

Lecture 25.

Radiation and climate.

Objectives:

1. Planetary radiative equilibrium and radiative energy distribution.
2. Radiative and radiative-convective equilibrium.
3. Examples of simple energy balance models.

Appendix. Derivation of the Eddington gray radiative equilibrium.

Required reading:

L02: 8.3; 8.5; 8.6.1

Trenberth, K. E., J. T. Fasullo, and J. Kiehl, 2009: Earth's global energy budget. *Bull. Amer. Meteor. Soc.*, **90**, No. 3, 311-324, doi: 10.1175/2008BAMS2634.1.

1. Planetary radiative equilibrium and radiative energy distribution

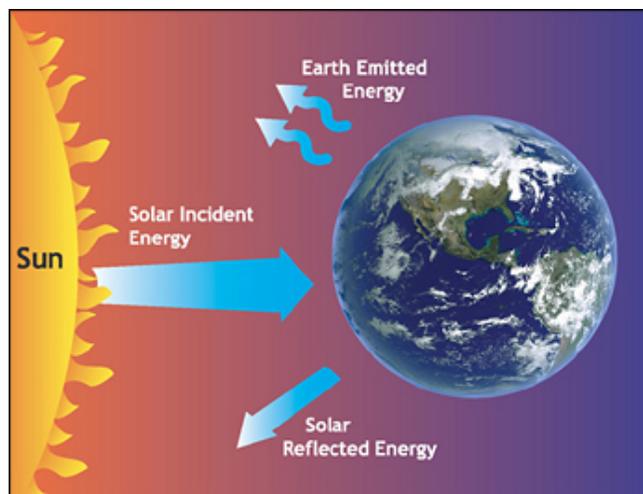
- ✓ Radiation is a key factor controlling the Earth's climate.
- ✓ Radiation equilibrium at the top of the atmosphere (TOA) represents the fundamental mode of the climate system.

Planetary radiative equilibrium (over the entire planet and long time interval):

$$\text{TOA incoming radiation} \equiv \text{TOA outgoing IR radiation}$$

TOA incoming radiation = incoming solar radiation – reflected solar radiation

TOA outgoing IR radiation = outgoing IR radiation (emitted by the atmosphere-surface system)



Global annual mean Earth's energy balance

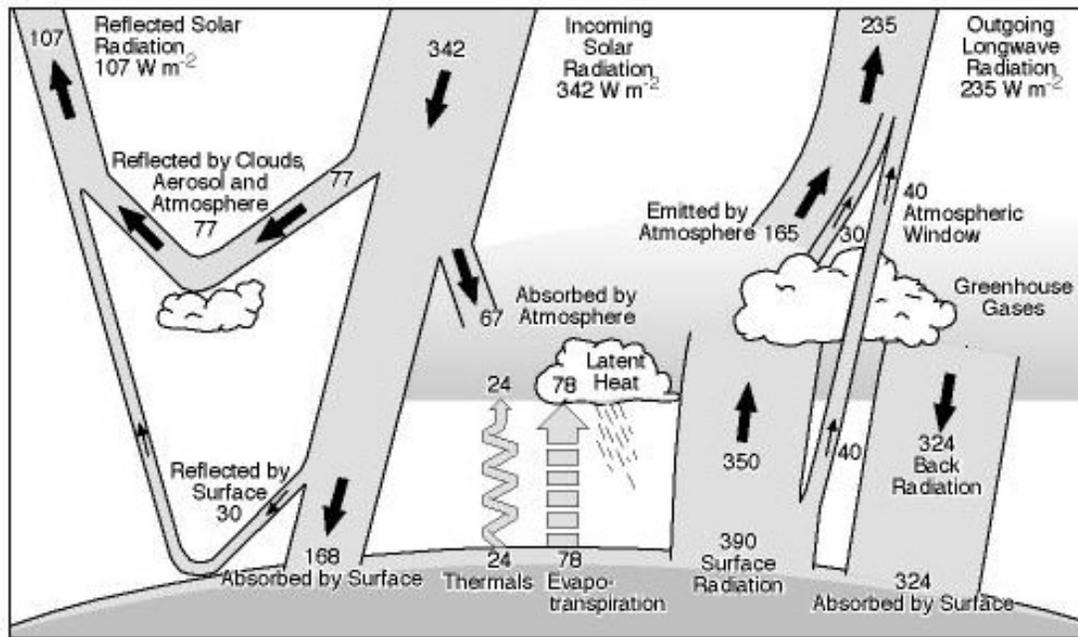


Figure 25.1 “Old” Earth’s energy balance diagram from Kiehl and Trenberth, 1997: Earth’s annual global mean energy budget. *Bull. Amer. Meteor. Soc.*, **78**, 197–208.

