**Lecture 15**

**Principles of passive remote sensing using emission and applications:**

Remote sensing of atmospheric path-integrated quantities (cloud liquid water content and precipitable water vapor).

**Remote sensing of SST.**

1. Radiative transfer with emission.
2. Microwave radiative transfer.
3. Measurements of atmospheric path-integrated quantities (precipitable water, cloud liquid water).
4. Remote sensing of SST.

**Required reading:**
S: 7.1, 7.2, 7.3

**Additional reading:**
Petty: 8

Microwave remote sensing (various satellite data products, including SSM/I):

http://www.remss.com/

Example: NOAA AMSU-A retrieval algorithm for total precipitable water and cloud liquid water:

http://www.star.nesdis.noaa.gov/corp/scsb/mspps/algorithms.html#ATPWCLW

SST satellite products - Physical Oceanography (PO DAAC)

http://podaac.jpl.nasa.gov/

NOAA SST data


1. Radiative transfer with emission.

According to the Kirchhoff’s law of thermal radiation:

\[ \text{emission} = \text{absorption} \]

for an ideal blackbody.

Recall the Beer-Bouguer-Lambert law for emission

\[ dI_\lambda = k_{e,\lambda} J_\lambda ds \]

where \( k_{e,\lambda} \) is the volume extinction coefficient along path \( ds \).

- For a non-scattering medium in LTE, the Planck function gives the source function

\[ J_\lambda = B_\lambda \] \[ [15.1] \]

Neglecting scattering \( \Rightarrow \) volume extinction coefficient = volume absorption coefficient

Thus, the net change of radiation along path \( ds \) is due to the combination of emission and extinction

\[ dI = dI(\text{extinction}) + dI(\text{emission}) \]

and thus the radiative transfer equation in the thermal region (ignoring scattering) is

\[ dI_\lambda = -k_\lambda I_\lambda ds + k_\lambda B_\lambda ds \] \[ [15.2] \]

or

\[ \frac{dI_\lambda}{ds} = -k_\lambda [I_\lambda - B_\lambda] \] \[ [15.3] \]

NOTE: Eqs.[15.2]-[15.3] are often called the differential forms of the radiative transfer equation.

Recall that by definition

\[ d\tau_\lambda = -k_\lambda(s) ds \]

Let’s re-arrange terms in Eq.[9.3] and multiply both sides by \( \exp(-\tau_\lambda) \)

\[ - \frac{\exp(-\tau_\lambda)dI_\lambda}{d\tau_\lambda} + \exp(-\tau_\lambda)I_\lambda = \exp(-\tau_\lambda)B_\lambda \] \[ [15.4] \]

and (using that \( d[I(x)\exp(-x)] = \exp(-x)dI(x) - \exp(-x)I(x)dx \)) we have
Integrating the above equation along a path extending from some point \( s' \) to the end point \( s'' \), it becomes

\[
I_\lambda(s'')e^{-\tau_\lambda(s'')} - I_\lambda(s')e^{-\tau_\lambda(s')} = \int_{\tau(s')}^{\tau(s'')} B_\lambda(s)e^{-\tau_\lambda(s)} d\tau(s)
\]

and, re-arranging terms, we have the solution of the radiative transfer in IR

\[
I_\lambda(s'') = I_\lambda(s')e^{-[\tau(s')-\tau(s'')]} + \int_{\tau(s')}^{\tau(s'')} B_\lambda(s)e^{-[\tau(s)-\tau(s'')]} d\tau(s)
\]

contribution from radiation incident at \( s' \)

and transmitted to \( s'' \)

and the contribution from radiation emitted along the path and transmitted to \( s'' \)

Let's consider a plane-parallel atmosphere \((dz=\mu ds \quad \text{and} \quad \pi(z) = \mu \pi(s))\)

**Upward intensity** \( I_\lambda^\uparrow \) is for \( 1 \geq \mu \geq 0 \) (or \( 0 \leq \theta \leq \pi / 2 \));

**Downward intensity** \( I_\lambda^\downarrow \) is for \( -1 \leq \mu \leq 0 \) (or \( \pi / 2 \leq \theta \leq \pi \))

(used that \( \cos(0)=1; \cos(\pi/2)=0 \) and \( \cos(\pi)=-1 \))
NOTE: For downward intensity, \( \mu \) is replaced by \(-\mu\).

