

Lecture 11.

Terrestrial infrared radiative processes. Part 4:

IR radiative heating/cooling rates .

Objectives:

1. IR radiative transfer revisited.
2. Infrared radiative heating/cooling rates in the cloud-free atmosphere.
3. Concept of the broadband flux emissivity.

Required reading:

L02: 4.2.2; 4.5, 4.7

1. IR radiative transfer revisited.

Recall Lecture 8 where we have derived the solutions of the radiative transfer equation for the **monochromatic upward and downward intensities** in the IR for a plane-parallel atmosphere consisting of absorbing gases (no scattering)

$$I_{\nu}^{\uparrow}(\tau; \mu) = B_{\nu}(\tau^*) \exp\left(-\frac{\tau^* - \tau}{\mu}\right) + \frac{1}{\mu} \int_{\tau}^{\tau^*} \exp\left(-\frac{\tau' - \tau}{\mu}\right) B_{\nu}(\tau') d\tau' \quad [8.3a]$$

$$I_{\nu}^{\downarrow}(\tau; -\mu) = \frac{1}{\mu} \int_0^{\tau} \exp\left(-\frac{\tau - \tau'}{\mu}\right) B_{\nu}(\tau') d\tau' \quad [8.3b]$$

and in terms of transmission function

$$I_{\nu}^{\uparrow}(\tau; \mu) = B_{\nu}(\tau^*) T_{\nu}(\tau^* - \tau; \mu) - \int_{\tau}^{\tau^*} B_{\nu}(\tau') \frac{dT_{\nu}(\tau' - \tau; \mu)}{d\tau'} d\tau' \quad [8.4a]$$

$$I_{\nu}^{\downarrow}(\tau; -\mu) = \int_0^{\tau} B_{\nu}(\tau') \frac{dT_{\nu}(\tau - \tau'; \mu)}{d\tau'} d\tau' \quad [8.4b]$$

Recall that Eq.[8.3a, b] and Eq.[8.4a, b] have been derived for the entire atmosphere with the optical depth τ_v^* for two boundary conditions:

Surface: assumed to be a blackbody in the IR emitting with the surface temperature T_s ,

$$I_v^\uparrow(\tau_v^*, \mu) = B_v(T_s) = B_v(T_s(\tau_v^*)) = B_v(\tau_v^*)$$

Top of the atmosphere (TOA), $\tau_v = 0$: no downward emission

$$I_v^\downarrow(0, -\mu) = 0$$

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EXAMPLE: Consider an atmosphere that is represented by a single layer with the effective temperature of T_{eff} (e.g., an isothermal atmosphere). Under this assumption, The Plack function is constant $B = B(T_{\text{eff}})$ and hence Eq.[8.3a] gives a simplified expression for the radiance at the top of the atmosphere:

$$I_v^\uparrow(0; \mu) = B_v(\tau^*) \exp\left(-\frac{\tau^*}{\mu}\right) + B_v(T_{\text{eff}}) [1 - \exp\left(-\frac{\tau^*}{\mu}\right)]$$

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In Lecture 3, upwelling and downwelling fluxes were defined as

$F_v^\uparrow = 2\pi \int_0^1 I_v^\uparrow(\mu) \mu d\mu$ $F_v^\downarrow = 2\pi \int_0^1 I_v^\downarrow(-\mu) \mu d\mu$	[11.1]
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NOTE: Eq.[11.1] assumes that there is no dependency on ϕ in a plane-parallel atmosphere.

Thus, we can re-write the radiative transfer equation and its solutions in terms of **the monochromatic upward and downward fluxes**. From Eq.[8.3a, b], we have

$$\begin{aligned}
F_{\nu}^{\uparrow}(\tau) &= 2\pi B_{\nu}(\tau^*) \int_0^1 \exp\left(-\frac{\tau^* - \tau}{\mu}\right) \mu d\mu \\
&+ 2\pi \int_0^1 \int_{\tau}^{\tau^*} \exp\left(-\frac{\tau' - \tau}{\mu}\right) B_{\nu}(\tau') d\tau' d\mu
\end{aligned}
\tag{11.2a}$$

and

$$F_{\nu}^{\downarrow}(\tau) = 2\pi \int_0^1 \int_0^{\tau} \exp\left(-\frac{\tau - \tau'}{\mu}\right) B_{\nu}(\tau') d\tau' d\mu
\tag{11.2b}$$