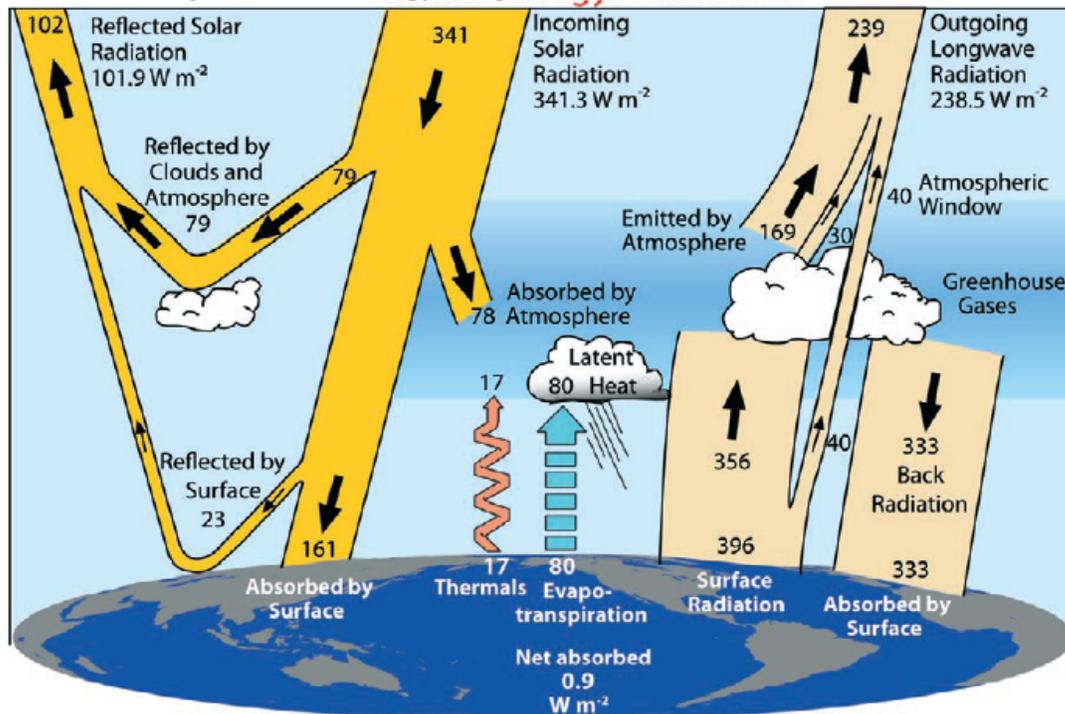


Figure 25.2 Earth’s energy balance diagram from Trenberth et al. (2009). The global annual mean Earth’s energy budget for the Mar 2000 to May 2004 period (W/m^2). The broad arrows indicate the schematic flow of energy in proportion to their importance.

Consider the radiation budget shown in Fig.25.1

What is TOA incoming solar radiation?

If F_0 is the solar constant (i.e., solar flux at the top of the atmosphere, W/m^2), then incoming solar radiation (per unit area of Earth's surface) is

$$F_0 \pi R_e^2 / 4\pi R_e^2 = F_0 / 4$$

where R_e is the radius of the Earth.

Assuming $F_0 = 1368 W/m^2$, incoming solar radiation is $1368 W/m^2 / 4 = \underline{342 W/m^2}$

What is reflected solar radiation?

Taking a global mean planetary albedo $\bar{r} = 0.31$:

$$\text{Reflected solar radiation} = \bar{r} F_0 / 4 = 107 \underline{W/m^2}$$

Table 25.1 TOA annual mean radiation budget quantities for the CERES period of Mar 2000 to May 2004 for global, global land, and global ocean from Trenberth et al. (2009). The downward solar (Solar in), reflected solar (Solar reflected), and net (NET down) radiation are given with the ASR and OLR (W/m^2), and albedo is given in percent. The values are from ISCCP-FD, the reanalyses NRA and JRA, and this paper.

Global	Solar in	Solar reflected	Albedo (%)	ASR	OLR	NET down
ISCCP-FD	341.7	105.2	30.8	236.5	235.6	0.9
NRA	341.8	117.0	34.2	224.5	237.8	-13.0
JRA	339.1	94.6	27.9	244.5	253.6	-9.1
This paper	341.3	101.9	29.8	239.4	238.5	0.9
Land						
ISCCP-FD	330.9	111.6	33.7	219.3	231.3	-12.0
NRA	330.6	116.4	35.2	214.2	234.7	-20.5
JRA	328.3	100.6	30.6	227.7	250.8	-23.1
This paper	330.2	113.4	34.4	216.8	232.4	-15.6
Ocean						
ISCCP-FD	345.7	102.9	29.8	242.8	237.2	5.6
NRA	345.9	117.3	33.9	228.7	238.9	-10.2
JRA	343.0	92.5	27.0	250.5	254.7	-4.2
This paper	345.4	97.8	28.3	247.7	240.8	6.9

Another view on Earth's energy balance:

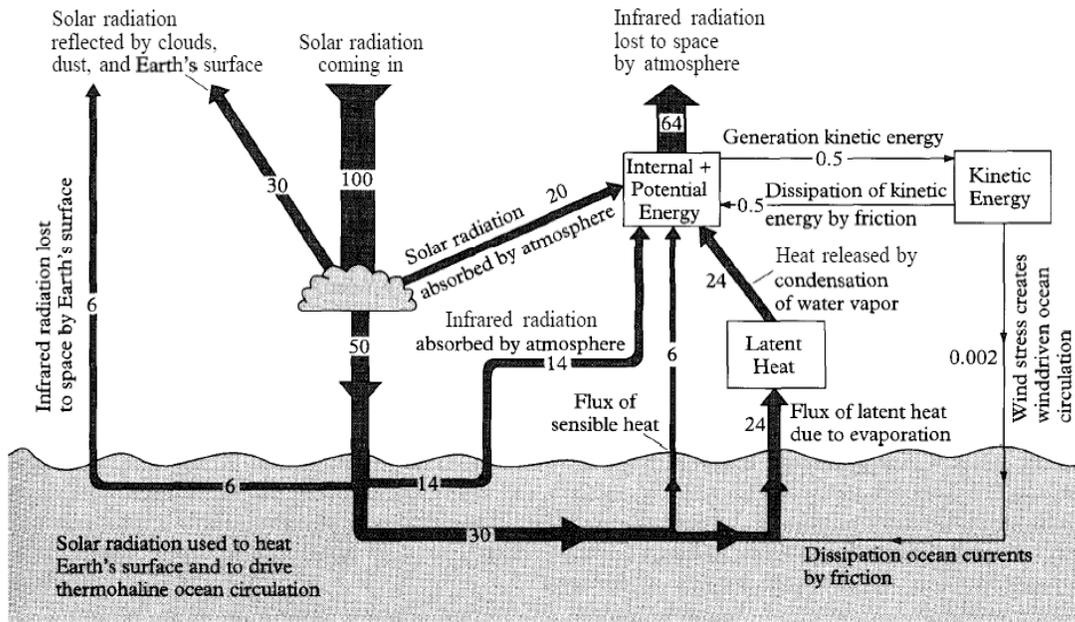


Figure 25.3 Earth's energy balance diagram from Piexoto and Oort (1992).

