

**Lecture 16.**

**Course Summary & Review for Exam II**

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## Upward/downward intensity in the plane-parallel atmosphere with scattering/absorption (Lecture 5):

$$I_{\lambda}^{\uparrow}(\tau, \mu, \varphi) = I_{\lambda}^{\uparrow}(\tau^*, \mu, \varphi) \exp\left(-\frac{\tau^* - \tau}{\mu}\right) + \frac{1}{\mu} \int_{\tau}^{\tau^*} \exp\left(-\frac{\tau' - \tau}{\mu}\right) J_{\lambda}^{\uparrow}(\tau', \mu, \varphi) d\tau'$$

$$I_{\lambda}^{\downarrow}(\tau, -\mu, \varphi) = I_{\lambda}^{\downarrow}(0, -\mu, \varphi) \exp\left(-\frac{\tau}{\mu}\right) + \frac{1}{\mu} \int_0^{\tau} \exp\left(-\frac{\tau - \tau'}{\mu}\right) J_{\lambda}^{\downarrow}(\tau', -\mu, \varphi) d\tau'$$

$$J(\tau, \mu, \varphi) = \frac{\omega_0}{4\pi} \int_0^{2\pi} \int_{-1}^1 I(\tau, \mu', \varphi') P(\mu, \varphi, \mu', \varphi') d\mu' d\varphi' + \frac{\omega_0}{4\pi} F_0 P(\mu, \varphi, \mu_0, \varphi_0) \exp(-\tau / \mu_0)$$

## Upward/downward intensity in the plane-parallel atmosphere with emission/absorption (Lecture 9):

$$I_{\lambda}^{\uparrow}(\tau; \mu; \varphi) = I_{\lambda}^{\uparrow}(\tau^*; \mu; \varphi) \exp\left(-\frac{\tau^* - \tau}{\mu}\right) + \frac{1}{\mu} \int_{\tau}^{\tau^*} \exp\left(-\frac{\tau' - \tau}{\mu}\right) B_{\lambda}(\tau') d\tau'$$

$$I_{\lambda}^{\downarrow}(\tau; -\mu; \varphi) = I_{\lambda}^{\downarrow}(0; -\mu; \varphi) \exp\left(-\frac{\tau}{\mu}\right) + \frac{1}{\mu} \int_0^{\tau} \exp\left(-\frac{\tau - \tau'}{\mu}\right) B_{\lambda}(\tau') d\tau'$$

## Clouds:

### *Cloud amount/coverage (cloud mask)*

*Visible+ IR => Lecture 12 and Lab 10*

*Principles: based on a combination of thresholds for solar reflectivity and brightness temperature (in the IR)*