Eq.[15.7] gives both the upward intensity in the plane-parallel atmosphere

\[
I_{\lambda}^{\uparrow}(\tau; \mu; \varphi) = I_{\lambda}^{\uparrow}(\tau^*; \mu; \varphi) \exp\left(-\frac{\tau^* - \tau}{\mu}\right)
+ \frac{1}{\mu} \int_{\tau}^{\tau^*} \exp\left(-\frac{\tau' - \tau}{\mu}\right) B_{\lambda}(\tau')d\tau'
\]

[15.8]

and the downward intensity in the plane-parallel atmosphere:

\[
I_{\lambda}^{\downarrow}(\tau; -\mu; \varphi) = I_{\lambda}^{\downarrow}(0; -\mu; \varphi) \exp\left(-\frac{\tau}{\mu}\right)
+ \frac{1}{\mu} \int_{0}^{\tau} \exp\left(-\frac{\tau - \tau'}{\mu}\right) B_{\lambda}(\tau')d\tau'
\]

[15.9]

- In the atmospheric conditions for IR radiation, one can consider that at the surface

\[
I_{\lambda}^{\uparrow}(\tau^*; \mu; \varphi) = B_{\lambda}(T_s) \text{ or } I_{\lambda}^{\downarrow}(\tau^*; \mu; \varphi) = \varepsilon_{\lambda} B_{\lambda}(T_s)
\]

no thermal incident radiation at the TOA

\[
I_{\lambda}^{\uparrow}(0; \mu; \varphi) = 0
\]

no dependence on azimuthal angle \( \varphi \).

Thus Eqs.[15.8] and [15.9] can be re-written as (in the wavenumber domain)

\[
I_{\nu}^{\uparrow}(\tau; \mu) = B_{\nu}(\tau^*) \exp\left(-\frac{\tau^* - \tau}{\mu}\right)
+ \frac{1}{\mu} \int_{\tau}^{\tau^*} \exp\left(-\frac{\tau' - \tau}{\mu}\right) B_{\nu}(\tau')d\tau'
\]

[15.10]

\[
I_{\nu}^{\downarrow}(\tau; -\mu) = \frac{1}{\mu} \int_{0}^{\tau} \exp\left(-\frac{\tau - \tau'}{\mu}\right) B_{\nu}(\tau')d\tau'
\]

[15.11]


Recall that
\[ T_v(\tau, \mu) = \exp\left(-\frac{\tau_v}{\mu}\right) \quad [15.12] \]

and the differential form is
\[ \frac{dT_v(\tau, \mu)}{d\tau} = -\frac{1}{\mu} \exp\left(-\frac{\tau_v}{\mu}\right) \quad [15.13] \]

- **Multiplication law of transmittance** states that when several gases absorb, the **monochromatic transmittance** is a product of the monochromatic transmittances of individual gases:
\[ T_{v,1,2,..N} = T_{v,1}T_{v,2}..T_{v,N} \quad [15.14] \]

Thus the general solutions for monochromatic upward and downward radiances in terms of transmittance are:
\[ I_{v}^\uparrow(\tau; \mu) = B_{v}(\tau^*)T_{v}(\tau^* - \tau, \mu) \]
\[ - \int_{\tau}^{\tau^*} B_{v}(\tau') \frac{dT_v(\tau' - \tau, \mu)}{d\tau'} d\tau' \quad [15.15] \]
\[ I_{v}^\downarrow(\tau; -\mu) = \int_{0}^{\tau} B_{v}(\tau') \frac{dT_v(\tau - \tau', \mu)}{d\tau'} d\tau' \quad [15.16] \]

The general solutions for monochromatic upward and downward radiances can be expressed in terms of the weighing function:
\[ I_{v}^\uparrow(\tau; \mu) = B_{v}(\tau^*)T_{v}(\tau^* - \tau, \mu) + \int_{\tau^*}^{\tau} B_{v}(\tau')W_v(\tau, \tau', \mu) d\tau' \quad [15.17] \]

where the **weighting function** is defined as
\[ W_v(\tau, \tau', \mu) = \left| \frac{dT_v(\tau, \tau', \mu)}{d\tau} \right| \quad [15.18] \]

