\tau'/\mu \tag{18}$$

For a satellite sensor at $z_{top} = 0$, we have,

$$I(0, \mu, \phi) = \frac{\omega_0}{4\pi} F_0 P(\Theta) \frac{\mu_0}{\mu + \mu_0} [1 - exp(-(\frac{1}{\mu_0} + \frac{1}{\mu})\tau^*)] \tag{19}$$

When the atmosphere is optically thin, $\tau^* < 1$,

$$I(0, \mu, \phi) = \frac{\omega_0}{4\pi} F_0 P(\Theta) \frac{\tau^*}{\mu} \tag{20}$$

2 Radiative transfer with emission

According to Kirchoff's Law,

$$Emission = Absorption \tag{21}$$

For a non-scattering medium in LTE, the Planck function gives the source function:

$$J_\lambda = B_\lambda \tag{22}$$

NOTE: If we neglect scattering, then the volume extinction coefficient is equal to the volume absorption coefficient.

$$k_a = k_e \tag{23}$$

Then the net changes of radiation along a path ds is due to the combination of emission and extinction (absorption),

$$dI = dI_{ext} + dI_{emission} \tag{24}$$

thus we have the RTE in the thermal region (neglecting scattering),

$$dI = -k_e I ds + k_e B ds \tag{25}$$

or

$$\frac{dI}{d\tau} = [I + B] \tag{26}$$

where $d\tau = -k_e ds$.
Now rearrange and multiply by $exp[-\tau]$,

$$-\frac{exp(-\tau)dI}{d\tau} + exp(-\tau)I = exp(-\tau)B \tag{27}$$

Using,

$$d[I(x)exp(-x)] = exp(-x)dI(x) - exp(-x)I(x)dx \tag{28}$$

We have,

$$-d[Iexp(-\tau)] = exp(-\tau)Bd\tau \tag{29}$$

Integrating the above equation gives along a path from s' to s",

$$I(s'')exp[-\tau(s'')] - I(s')exp[-\tau(s')] = \int_{\tau(s'')}^{\tau(s')} B(s)exp[-\tau(s)]d\tau(s) \tag{30}$$

rearranging once again gives the solution to the RTE in IR,

$$I_\lambda(s'') = I_\lambda e^{-[\tau(s')-\tau(s'')]} + \int_{\tau(s'')}^{\tau(s')} B_\lambda(s)e^{-[\tau(s)-\tau(s'')]}d\tau(s) \tag{31}$$

Where the first term is the contribution from the radiation incident on s' and transmitted to s", and the second term is the contribution from radiation emitted along the path and transmitted to s".