*Active (CALIPSO, CloudSat) => Lectures 13-14*

### *Cloud liquid water content (column integrated)*

*Microwave => Lecture 9*

### *Cloud type*

*ISCCP classification => Lecture 12*

### *Cloud particle size distribution (effective size) and optical depth*

*MODIS retrieval technique => Lecture 12 and Lab 10*

### *Cloud thermodynamic phase*

*MODIS retrieval technique => Lecture 12*

### *Cloud-top pressure*

*O2 absorption technique” and “CO2 slicing technique = > (see textbook)*

### *Cloud height and cloud detection*

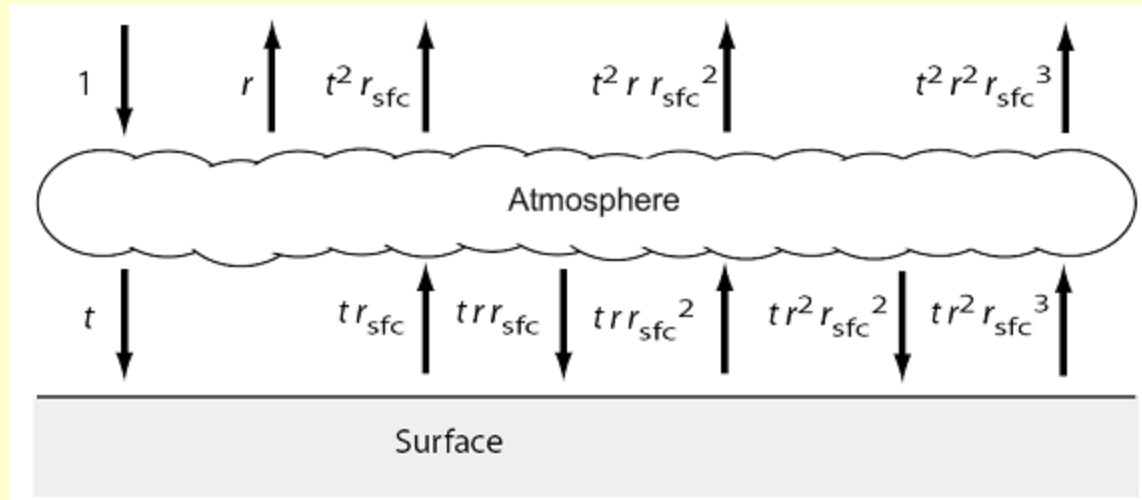
*Lidars/Radars => Lectures 13-14 and Lab 12*

## Problem solving example

You analyze a satellite image of two clouds with one appearing brighter at the visible wavelengths. In general, would you expect more, less, or unknown infrared radiance emitted by the brighter-looking cloud?

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Correct answer: unknown



$$R_{total} = R_{atm} + \frac{R_{sur} T_{atm}^2}{1 - R_{sur} R_{atm}}$$

For optically thin atmosphere ( $R_{atm} \ll 1$  and  $T_{atm} \sim 1$ ):

$$R_{total} \approx R_{atm} + R_{sur}$$

## Approximate eqs:

### **SOLAR SPECTRUM**

**Cloud reflectivity (conservative scattering:  $\omega_0 = 1$ ):**

$$R_{cld} = \frac{(1 - g)\tau}{1 + (1 - g)\tau}$$

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### **THERMAL SPECTRUM**

**Atmospheric features (cloud) with emission/absorption**

$$I_{\nu}^{\uparrow}(0; \mu) = B_{\nu}(T_{src})(1 - \varepsilon_{cld}) + B_{\nu}(T_{cld})\varepsilon_{cld}$$

## Problem solving example

Consider a cloud with temperature of 220 K overlying a surface with  $T=285$  K. Assume that the atmosphere above and below the cloud is transparent to the radiation at  $11 \mu\text{m}$ .

If the cloud emissivity is 1, what is the brightness temperature that will be measured by a nadir looking satellite radiometer at  **$11 \mu\text{m}$** ?

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**Solution:** Use the following Eq., and then find BT from I by inverting the Planck function :

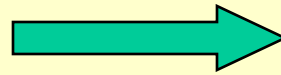
$$I_{\lambda}^{\uparrow}(0; \mu) = B_{\lambda}(T_{sur})T_{\lambda}(\tau^*, \mu) + B_{\lambda}(T_{cloud})[1 - T_{\lambda}(\tau^*, \mu)]$$

## Brightness temperature (Lecture 2)

**Brightness temperature,  $T_b$** , is defined as the temperature of a blackbody that emits the same intensity as measured.

Brightness temperature is found by inverting the Planck function.

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5 (\exp(hc / k_B T \lambda) - 1)}$$



$$T_b = \frac{C_2}{\lambda \ln\left[1 + \frac{C_1}{\lambda^5 I_\lambda}\right]}$$

where  $C_1 = 2hc^2 = 1.1911 \times 10^8 \text{ W m}^{-2} \text{ sr}^{-1} \mu\text{m}^4$

$C_2 = hc/k_B = 1.4388 \times 10^4 \text{ K } \mu\text{m}$



## Aerosols:

### *Aerosol optical depth/particle size distribution/Angstrom exponent*

Sunphotometers (AERONET) => Lecture 5 and Lab 4

Principles: based on measurements of direct solar radiation that permit to retrieve the aerosol optical depth

Visible-near IR satellite remote sensing (MODIS, MISR, AVHRR, SeaWiFS) =>

Lecture 5

Principles: based on measurements of reflected solar radiation and look-up tables for pre-defined aerosol models (size distribution and refractive index)