Zonal mean radiation budget:

When globally averaged over a year, net energy gains are balanced by energy losses, or nearly so. This is not the case when the radiation gains and losses are averaged as a function of latitude. Regions between approximately 30°N and 30°S gain radiant energy while the polar regions are losing energy through radiation processes. Transport of energy from the tropics to the poles is accomplished by the atmosphere and ocean currents and is the reason we have weather!

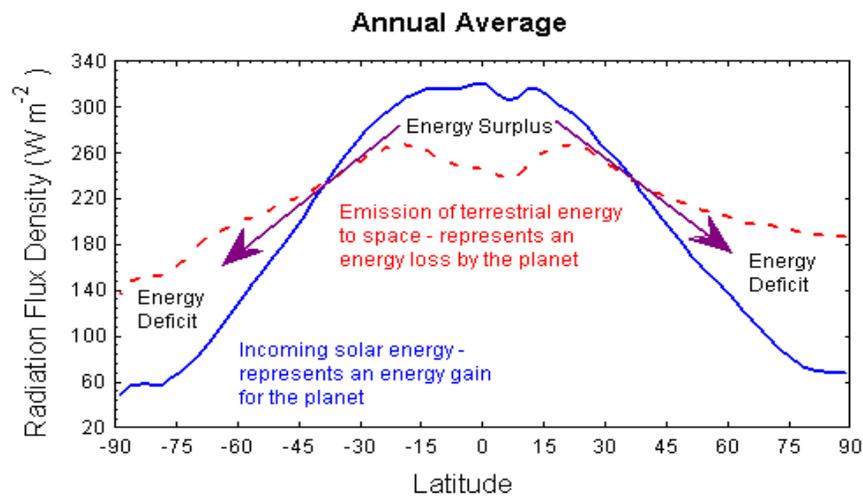


Figure 25.4 Schematic representation of zonal mean radiation budget.

Spatial and temporal distribution of radiation

✓ The **planetary albedo** is a key climate variable as it, combined with the solar insolation determines the radiative energy input to the planet. The albedo varies quite markedly with geographic region and time of year. Oceans have a low albedo, snow a high albedo. While the Northern Hemisphere has more land than the Southern Hemisphere, the annual average albedo of the two hemispheres is nearly the same, demonstrating the important influence of clouds in determining the albedo.

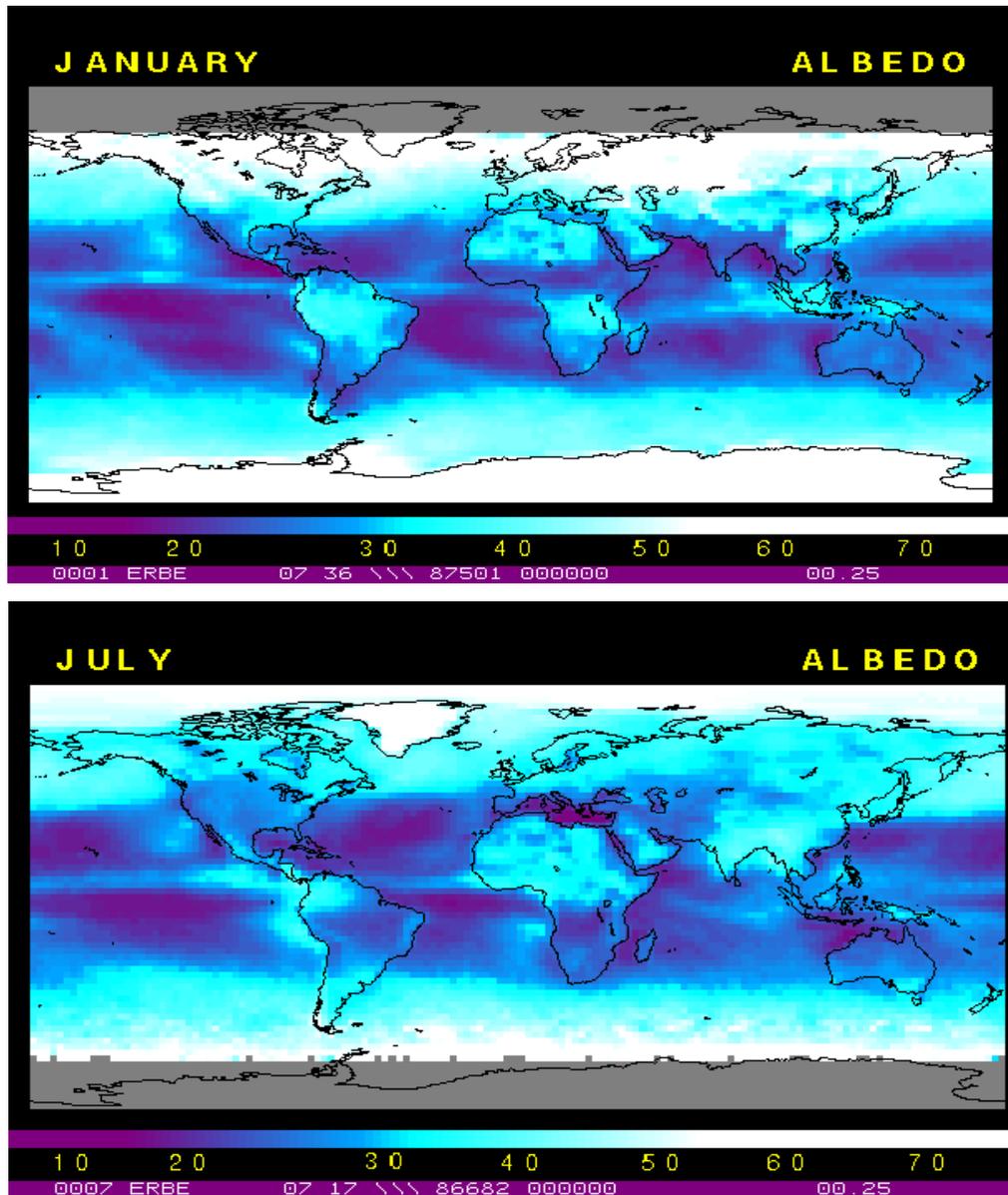


Figure 25.5 Examples of monthly mean albedo measured by the NASA Earth Radiation Budget Experiment (ERBE) satellite.

