<u>Homework Assignment 5</u>

Due: Nov.12

Problem 1 (30 points)

Rayleigh scattering for spherical particles is a limiting case of Mie scattering as the size parameter x -> 0. In this limit the scattered spherical wave Mie coefficients $\boldsymbol{a_n}$ and $\boldsymbol{b_n}$ (see Lecture 15) are all negligible except for a_1 which is

$$a_1 = \frac{2i}{3}\frac{m^2 - 1}{m^2 + 2}x^3$$

Using the Mie theory results (Lecture 15), derive the following quantities for Rayleigh scattering

a) scattering efficiency Q_s and the extinction efficiency Q_e;

b) scattering amplitudes $\mathbf{S_1(\Theta)}$ and $\mathbf{S_2(\Theta)}$**;**

c) scattering phase function $\mathbf{P(\Theta)}$**;**

d) Consider the case of a nonabsorbing sphere in this limit. What is the extinction efficiency derived above in this case? Is this result physical (compare with the scattering)? What might be the cause of this dilemma? In this limit of Mie theory what process does Q_e really measure?

Problem 2 (40 points)

Using link to run Mie code provided in Lecture 16, compute and analyze the spectral behavior of the extinction Q_e and absorption Q_a efficiencies. Consider the spectral region from 300 nm to 1 μm, and particle sizes of 0.1, 1 and 10 μm. Perform your analysis for three types of aerosols: non-absorbing (m = 1.5), absorbing (m =1.5-i0.01), and internal (50%/50%) mixing of these aerosols.

Problem 3 (10 Points)

Determine the effective optical depth, single scattering albedo and asymmetry parameter of an atmospheric layer containing water vapor and aerosol at a particular wavelength. At this wavelength mass absorption coefficient of water vapor is $k_\lambda = 0.03\ m^2/kg$ and aerosol volume extinction coefficient is β_λ = 0.03 km^{-1}, aerosol single scattering albedo is ω_o=0.9 and aerosol asymmetry parameter is g=0.75. The water vapor density is ρ_v = 10 g/m^3.

Problem 4 (40 points)

A volcanic eruption results in a layer of sulfuric acid aerosol droplets in the Earth's lower stratosphere. The layer is 5 km thick and has the particle size distribution characterized by a lognormal function with N = 100 cm^{-3} number concentration, r_o= 0.2 μm median radius and lnσ =0.5 standard deviation.

a) Calculate the effective radius of this size distribution. What is the size parameter of the effective radius for a wavelength of λ =10 μm?

b) Calculate the optical depth of the aerosol layer at λ =10 μm. The refractive index of sulfuric acid at this wavelength is m = 2.094-0.306i.

c) If the temperature of the stratospheric aerosol layer is 215 K and the surface/lower atmosphere at 10 μm radiates like a blackbody at 290 K, what is the fractional change in upwelling longwave flux at 10 μm due to the volcanic aerosols?