Let's introduce **the transmission function** for the radiative flux (called **diffuse transmission function** or **slab transmission function** or **flux transmission function**) as

$$T_{\nu}^f(\tau) = 2 \int_0^1 T_{\nu}(\tau; \mu) \mu d\mu
\tag{11.3}$$

where $T_{\nu}(\tau; \mu)$ is the monochromatic transmittance defined in Lecture 7, Eq.[7.2]

Spectral diffuse transmission function (or **transmittance**) may be defined as:

$$T_{\nu}^f(\tau) = 2 \int_0^1 T_{\nu}(\tau; \mu) \mu d\mu
\tag{11.4}$$

Using the definition of monochromatic diffuse transmittance and solution of the radiative transfer equation expressed via **the transmittance** Eq.[8.4a, b], the solution for fluxes can be written as

$$\begin{aligned}
F_{\nu}^{\uparrow}(\tau) &= \pi B_{\nu}(\tau^*) T_{\nu}^f(\tau^* - \tau) \\
&- \int_{\tau}^{\tau^*} \pi B_{\nu}(\tau') \frac{dT_{\nu}^f(\tau' - \tau)}{d\tau'} d\tau'
\end{aligned}
\tag{11.5a}$$

and

$$F_{\nu}^{\downarrow}(\tau) = \int_0^{\tau} \pi B_{\nu}(\tau') \frac{dT_{\nu}^f(\tau - \tau')}{d\tau'} d\tau'
\tag{11.5b}$$

NOTE: On the right side of Eq.[11.5a] for the upward flux, the first term gives the surface emission that is attenuated to the level τ and the second term gives the emission from the atmospheric layers characterized by the Planck function multiplied by the **weighting function** $dT_v^f / d\tau$. Likewise, the downward flux at a given layer (Eq.[11.5b]) is produced by the emission from the atmospheric layers.

2. Infrared radiative heating/cooling rates in the cloud-free atmosphere.

- ✓ Radiative processes may affect the dynamics and thermodynamics of an atmosphere through the generation of **radiative heating/cooling rates**.

The **radiative heating (or cooling) rate** is defined as the rate of temperature change of the layer dz due to radiative energy gain (or loss):

$$\boxed{\left(\frac{dT}{dt} \right) = - \frac{1}{c_p \rho} \frac{dF_{net}}{dz} = \frac{g}{c_p} \frac{dF_{net}}{dp}} \quad [11.6]$$

where c_p is the specific heat at the constant pressure ($c_p = 1004.67 \text{ J/kg/K}$) and ρ is the air density in a given layer.

NOTE: The thermodynamic equation for the temperature changes in the atmosphere (i.e. the first law of thermodynamic for moist air) includes **the radiative energy exchange term (i.e. total radiative heating/cooling rates** which are a sum of solar and infrared heating/cooling rates). In this lecture we discuss IR radiative rates only (solar will be discussed later in the course).

Recall the definition of the **monochromatic net flux** (net power per area at a given height) (see Lecture 3):

$$F_v(z) = F_v^{\uparrow}(z) - F_v^{\downarrow}(z) \quad [11.7]$$

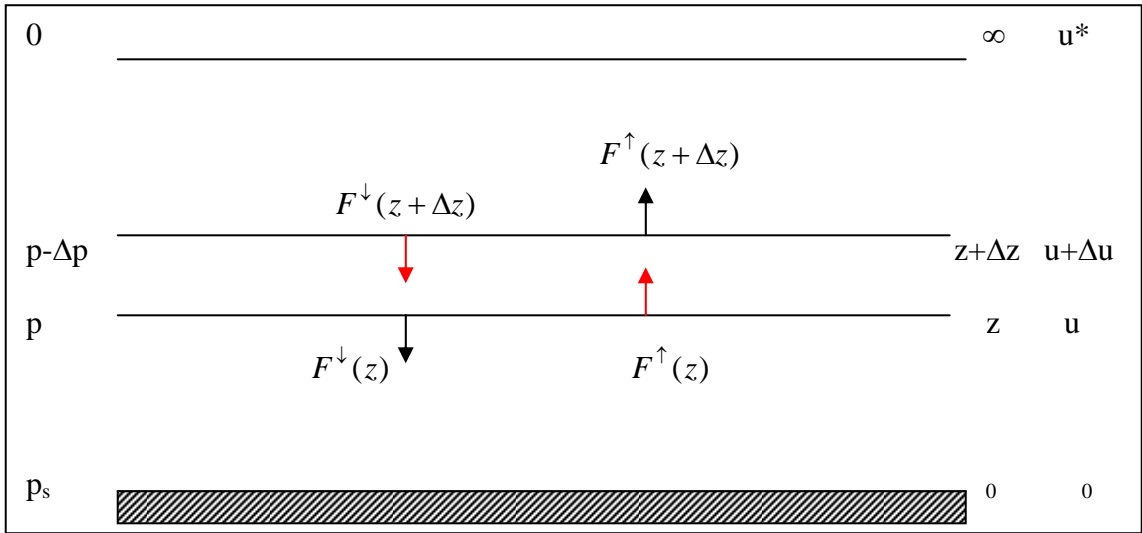
Similar, we can define the **total net flux**:

$$F(z) = F^{\uparrow}(z) - F^{\downarrow}(z) \quad [11.8]$$

Introducing the net flux $F(z+\Delta z)$ at the level $z+\Delta z$ (see figure below), we find the **net flux divergence** for the layer Δz as

$$\Delta F = F(z + \Delta z) - F(z)$$

$F(z+\Delta z) < F(z)$ (hence $\Delta F < 0$) => a layer gains radiative energy => heating
 $F(z+\Delta z) > F(z)$ (hence $\Delta F > 0$) => a layer loses radiative energy => cooling



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EXAMPLE Calculate longwave cooling at night for an atmospheric layer from 0 to 1 km using the upwelling and downwelling fluxes calculated with MODTRAN for US Standard Atmosphere 1976.

Altitude (km)	IR Upwelling flux (W/m ²)	IR Downwelling flux (W/m ²)
0	390	285
1	375	250

SOLUTION:

Need to find net fluxes at each altitude

$$F(z) = F^{\uparrow}(z) - F^{\downarrow}(z)$$

At 0 km: $F_{\text{net}} = 390 - 285 = 105 \text{ W/m}^2$

At 1 km: $F_{\text{net}} = 375 - 250 = 125 \text{ W/m}^2$

Thus $\Delta F = 20 \text{ W/m}^2$

$$\left(\frac{dT}{dt} \right)_{IR} = - \frac{1}{c_p \rho} \frac{dF_{\text{net}}}{dz} = \frac{-20 \text{ Js}^{-1} \text{m}^{-2}}{(1.17 \text{ kg / m}^3)(1004 \text{ Jkg}^{-1} \text{K}^{-1})(1000 \text{ m})}$$

$$dT/dt = -1.7 \times 10^{-5} \text{ K/s} = -1.5 \text{ K/day}$$

Thermal infrared region:

Each part of the atmosphere emits and absorbs radiation. Where absorption dominates, there is the net radiative heating; where emission dominates, there is net cooling. The latter is more common. Thus one can talk about IR radiative cooling rates (giving positive values of $\left(\frac{dT}{dt} \right)_{IR}$) or IR radiative heating rates (giving negative values of

$\left(\frac{dT}{dt} \right)_{IR}$).

To calculate the IR heating/cooling rates one needs to know:

i) Profiles of IR upwelling and downwelling fluxes (to calculate the profile of the IR net fluxes);

To compute the IR downward and upward fluxes one needs to know:

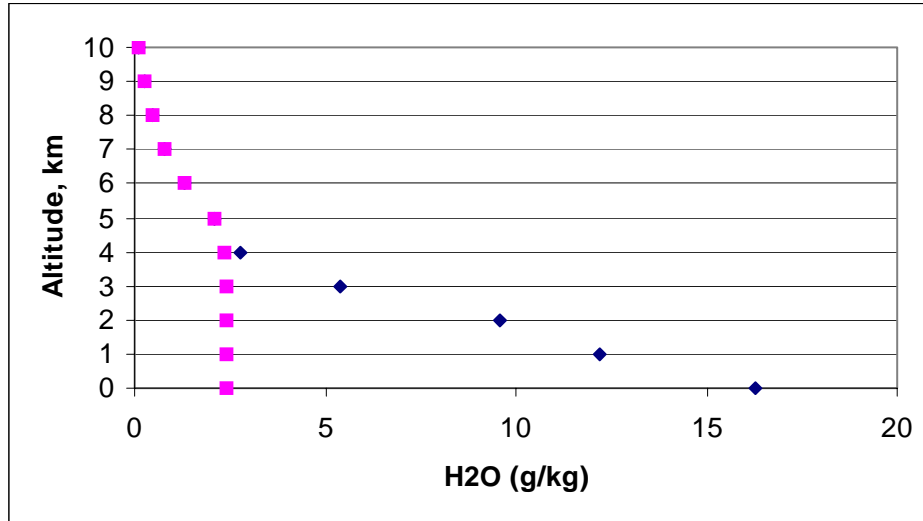
- 1) Atmospheric characteristics: vertical profiles of T, P and air density; and
- 2) The vertical profiles of IR radiatively active gases, clouds and aerosols.