Outgoing longwave (infrared) radiation

Low values usually indicate cold temperatures while high values are warm areas of the globe. The minimum in OLR near the equator is due to the high cloud tops associated with the inter-tropical convergence zone. This minimum migrates about the equator as seen in the monthly mean maps, and is also seen as a maximum in albedo (see Figure 2.4). Notice how it is difficult to observe the oceanic stratus regions we observed in the albedo maps. This is because the temperature of the clouds is similar to the surrounding oceans, making it difficult to observe. Note how the major deserts have their largest OLR during their summer. Note also the large emission in the vicinity of the oceanic subtropical highs (30N and 30S).

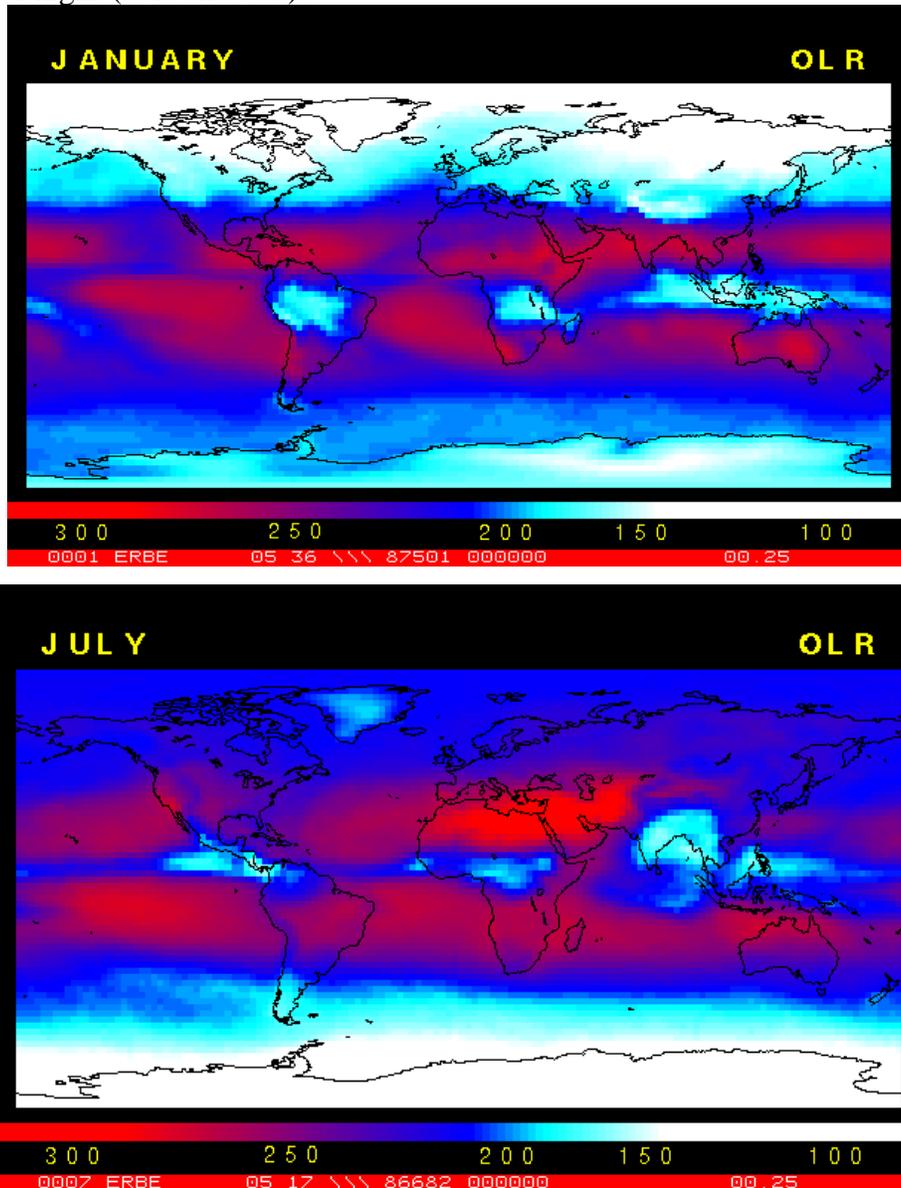


Figure 25.6 Examples of monthly mean outgoing longwave radiation (OLR) measured by the NASA Earth Radiation Budget Experiment (ERBE) satellite.

2. Radiative and radiative-convective equilibrium models.

A hierarchy of climate models:

- Climate models can be classified by their dimensions:

Zero Dimensional Models (0-D):

consider the Earth as a whole (no change by latitude, longitude, or height)

One Dimensional Models (1-D):

allow for variation in one direction only (e.g., resolve the Earth into latitudinal zones or by height above the surface of the Earth)

Two Dimensional Models (2-D):

allow for variation in two directions at once (e.g., by latitude and by height)

Three Dimensional Models (3-D)

allow for variation in three directions at once (i.e., divide the earth-atmosphere system into domains, each domain having its own independent set of values for each of the climate parameters used in the model.

- Climate models can be classified by the basic physical processes included into the consideration:

Energy Balance Models:

0-D or 1-D models (e.g., allow to change the albedo by latitude) calculate a balance between the incoming and outgoing radiation of the planet;

Radiative Convective Models:

1-D models to model the temperature profile the atmosphere by considering radiative and convective energy transport up through the atmosphere.