### *Vertical profile of backscattering, extinction and optical depth (lidars) =>*

Lecture 14 and Lab 12

**Assuming no surface reflection** (dark surface), the upwelling intensity at the level Z (or  $\tau$ ) is

$$I^\uparrow(\tau, \mu, \varphi) = \int_{\tau}^{\tau^*} J(\tau', \mu, \varphi) \exp[-(\tau' - \tau) / \mu] d\tau' / \mu$$

Substituting in the source function

$$I^\uparrow(\tau, \mu, \varphi) = \frac{\omega_0}{4\pi} F_0 P(\Theta) \int_{\tau}^{\tau^*} \exp[-(\tau' - \tau) / \mu - \tau' / \mu_0] d\tau' / \mu$$

Satellite sensor measures

$$I^\uparrow(0, \mu, \varphi) = \frac{\omega_0}{4\pi} F_0 P(\Theta) \frac{\mu_0}{\mu + \mu_0} \left[ 1 - \exp\left(-\left(\frac{1}{\mu_0} + \frac{1}{\mu}\right)\tau^*\right) \right]$$

**In the single scattering approximation when  $\tau^* \ll 1$ :**

$$I^\uparrow(0, \mu, \varphi) = \frac{\omega_0}{4\pi} F_0 P(\Theta) \frac{\tau^*}{\mu}$$

# Lidar equations

**MIE lidar:**

$$P_r(R) = \frac{C}{R^2} \frac{h}{2} \frac{k_b}{4\pi} \exp\left(-2 \int_0^R k_e(r') dr'\right)$$

**RAMAN lidar:**

$$P_r(R, \lambda_L, \lambda_R) = \frac{C}{R^2} \frac{h}{2} \frac{k_b(R, \lambda_L, \lambda_R)}{4\pi} \exp\left(-\int_0^R [k_e(r', \lambda_L) + k_e(r', \lambda_R)] dr'\right)$$

## Scattering domains:

**Rayleigh scattering regime:**  $2\pi r/\lambda \ll 1$ , and the refractive index  $m$  is arbitrary (applies to scattering by molecules and small aerosol particles)

**Rayleigh-Gans scattering:**  $(m - 1) \ll 1$  (not useful for atmospheric application)

**Mie-Debye scattering:**  $2\pi r/\lambda$  and  $m$  are both arbitrary but for spheres only (applies to scattering by aerosol and cloud particles)

**Geometrical optics:**  $2\pi r/\lambda$  is very large and  $m$  is real (applies to scattering by large cloud droplets).

**The size parameter  $x = 2\pi r/\lambda$  is a key factor determining how a particle interacts with EM radiation**

## MIE theory:

Efficiencies (or efficiency factors) for extinction, scattering and absorption are defined as

$$Q_e = \frac{\sigma_e}{\pi r^2}$$

$$Q_s = \frac{\sigma_s}{\pi r^2}$$

$$Q_a = \frac{\sigma_a}{\pi r^2}$$

$$Q_a = Q_e - Q_s$$

$$Q_b = \frac{\sigma_b}{\pi r^2}$$

$$\sigma_b = \sigma_s P(\Theta = 180^\circ)$$

(Lecture 13)

From Mie theory:

$$Q_e = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1) \operatorname{Re}[a_n + b_n]$$

$$Q_s = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1) [|a_n|^2 + |b_n|^2]$$

## In Rayleigh scattering regime:

$$Q_a \approx -4x \operatorname{Im} \left( \frac{m^2 - 1}{m^2 + 2} \right)$$

$$Q_s \approx \frac{8}{3} x^4 \left| \frac{m^2 - 1}{m^2 + 2} \right|^2$$

$$Q_b \approx 4x^4 \left| \frac{m^2 - 1}{m^2 + 2} \right|^2$$

## Integration over the size distribution:

For a given type of particles characterized by the size distribution  $N(r)dr$ , the **extinction, scattering and absorption coefficients** (in units  $\text{LENGTH}^{-1}$ ) are determined as

$$k_e = \int_{r_1}^{r_2} \sigma_e(r)N(r)dr$$

$$k_s = \int_{r_1}^{r_2} \sigma_s(r)N(r)dr$$

$$k_a = \int_{r_1}^{r_2} \sigma_a(r)N(r)dr$$

### **Backscattering coefficient (Lecture 14)**

$$k_b = \int_{r_1}^{r_2} \sigma_b(r)N(r)dr$$

## Gases:

### **Absorption (emission):**

- depends on molecular structure (dipole!)
- wavelength-selective

### **Scattering:**

- a point dipole approach – Rayleigh scattering
- $\sim \text{wavelength}^{-4} \Rightarrow$  important in UV-vis  
negligible in IR&microwave