**NOTE:** The concept of the **weighting function** plays a central role in sounding techniques (i.e., retrievals of the vertical profile of T and gas (H\(_2\)O, O\(_3\)) concentrations from hyperspectral passive remote sensing.
Consider an isothermal atmosphere. Then \( B(\tau) = B(z) = \text{constant} = B(T_{\text{atm}}) \)

The radiance at the top of the isothermal atmosphere (from Eq. [15.15]) is

\[
I_v^\uparrow (0; \mu) = B_v(\tau^*) T_v(\tau^*, \mu) + B_v(T_{\text{atm}})(1 - T_v(\tau^*, \mu))
\]

[15.19]

where \( T_v(\tau^*, \mu) \) is the transmission function of the entire atmosphere.

If no scattering, then

\[ \text{Emissivity} = \text{Absorptivity} = 1 - \text{Transmission} \]

Thus Eq. [15.19] can be re-written in terms of emissivity (or absorptivity).

2. Microwave radiative transfer.

According to the Rayleigh-Jeans distribution (see Lecture 4):

\[ \text{Brightness temperature is linearly proportional to the radiance (when wavelengths are in the microwave spectrum)} \]

- In the microwave, surface emissivities are low \( \Rightarrow \) need to account for reflection (i.e., the portion of microwave radiation emitted by the atmosphere toward the ocean is reflected back to the atmosphere and can be polarized depending on the viewing direction).

Eq. [15.8] can be modified to give the brightness temperature \( T_{b,v} \) measured by a satellite passive microwave instrument at a wavenumber \( v \)

\[
T_{b,v} = \varepsilon_v^p T_{\text{sur}} \exp(-\tau^* / \mu) + \int_0^{\tau^*} T_{\text{atm}}(\tau') \exp(-\tau' / \mu) d\tau' / \mu
\]

\[
+ R_v^p \exp(-\tau^* / \mu) \int_0^{\tau^*} T_{\text{atm}}(\tau') \exp(-\tau^* - \tau') / \mu d\tau' / \mu
\]

[15.20]

where

\( T_{\text{sur}} \) is the surface temperature, \( T_{\text{atm}} \) is the atmospheric temperature,

\( \varepsilon_v^p \) is the emissivity of the ocean surface with the given polarization state \( p \), and
\[ R_p^p = (1 - \varepsilon_p^p) \] is reflectivity of the ocean surface with the given polarization state \( p \).

Let’s assume that there is absorption by water vapor only in the boundary layer

\[
\int_0^\infty T_{\text{atm}}(\tau') \exp(-\tau' / \mu)d\tau' / \mu \approx T_{\text{sur}} [1 - \exp(-\tau^* / \mu)]
\]  \[15.21\]

Thus we have from Eq.[15.20]

\[
T_{h,v} = T_{\text{sur}} [1 - T_v^2 (\tau^*, \mu)(1 - \varepsilon_v^p)]
\]  \[15.22\]

where \( T_v (\tau^*, \mu) = \exp(-\frac{\tau^*}{\mu}) \) is the transmission function.


Let’s consider brightness temperature measured at 19.35 GHz and 37 GHz for two polarization state (horizontal, \( H \), and vertical \( V \), polarization states)

Using Eq.[15.22], we have at each frequency

\[
\Delta T_{h,v} = T_{\text{sur}} (R_v^V - R_v^H) \exp(\frac{\tau^*}{\mu})
\]  \[15.23\]

where \( R_v^{H,V} = (1 - \varepsilon_v^{H,V}) \)

The atmospheric transmission can be represented as a combination of transmission for \( \text{O}_2 \), \( T_{\text{O}_2} \), cloud liquid water, \( T_w \), and water vapor, \( T_\text{w} \), at each frequency

\[
T^{2} = T_{\text{O}_2}^2 T_w^2 T_\text{w}^2
\]  \[15.24\]

\( T_w = \exp(-k_{a,w} \text{LWP} / \mu) \) where \( k_{a,w} \) is the mass absorption coefficient of liquid water (cloud drops) and \( \text{LWP} \) is the liquid water path defined as liquid water content \( (LWC, \text{Lecture 9, Eq.[9.5])} \) integrated over the path.