General Circulation Climate Models:

2-D (longitude-averaged) or 3-D climate models solve a series of equations and have the potential to model the atmosphere very closely.

Radiative and radiative-convective equilibrium models.

In a real atmosphere, solar heating rates do not equal to IR cooling rates. This imbalance is the key driver of atmospheric dynamics.

Let's consider a hypothetical motionless atmosphere, radiative transfer processes only. Then the climate state (temperature profile) is determined by the radiative equilibrium. The radiative equilibrium climate model is a model that predicts the atmosphere

temperature profile of an atmosphere in radiative equilibrium $\frac{dF_{net}}{dz} = 0$

- ✓ Under the “gray atmosphere” assumption, we can solve for the temperature profile analytically.

Eddington gray radiative equilibrium results: (see Appendix for the full derivation).

Assumptions:

- 1) Radiative equilibrium: $\frac{dF_{net}}{dz} = 0$
- 2) Gray atmosphere in longwave
- 3) No scattering and black surface in longwave
- 4) No solar absorption in the atmosphere
- 5) Eddington approximation: $I(\mu) = I_0 + I_1\mu$

Longwave flux profile:

$$F^\uparrow(\tau) = F_{sun} \left(1 + \frac{3}{4}\tau\right) \quad \text{and} \quad F^\downarrow(\tau) = F_{sun} \left(\frac{3}{4}\tau\right) \quad [25.1]$$

where $F_{sun} = (1 - \bar{r})F_0 / 4$

Atmosphere blackbody emission and temperature profiles:

$$B(\tau) = \frac{F_{sun}}{2\pi} \left(1 + \frac{3}{2}\tau\right) \quad \text{and} \quad T^4(\tau) = T_e^4 \left(\frac{1}{2} + \frac{3}{4}\tau\right) \quad [25.2]$$

Surface temperature is discontinuous with the atmosphere (hotter):

$$B_s = B(\tau^*) + \frac{F_{sun}}{2\pi} \quad \text{and} \quad T_s^4 = T_e^4 \left(1 + \frac{3}{4}\tau^*\right) \quad [25.3]$$

Implications:

- ✓ Greenhouse effect – larger τ^* increases surface temperature
- ✓ Runaway greenhouse effect - τ^* increases $\Rightarrow T_s$ increases
- ✓ Positive feedback – higher temperature \Rightarrow greenhouse gases

Eddington gray radiative equilibrium temperatures

If one wants to have the temperature profile in terms of height, one needs to relate optical depth to height.

Assume that an absorber has the exponential profile

$$\rho_a = \rho_0 \exp(-z/H_a) \quad [25.4]$$

So the profile of optical depth is

$$\tau(z) = \bar{k}_a \int_z^\infty \rho_a(z) dz = \bar{k}_a \rho_0 H_a \exp(-z/H_a) = \tau^* \exp(-z/H_a) \quad [25.5]$$

Temperature profile

$$T^4(z) = T_e^4 \left(1 + \frac{3}{4} \tau^* \exp(-z/H_a)\right) \quad [25.6]$$

Lapse rate

$$\frac{dT}{dz}(z) = -\frac{3}{8} \frac{\tau^*}{1 + \frac{3}{2} \tau^*} \frac{T(z)}{H_a} \exp(-z/H_a) \quad [25.7]$$

Implications:

- ✓ Low τ^* \Rightarrow stable atmosphere
- ✓ Smaller scale height H_a of the absorber causes steeper lapse rate
- ✓ Steepest lapse rate near the surface ($z=0$)

Radiative equilibrium models:

- Radiative equilibrium climate models solve for the vertical profile of temperature using accurate broadband radiative transfer models.
- Model inputs vertical profile of gases, aerosols and clouds. Iterates the temperature profile to archive equilibrium (i.e., zero heating rates or $\frac{dF_{net}}{dz} = 0$)
- Climate feedbacks can be included by having water vapor, surface albedo, clouds, etc. depend on temperature.

Solving for radiative equilibrium:

Iterate the temperature profile $T(z)$ to get zero heating rates $\frac{\partial T}{\partial t} = 0$

1. ***Time marching method:***

T at t+1 time step from heating rate at time t:

$$T^{t+1}(z_k) = T(z_k) + \left(\frac{\partial T(z_k)}{\partial t} \right)^t \Delta t \quad [25.8]$$

2. ***Direct solve:***

Use gradient information in nonlinear root solver (faster, but more complex than time marching)

Radiative equilibrium temperature profiles show (see Figure 25.7 below):

- ✓ CO₂ –only-atmosphere has less steep profile.
- ✓ Earth’s stratosphere warms due to UV absorption by ozone.
- ✓ Most greenhouse effect from water vapor.