**Ozone and trace gases ( $\text{NO}_2$ ,  $\text{SO}_2$ ,  $\text{BrO}$ ,  $\text{OClO}$ ):**

*Ozone profile*

*Sounding => Lecture 10*

*Other gases => see Table 15.1 in Lecture 15*

*Lidars (profile) => Lecture 14*

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**Water vapor:**

*Integrated column (total precipitable water) from microwave =>  
Lecture 9*

*Profile from IR sounding => Lecture 10*

*Profile from microwave sounding => Lecture 10*

*Profile from Raman lidar, DIAL => Lecture 14*

$$\tau_v = \int_{u_1}^{u_2} k_v du$$



$$\tau_v = \int_{z'}^z k_v \rho_{gas} dz''$$



$$T_v(z, z', \mu) = \exp\left(-\frac{1}{\mu} \int_{z'}^z k_v \rho_{gas} dz''\right)$$



$$\frac{dT_v(z, z', \mu)}{dz'} = -\frac{k_v \rho_{gas}}{\mu} \exp\left(-\frac{1}{\mu} \int_{z'}^z k_v \rho_{gas} dz''\right)$$



$$I_v^\uparrow(z, \mu) = I_v^\uparrow(0, \mu) T_v(z, 0, \mu) + \int_0^z B_v(T(z')) \left| \frac{dT_v(z, z', \mu)}{dz'} \right| dz'$$



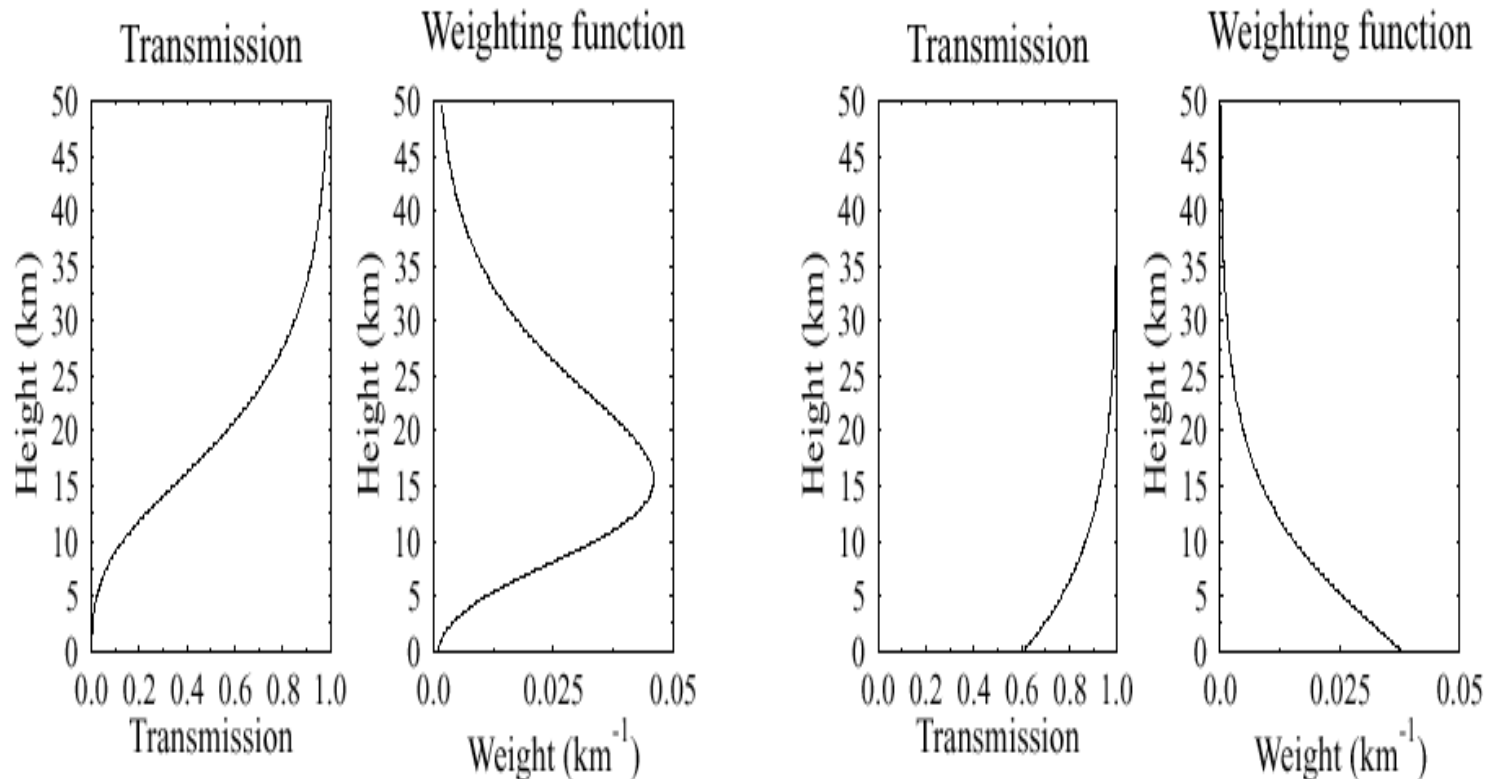
$$I_v^\uparrow(z, \mu) = I_v^\uparrow(0, \mu) \exp\left[-\frac{1}{\mu} \int_0^z k_v \rho_{gas} dz'\right] + \frac{1}{\mu} \int_0^z \exp\left[-\frac{1}{\mu} \int_{z'}^z k_v \rho_{gas} dz''\right] B_v(T(z')) k_v \rho_{gas} dz'$$