\( T_\text{w} = \exp(-k_{a} \sigma / \mu) \) where \( k_{a} \) is the absorption coefficient of water vapor and \( \sigma \) is the amount of water vapor integrated over the part (called precipitable water, often reported in mm).
From Eq.[15.23] we have
\[ k_{n,w}LWP + k_{\omega, \sigma} = -\frac{\mu}{2} \ln \left[ \frac{\Delta T_b}{T_{sw}(R^V - R^H)T_{O2}} \right] \]  

[15.25]

- Eq.[9.25] for two channels \(=\) we have two equations to solve for \(LWP\) and \(\sigma\) given the values of \(T_{O2,19}, T_{O2,37}, k_{n,w}, k_{\omega, \sigma}\), and \(R^H, V\)

Problems:
1) Need to know absorption coefficients
2) \(R^H, V\) are functions of wind speed

Example: The above principle provides a basis for the retrieval algorithm of Special Sensor Microwave/Imager (SSM/I). SSM/I is a passive microwave sensor aboard the DMSP satellite series. The SSM/I time series consists of 6 satellites covering the period from 1987 to the present. http://www.remss.com/

Example of SSM/I products

Daily (UTC AM) retrievals of precipitable water vapor
**Weekly** retrievals of precipitable water vapor

![Weekly retrievals of precipitable water vapor](image1)

**Daily** (UTC AM) retrievals of cloud liquid water - compare to the above image for precipitable water

![Daily retrievals of cloud liquid water](image2)
Weekly retrievals of cloud liquid water - compare to the above image for precipitable water


The concept of SST retrievals from passive infrared remote sensing:
measure IR radiances in the atmospheric window and correct for contribution from “clear” sky by using multiple channels (called a split-window technique)

Using Eq.[15.10], we can write the IR radiance at TOA:

\[
I^\uparrow(\theta; \mu) = B_\lambda(\tau^*) \exp\left(-\frac{\tau^*}{\mu}\right) + \frac{1}{\mu} \int_0^\infty \exp\left(-\frac{\tau'}{\mu}\right) B_\lambda(\tau')d\tau'
\]  

[15.26]

Let’s re-write this equation using the transmission function \( T_\lambda(\tau^*, \mu) = \exp\left(-\frac{\tau^*}{\mu}\right) \)

and that

\[
I^\uparrow(\theta; \mu) = B_\lambda(T_\text{sur})T_\lambda(\tau^*, \mu) + B_\lambda(T_\text{atm})[1 - T_\lambda(\tau^*, \mu)]
\]  

[15.27]
where $T_{atm}$ is an “effective” blackbody temperature which gives the atmospheric emission

$$B_{\lambda}(T_{atm}) = [1 - T_{\lambda}^{*}(\tau^{*}, \mu)]^{-1} \frac{1}{\mu} \int_{0}^{\tau^{*}} \exp(-\frac{\tau'}{\mu}) B_{\lambda}(\tau') d\tau'$$  \[15.28\]

We want to eliminate the term with $T_{atm}$ in Eq.[15.28]. Suppose we can measure IR radiances $I_1$ and $I_2$ at two adjacent wavelengths $\lambda_1$ and $\lambda_2$

$$I_{1}^{\uparrow} = B_{1}(T_{\text{sur}})T_{1}(\tau_{1}^{*}, \mu) + B_{1}(T_{atm})[1 - T_{1}(\tau_{1}^{*}, \mu)]$$  \[15.29\]

$$I_{2}^{\uparrow} = B_{2}(T_{\text{sur}})T_{2}(\tau_{2}^{*}, \mu) + B_{2}(T_{atm})[1 - T_{2}(\tau_{2}^{*}, \mu)]$$  \[15.30\]

**NOTE:** Two wavelengths need to be close to neglect the variation in $B_{\lambda}(T_{atm})$ as a function of $\lambda$.