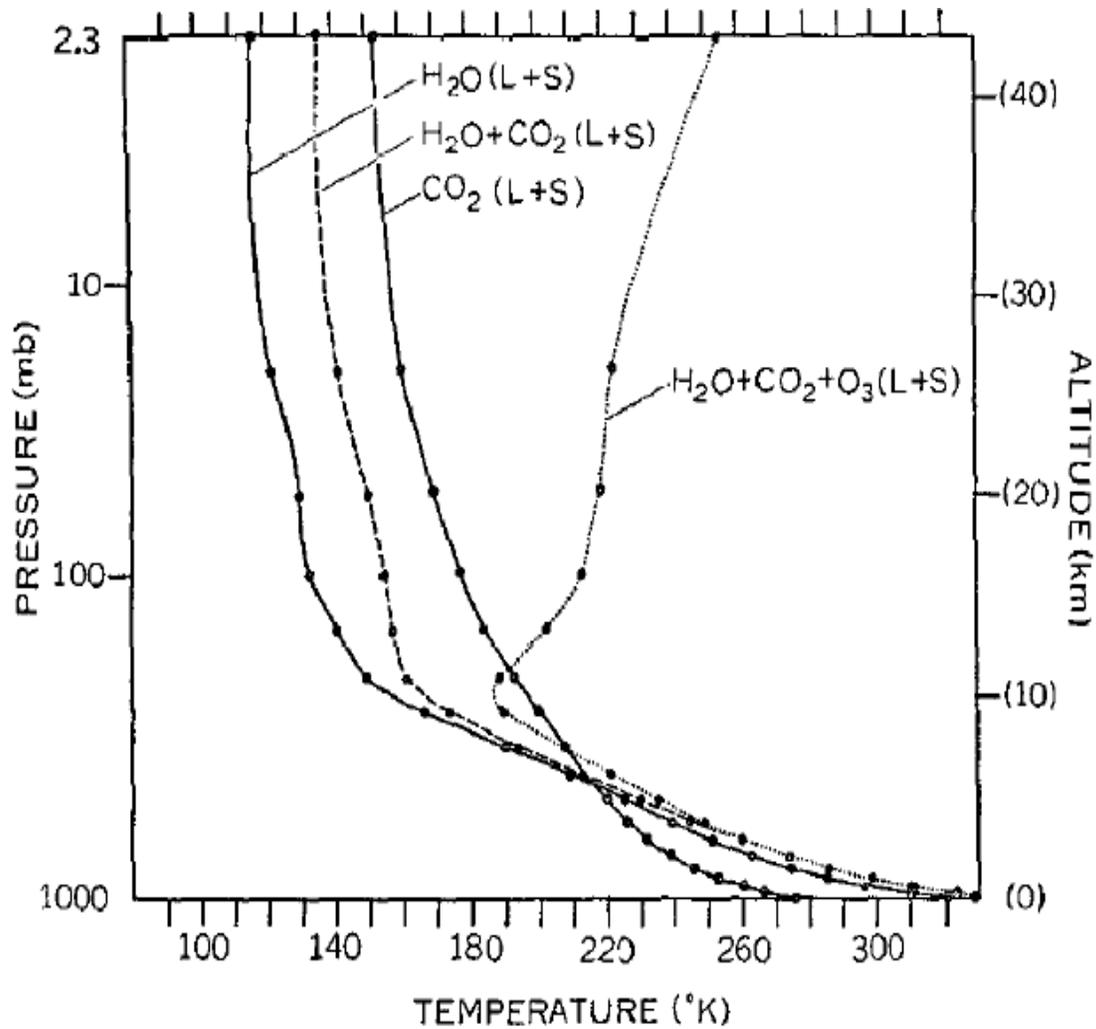


Figure 25.7 Pure radiative equilibrium temperature profiles for various atmospheric gases in a clear sky at 35 N in April. L+S means that the effects of both longwave and shortwave radiation are included (from Manabe and Strickler, 1964).

Results: the radiative equilibrium surface temperature is too high and the temperature profile is unrealistic.

Problem: radiative equilibrium surface temperature lapse rate near the surface exceeds threshold for convection

Fix: assume convection limits lapse rates to $< \gamma_c$ (e.g., 6.5 K/km)

- Radiative-convective equilibrium is equilibrium of radiative and convective fluxes

Convective adjustment methods:

1. Move heat like convection: if γ_c exceeded, adjust temperature so γ_c achieved and heat is conserved
2. Parameterize convective flux, e.g.

$$F_{conv} = C \left(\left| \frac{dT}{dz} \right| - \gamma_c \right) \quad \text{if} \quad \left(\left| \frac{dT}{dz} \right| - \gamma_c \right) > 0$$

Results of the RCE model developed by Manabe and Strickler (1964):