# Weighting functions for near-nadir sounding:

For a satellite sensor looking down:

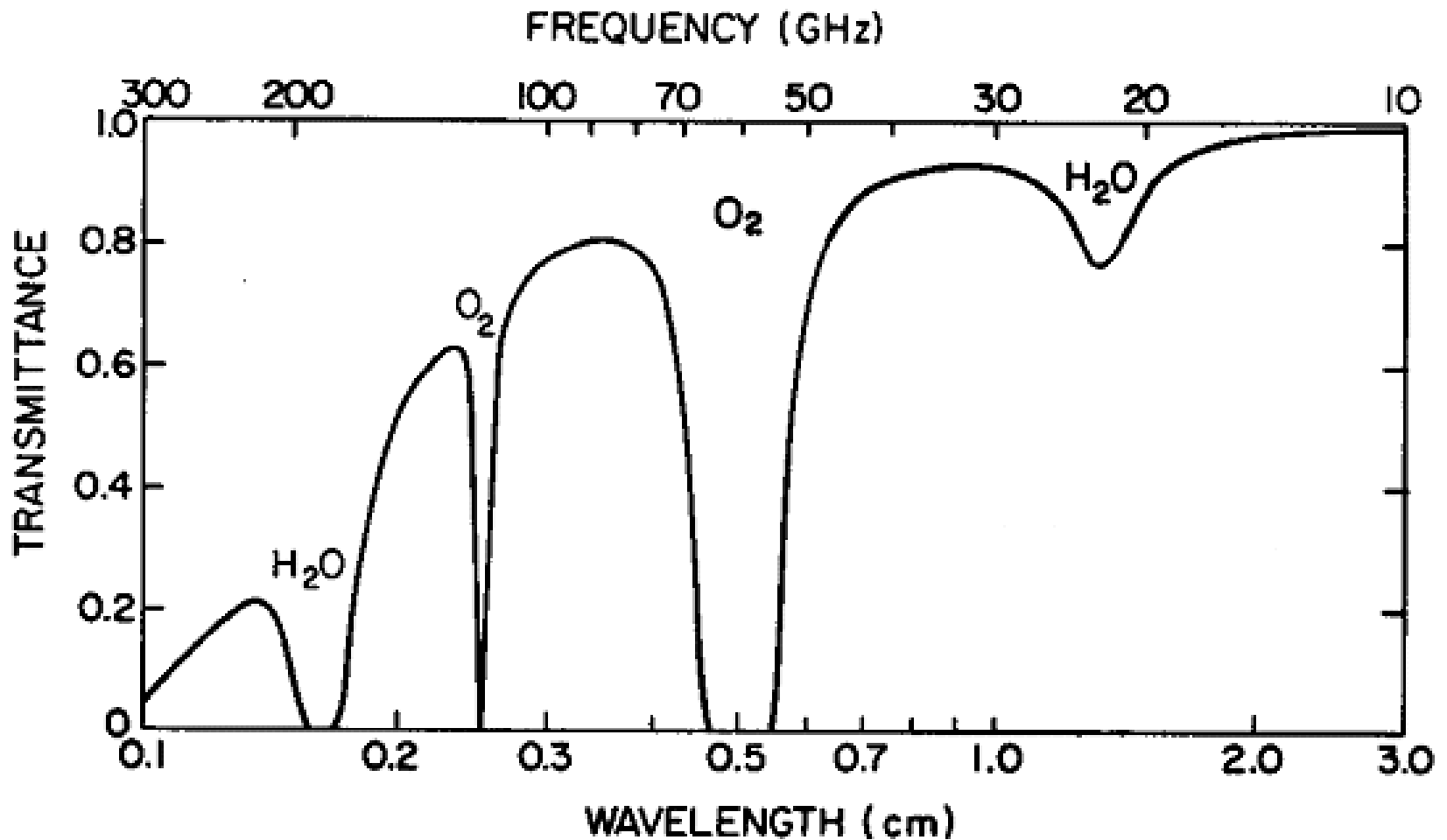
$$I_v^\uparrow(\infty, \mu) = I_v^\uparrow(0, \mu)T_v(\infty, 0, \mu) + \int_0^\infty B_v(T(z')) \left| \frac{dT_v(\infty, z', \mu)}{dz'} \right| dz'$$