Let’s apply the Taylor’s expansion to $B_{\lambda}(T)$ at temperature $T = T_{atm}$

$$B_{\lambda}(T) \approx B_{\lambda}(T_{atm}) + \frac{\partial B_{\lambda}(T)}{\partial T} (T - T_{atm})$$  \[15.31\]

Applying this expansion for both wavelengths, we have

$$B_{1}(T) \approx B_{1}(T_{atm}) + \frac{\partial B_{1}(T)}{\partial T} (T - T_{atm})$$  \[15.32\]

$$B_{2}(T) \approx B_{2}(T_{atm}) + \frac{\partial B_{2}(T)}{\partial T} (T - T_{atm})$$  \[15.33\]

and thus, eliminating $T - T_{atm}$, we have

$$B_{2}(T) \approx B_{2}(T_{atm}) + \frac{\partial B_{2}(T)}{\partial B_{1}(T)} \frac{\partial B_{1}(T)}{\partial T} [B_{1}(T) - B_{1}(T_{atm})]$$  \[15.34\]

Let’s introduce brightness temperatures for these two channels $T_{b,1}$ and $T_{b,2}$

$$I_{1} = B_{1}(T_{b,1}) \text{ and } I_{2} = B_{2}(T_{b,2})$$

and apply [15.34] to $B_{2}(T_{b,2})$ and to $B_{2}(T_{\text{sur}})$
\[ B_2(T_{b,2}) \approx B_2(T_{\text{atm}}) + \frac{\partial B_2(T)}{\partial B_1(T)} \frac{\partial T}{\partial T} [B_1(T_{b,2}) - B_1(T_{\text{atm}})] \] \hspace{1cm} [15.35]

and

\[ B_2(T_{\text{sur}}) \approx B_2(T_{\text{atm}}) + \frac{\partial B_2(T)}{\partial B_1(T)} \frac{\partial T}{\partial T} [B_1(T_{\text{sur}}) - B_1(T_{\text{atm}})] \] \hspace{1cm} [15.36]

Let’s substitute the above expressions for \( B_2(T_{b,2}) \) and \( B_2(T_{\text{sur}}) \) in Eq.[15.30]

\[ B_2(T_{\text{atm}}) + \frac{\partial B_2(T)}{\partial B_1(T)} \frac{\partial T}{\partial T} [B_1(T_{b,2}) - B_1(T_{\text{atm}})] = \]

\[ = T_2 \{ B_2(T_{\text{atm}}) + \frac{\partial B_2(T)}{\partial B_1(T)} \frac{\partial T}{\partial T} [B_1(T_{\text{sur}}) - B_1(T_{\text{atm}})] \} + B_2(T_{\text{atm}})[1 - T_2] \]

where \( T_1 \) and \( T_2 \) are transmissions in the channels 1 and 2.

Eq.[15.37] becomes

\[ B_1(T_{b,2}) = B_1(T_{\text{sur}})T_2 + B_1(T_{\text{atm}})[1 - T_2] \] \hspace{1cm} [15.38]

Using Eq.[15.29], we can eliminate \( B_1(T_{\text{atm}}) \)

\[ B_1(T_{\text{sur}}) = I_1 + \gamma[I_1 - B_1(T_{b,2})] \] \hspace{1cm} [15.39]

where \( \gamma = \frac{1 - T_1}{T_1 - T_2} \); (\( T_1 \) and \( T_2 \) are transmissions in the channels 1 and 2).

Performing linearization of Eq.[15.39]

\[ T_{\text{sur}} \approx T_{b,1} + \gamma[T_{b,1} - T_{b,2}] \] \hspace{1cm} [15.40]

**Principles of a SST retrieval algorithm:**

SST is retrieved based on the linear differences in brightness temperatures at two IR channels. Two channels are used to eliminate the term involving \( T_{\text{atm}} \) and solve for \( T_{\text{sur}} \).