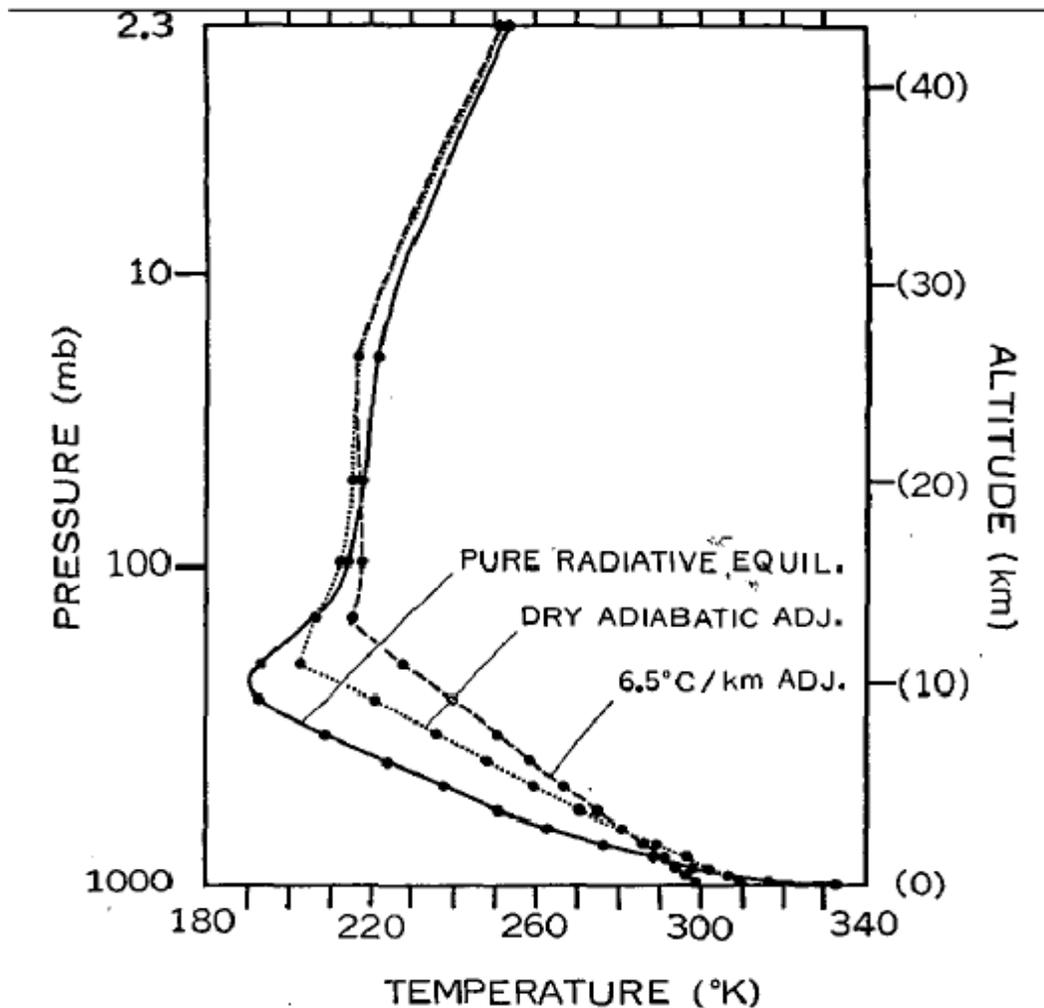


Figure 25.8 Pure radiative equilibrium and radiative-convective equilibrium temperature profiles for two values of γ_c for clear sky.

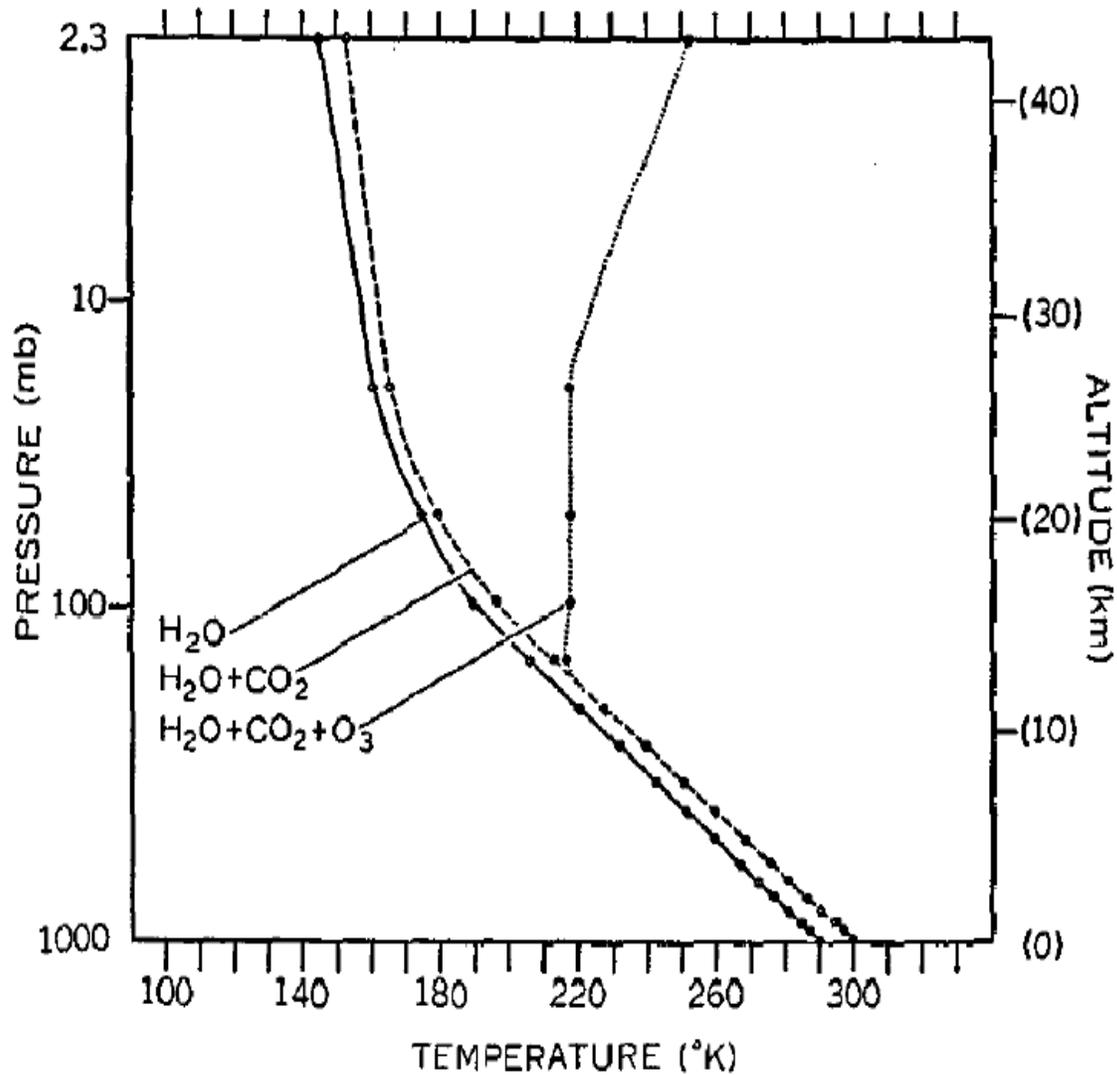


Figure 25.9 Radiative-convective equilibrium temperature profiles for various atmospheric gases in a clear sky at 35 N in April.

Comparing figures 25.7 and 25.9:

- ✓ Radiative equilibrium is fairly accurate for the stratosphere (though latitudinal and seasonal dependence is not correct)
- ✓ Convection required for to get reasonable tropospheric temperatures

3. Examples of simple energy balance models.

Let's estimate the **effective temperature** assuming that the Earth is in the radiative equilibrium. The sun emits $F_s = 6.2 \times 10^7 \text{ W/m}^2$ (a blackbody with about $T = 5800\text{K}$).