ii) Using the profile of net fluxes and air density, one calculates the IR radiative heating/cooling rates as

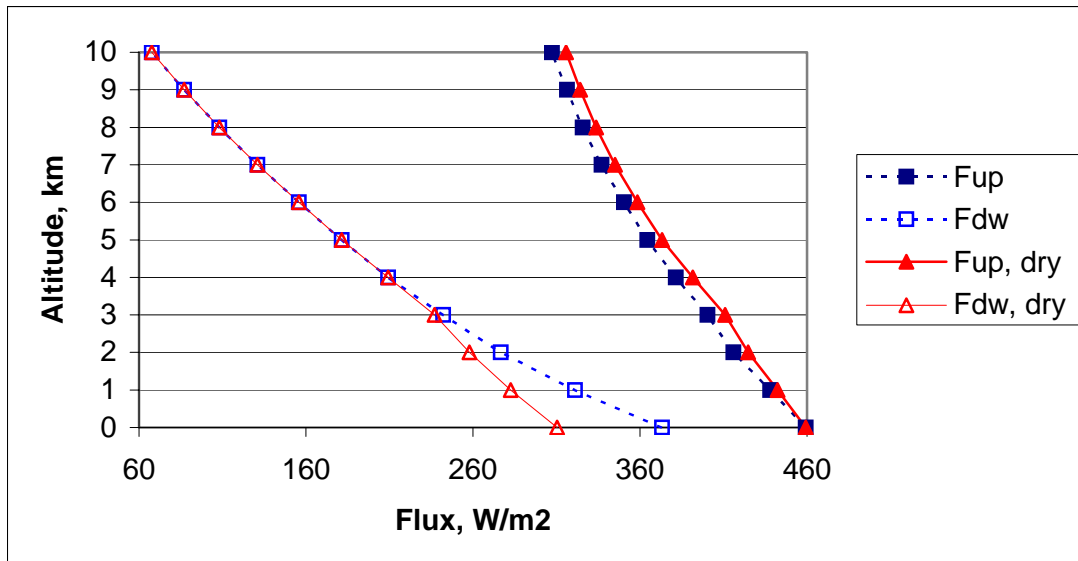
$$\left(\frac{dT}{dt} \right)_{IR} = - \frac{1}{c_p \rho} \frac{dF(z)}{dz}$$

Effect of the varying amount of a gas on IR radiation under the same atmospheric condition

Consider the standard tropical atmosphere and “dry” tropical atmosphere: same atmospheric characteristics, except the amount of H₂O (see Figure below).



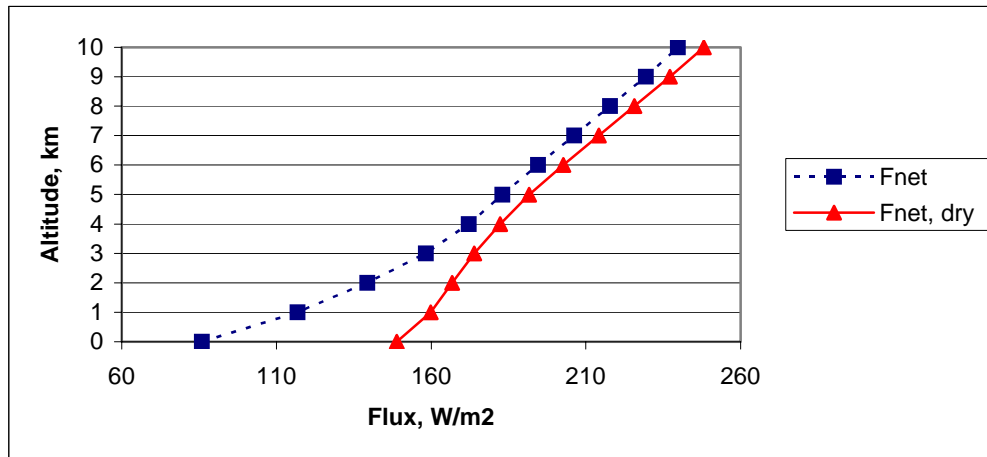
Calculated IR fluxes for tropical (dotted lines) and dry tropical atmospheres (solid lines)