$$W_v(\infty, z, \mu) = \left| \frac{T_v(\infty, z, \mu)}{dz} \right| = \frac{k_v \rho_{gas}}{\mu} \exp\left(-\frac{1}{\mu} \int_z^\infty k_v \rho_{gas} dz\right)$$



*Microwave*

$$\lambda(\text{in cm}) = \frac{30}{\tilde{\nu}(\text{in GHz})}$$



## Precipitation

Visible/IR techniques => Lecture 12

*Principles: indirect method that relates properties of clouds to precipitation*

Microwave techniques => Lecture 12

*Principles: direct method that relates the optical depth associated with the emitting rain drops and brightness temperature measured by a passive microwave radiometer.*

Radar => Lecture 13 and Lab 11

*Principles: measured backscattering from rain drops is related to the Z factor (size distribution) and then to precipitation via the Z-R relationship*

## Radar equation

$$\frac{P_r}{P_t} = \pi^2 G^2 \frac{h\theta_{HP}\varphi_{HP}}{128R^2} |K|^2 \int N(D)D^6 dD$$

$$P_r = C \frac{|K|^2}{R^2} Z$$

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$$P(\text{dBz}) = 10 \log(Z) \quad \rightarrow \quad Z = 10^{(P(\text{dBz})/10)}$$

## **Problem solving example:**

**Precipitation is a key component of the hydrological cycle. Briefly explain the principles and discuss advantages and disadvantages of the following remote sensing techniques:**

- **passive IR sensing of precipitation**
- **passive microwave sensing of precipitation**
- **active microwave sensing of precipitation**

### *Atmospheric temperature (profile)*

IR (or microwave) sounding techniques => Lecture 9 and Lab 7

*Principles: multi-spectral remote sensing in the 15  $\mu\text{m}$  CO<sub>2</sub> absorbing band  
(in microwave in the O<sub>2</sub> absorbing region)*

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### *Sea Surface Temperature*

IR split-window technique => Lecture 9

Microwave techniques => Lecture 9

### *Ocean color mapping*

Solar remote sensing (MODIS, SeaWiFS) => Lecture 6

### *Sea ice*

Passive microwave => Lecture 2 and Lab 1

Active microwave (radars) => (see textbook)