**NOTE:** Clouds cause a serious problem in SST retrievals => need a reliable algorithm to detect and eliminate the clouds (called a **cloud mask**).
NOTE: One needs to distinguish the bulk sea surface temperature from skin sea surface temperature:

**Bulk** (1-5 m depth) SST measurements:

1. Ships
2. Buoys (since the mid-1970s): buoy SSTs are much less noisy than ship SSTs
   
   Data from buoys are included in SST retrieval algorithms

**Skin** SST from infrared satellite sensors:

- SR (Scanning Radiometer) and VHRR (Very High Resolution Radiometer) both flown on NOAA polar orbiting satellites: since mid-1970
- AVHRR (Advanced Very High Resolution Radiometer) series: provide the longest data set of SST: since 1978 (4 channels, started on NOAA-6) and since 1988 (5 channels, started on NOAA-11)

<table>
<thead>
<tr>
<th>Table 12.1 AVHRR CHANNELS</th>
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<tr>
<td>AVHRR Channel</td>
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<tr>
<td>----------------</td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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<td>3</td>
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<tr>
<td>4</td>
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<tr>
<td>5</td>
</tr>
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</table>

**AVHRR MCSST (Multi-Channel SST) algorithm:**

\[
\text{SST} = a T_{b,4} + \gamma(T_{b,4} - T_{b,5}) + c \tag{15.41}
\]

where \(a\) and \(c\) are constants.

\[
\gamma = \frac{1 - T_4}{T_4 - T_5}, \quad T_4 \text{ and } T_5 \text{ are transmission function at AVHRR channels 4 and 5}
\]

**Reynolds Optimal Interpolation Algorithm:**


1) Merges Advanced Very High Resolution Radiometer (AVHRR) infrared satellite SST data and data from Advanced Microwave Scanning Radiometer (AMSR) on the NASA

2) Data products have a spatial grid resolution of 0.25° and a temporal resolution of 1 day.

![Daily SST Optimum Interpolation (OI) Analysis Diagram]

- **Microwave vs. IR SST retrievals**

<table>
<thead>
<tr>
<th>Factor affecting radiometry</th>
<th>Infrared radiometry</th>
<th>Microwave radiometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude of emitted radiation from the sea surface</td>
<td>[+] large $B(\lambda, T)$</td>
<td>[-] small $B(\lambda, T)$</td>
</tr>
<tr>
<td>Sensitivity of brightness to SST</td>
<td>[+] large $\frac{1}{B} \frac{\partial B}{\partial T}$</td>
<td>[-] small $\frac{1}{B} \frac{\partial B}{\partial T}$ (B is proportional to T)</td>
</tr>
<tr>
<td>Emissivity</td>
<td>[+</td>
<td>$\varepsilon$ = about 1</td>
</tr>
<tr>
<td>Clouds</td>
<td>[-</td>
<td>Not transparent</td>
</tr>
<tr>
<td></td>
<td>Improvement at longer wavelengths</td>
<td></td>
</tr>
<tr>
<td>--------------------------------</td>
<td>----------------------------------</td>
<td></td>
</tr>
<tr>
<td>Sea state (e.g., roughness)</td>
<td>[+ Independent</td>
<td>[-] $\varepsilon$ varies with sea state</td>
</tr>
<tr>
<td>Atmospheric interference</td>
<td>[- Requires complex correction</td>
<td>[+ Easily corrected with multichannel radiometer</td>
</tr>
<tr>
<td>Spatial resolution</td>
<td>[+ A narrow beam can be focused. Diffraction is not a problem in achieving high spatial resolution with a small instrument</td>
<td>[+ Diffraction controls the beam at large wavelengths. Large antenna required for high spatial resolution</td>
</tr>
<tr>
<td>Viewing direction on surface</td>
<td>[+ Surface radiance largely independent of viewing direction</td>
<td>[-] $\varepsilon$ varies with viewing direction</td>
</tr>
<tr>
<td>Absolute calibration</td>
<td>[+ Readily achieved using heated on-board target</td>
<td>[-] Absolute calibration target not readily achieved</td>
</tr>
<tr>
<td>Presently achievable sensitivity</td>
<td>0.1 degree K</td>
<td>1.5 degree K</td>
</tr>
<tr>
<td>Presently achievable absolute accuracy</td>
<td>0.6 degree K</td>
<td>2 degree K</td>
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