✓ H₂O increases in a layer => F[↓] increases because more IR radiation emitted in a layer => F[↓](surface) increases

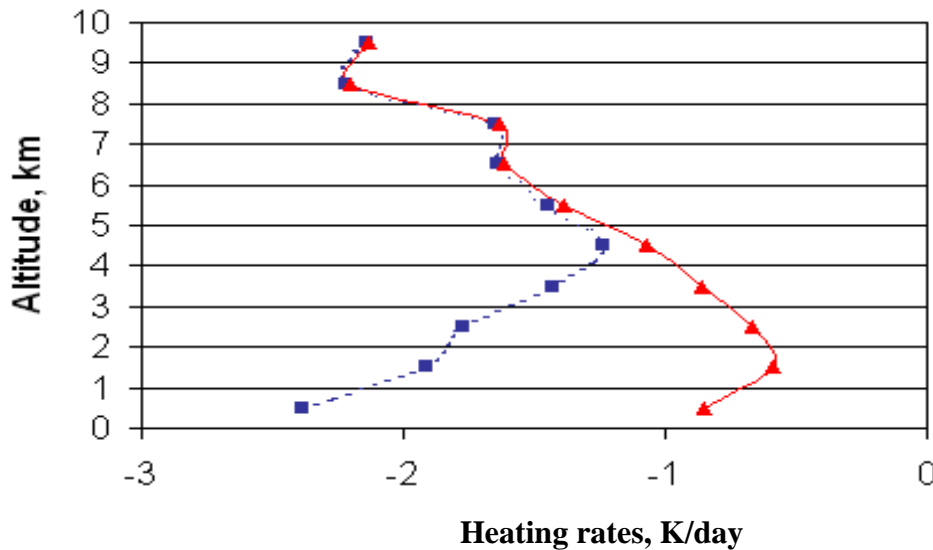
- ✓ H_2O increases in a layer $\Rightarrow F^\uparrow$ decreases because more IR radiation absorbed but reemitted at the lower temperature $\Rightarrow F^\uparrow(TOA)$ decreases
- ✓ Increase of an IR absorbing gas contributes to the greenhouse effect.

Calculated IR net fluxes for tropical (dotted lines) and dry tropical atmospheres (solid lines)



- ✓ The larger changes of the net flux from one level to another (i.e., the larger slope of $F(Z)$ vs Z), the larger IR cooling rates.

Calculated IR cooling rates for tropical (dotted lines) and dry tropical atmospheres (solid lines)



NOTE: The largest IR cooling rates for the standard tropical atmosphere are occurred in the surface layer.

IR cooling rates of individual gases:

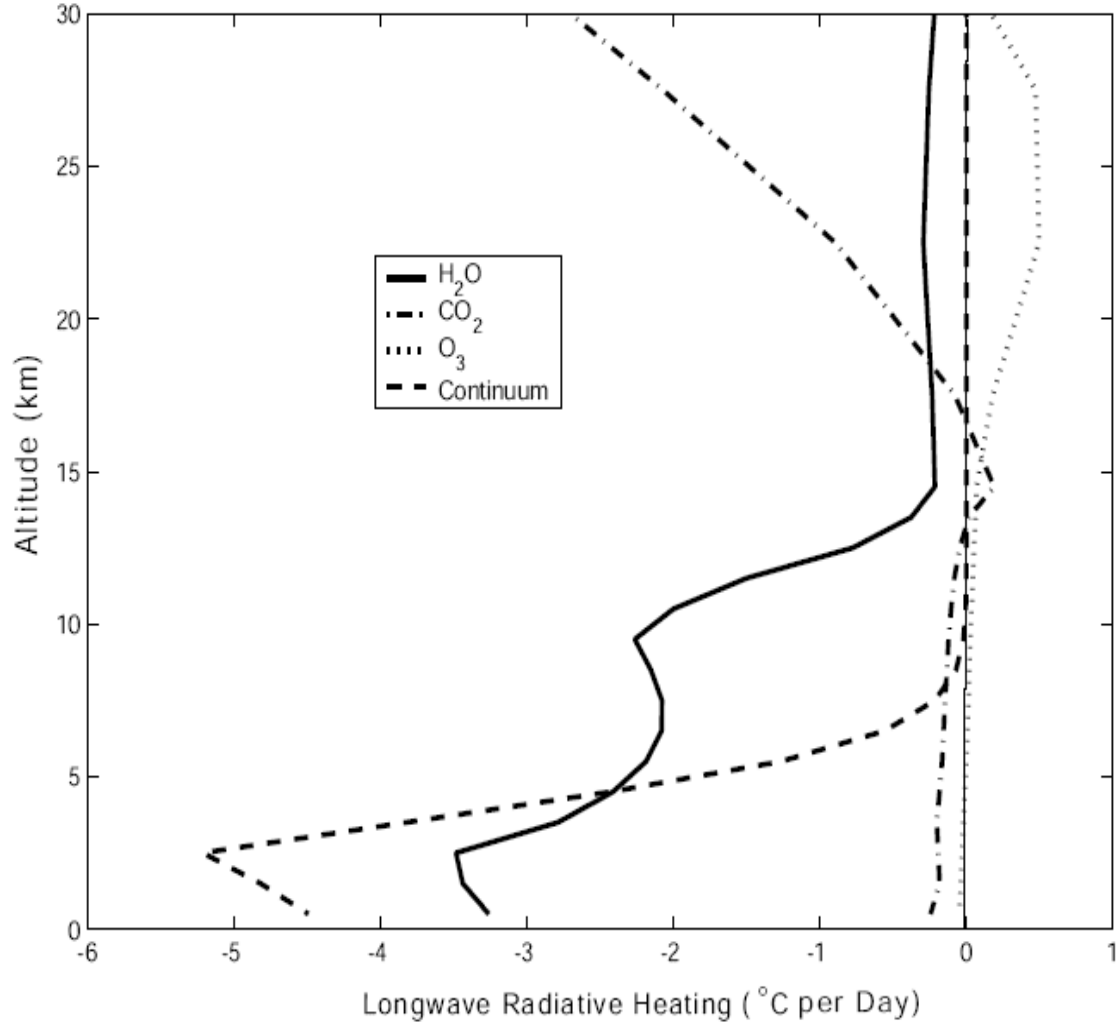


Figure 11.1 The thermal IR heating rate profiles in a cloud-free standard tropical atmosphere, segregated according to main absorbing gases. Note that negative values represent cooling.

NOTE: CO₂ has very small radiative heating rates. Radiation emitted at one level is absorbed at nearby level having almost the same temperature. Only at the tropopause (near 15 km), where the temperature profile has a minimum, there is a small amount of heating. At higher altitudes, pressure broadening is much weaker allowing emitted radiation to escape to space with little compensating radiation downward from higher levels.

H₂O is concentrated in the lower atmosphere resulting in the high cooling rates between 3 and 10 km. The two peaks in the profiles are associated with different absorption bands, the stronger of these being associated with the higher altitude peak.

H₂O continuum absorption is very sensitive to pressure. The mass absorption coefficient falls off rapidly with height. Consequently, the atmosphere in the upper troposphere and about is effectively transparent in this spectral band, while lower altitudes see fairly strong absorption.

O₃ is responsible for only instance of significant IR warming in the atmosphere below 30 km, with the peak warming between 20 and 30 km. This heating is due to the absorption at the base of the ozone layer of radiation emitted by the ground in the 9.6 μm band.

IR cooling rates in different cloud-free atmospheres:

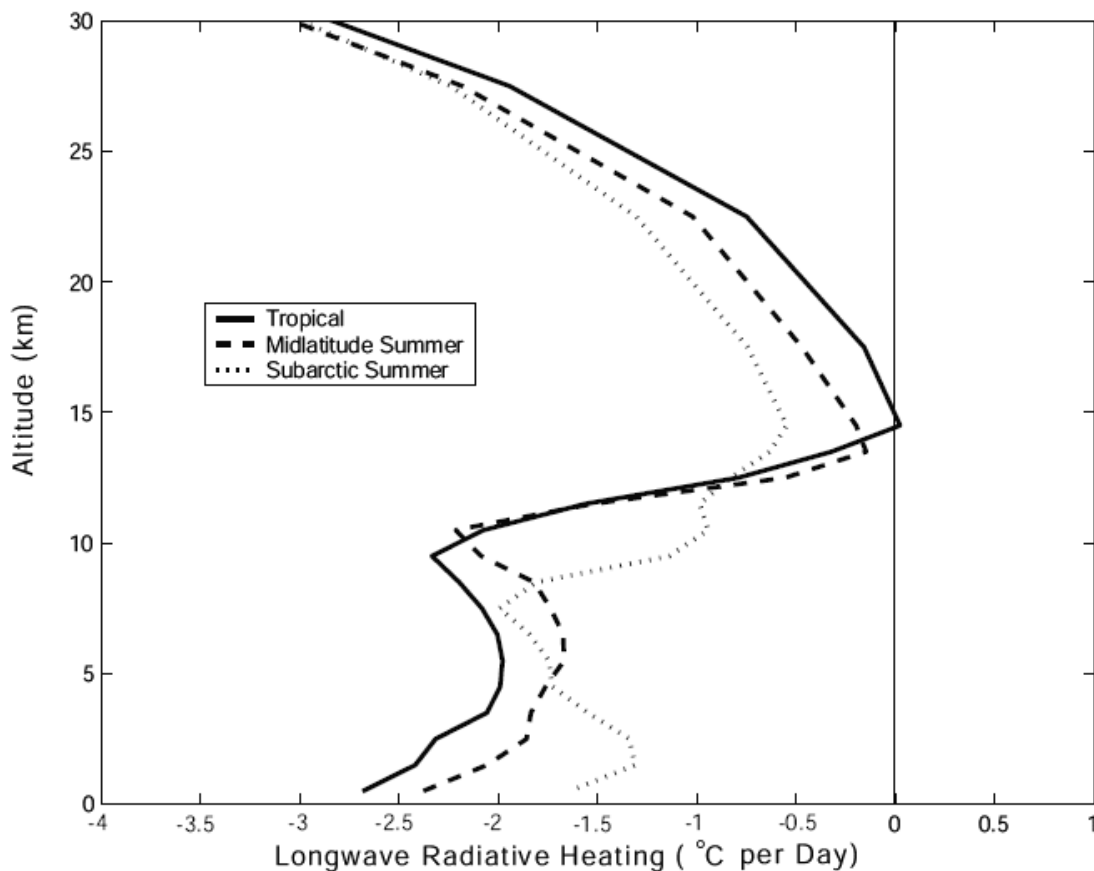


Figure 11.2 The total IR heating rates profiles calculate for three different model atmospheres.

3. Concept of the broadband flux emissivity

The **broadband flux emissivity** approach allows calculation of infrared fluxes and heating/cooling rates utilizing the temperature in terms of the Stefan-Boltzmann law instead of the Planck function.

Based on Eq.[11.5 a, b], the total upward and downward fluxes in the path length u coordinates may be expressed as

$$F^{\uparrow}(u) = \int_0^{\infty} \pi B_{\nu}(T_s) T_{\nu}^f(u) d\nu + \int_0^{\infty} \int_0^u \pi B_{\nu}(T(u')) \frac{dT_{\nu}^f(u-u')}{du'} du' d\nu \quad [11.9a]$$

and

$$F^{\downarrow}(u) = \int_0^{\infty} \int_{u^*}^u \pi B_{\nu}(T(u')) \frac{dT_{\nu}^f(u'-u)}{du'} du' d\nu \quad [11.9b]$$

From the Stefan-Boltzmann law (see Lecture 3), we have

$$\int_0^{\infty} \pi B_{\nu}(T) d\nu = \sigma_B T^4$$

Let's define the **isothermal broadband emissivity** as

$$\varepsilon^f(u, T) = \frac{\int_0^{\infty} \pi B_{\nu}(T)(1 - T_{\nu}^f(u)) d\nu}{\sigma_B T^4} \quad [11.10]$$

Using the **isothermal broadband emissivity**, Eq.[11.9a, b] may be approximated as

$$\boxed{F^{\uparrow}(u) \cong \sigma_B T_s^4 (1 - \varepsilon^f(u, T_s)) - \int_0^u \sigma_B T^4(u') \frac{d\varepsilon^f(u-u', T(u'))}{du} du} \quad [11.11a]$$

and

$$F^{\downarrow}(u) \cong \int_u^{u^*} \sigma_B T^4(u') \frac{d\varepsilon^f(u' - u, T(u'))}{du'} du' dv \quad [11.11b]$$

NOTE: If the **isothermal broadband emissivity** is known, the broadband fluxes and heating/cooling rates can be easily calculated from Eq.[11.11a, b].