From the energy conservation law, we have

$$F_s 4\pi R_s^2 = F_{s0} 4\pi D_0^2$$

where R_s is the radius of the sun ($6.96 \times 10^5 \text{ km}$);

F_{s0} is the solar flux reaching the top of the atmosphere (called the **solar constant** = about 1368 W/m^2) at the average distance of the Earth from the sun, $D_0 = 1.5 \times 10^8 \text{ km}$.

$$\text{Thus we have } F_{s0} = F_s R_s^2 / D_0^2$$

If the instantaneous distance from the Earth to sun is D , then the total sun energy flux F_0 reaching the Earth is

$$F_0 = F_{s0} (D_0/D)^2$$

The total sun energy intercepted by the cross section of the Earth is $F_{s0} \pi R_e^2$ where R_e is the radius of the Earth. This energy is spread uniformly over the entire planet (with surface area $4\pi R_e^2$). Thus the amount of received energy per unit surface becomes

$$F_{s0} \pi R_e^2 / 4\pi R_e^2 = F_{s0} / 4$$

Therefore, the total energy Q_{in} (in W/m^2) absorbed by the earth-atmosphere system is:

$$Q_{in} = (1 - \bar{r}) F_{s0} / 4$$

where \bar{r} is the spherical (or global) albedo. Spherical albedo of the earth is about 0.3.

Assuming that the Earth is a blackbody with temperature T_e , we have:

$$Q_{out} = F_b = \sigma_B T_e^4$$

where σ_B is the Stefan-Boltzmann constant.

From the balance of incoming and outgoing energy, the **effective temperature** of the Earth is:

$$Q_{in} = Q_{out}$$

$$F_{s0} (1 - \bar{r}) / 4 = \sigma_B T_e^4$$

$$T_e^4 = F_{s0} (1 - \bar{r}) / 4 \sigma_B$$

$$T_e = 255 \text{ K} = -18^0\text{C is very low!!!}$$

T_e is much lower than the global average surface temperature (about 288 K)

Why? Because we didn't include the greenhouse effect and ignore the temperature structure.

Table 25.2 Effective temperatures of some planets in the radiative equilibrium.

Planet	Relative distance to the sun with respect to the Earth	Global albedo	T_e (K)
Mercury	0.39	0.06	441
Venus	0.72	0.78	226
Earth	1	0.3	255
Mars	1.52	0.17	217
Jupiter	5.2	0.45	106

How can we measure the greenhouse effect?

Upwelling flux at the surface

$$F_s^\uparrow = \sigma T_s^4$$

Upwelling flux F_{TOA}^\uparrow at the top of the atmosphere (TOA): from satellite observations

The difference between the upwelling fluxes at the surface and TOA gives a measure of greenhouse effect G:

$$G = \sigma T_s^4 - F_{TOA}^\uparrow \quad [25.9]$$

NOTE: G is the amount of heat (e.g., measured in Watts) per unit area of the Earth.

What is a reasonable estimate of the greenhouse effect?

$$F_{TOA}^\uparrow = 235 \text{ W/m}^2$$

$$T_s = 288 \text{ K}$$

$$\rightarrow G = 390 - 235 = 155 \text{ W/m}^2$$

$$\sigma T_s^4 = 390 \text{ W/m}^2$$

Simple model of greenhouse effect #1: single layer gray energy balance model:

Let's include the atmosphere assuming that it emits (absorbs) as a gray body. Assume that the atmosphere does not absorb solar radiation - all is absorbed at the surface.

TOA balance:
$$\frac{F_0}{4}(1 - \bar{r}) = (1 - \varepsilon)\sigma T_s^4 + \varepsilon\sigma T_a^4 \quad [25.10]$$

Surface balance:
$$\frac{F_0}{4}(1 - \bar{r}) + \varepsilon\sigma T_a^4 = \sigma T_s^4 \quad [25.11]$$

where σ is the Stefan-Boltzmann constant and ε is the emissivity of the atmosphere.

↓

$$T_s^4 = \frac{F_0(1 - \bar{r})}{2\sigma(2 - \varepsilon)} \quad [25.12]$$

for $\varepsilon = 0.6 \Rightarrow T_s = 278 \text{ K}$

NOTE: increasing for ε increases T_s . This is so-called “**runaway greenhouse effect**” : warmer $T_s \Rightarrow$ more evaporation \Rightarrow more water vapor \Rightarrow higher emissivity \Rightarrow warmer T_s
Why? Because we assume that atmosphere is a gray body.

Simple model of greenhouse effect #2: single black layer with spectral window energy balance model:

A more realistic way to deal with partial longwave transparency of the atmosphere is to assume that a fraction of the spectrum is clear.

Assume that

$f = \frac{1}{\sigma T_a^4} \int_{\nu_1}^{\nu_2} B_\nu(T) d\nu$ is the fraction of LW spectrum which is completely transparent

and the remainder of the LW spectrum is black. Surface is black and no SW absorption by the atmosphere.

TOA balance:
$$\frac{F_0}{4}(1 - \bar{r}) = f\sigma T_s^4 + (1 - f)\sigma T_a^4 \quad [25.13]$$

Surface balance:
$$\frac{F_0}{4}(1 - \bar{r}) + (1 - f)\sigma T_a^4 = \sigma T_s^4 \quad [25.14]$$

Thus we can express T_s and T_a via T_e

$$T_s = \left(\frac{2}{1+f} \right)^{1/4} T_e \quad \text{and} \quad T_a = \left(\frac{1}{1+f} \right)^{1/4} T_e \quad [25.15]$$

Limits:

No window $f=0 \Rightarrow T_s = 2^{1/4} T_e$ and $T_a = T_e$

All window $f=1 \Rightarrow T_s = T_e$ and $T_a = T_e / 2^{1/4}$

Earth: f is about 0.3 $\Rightarrow T_s = 284 \text{ K}$ and $T_a = 239 \text{ K}$

How to make the model more realistic:

Tropics: radiation excess

North poles and high latitudes: radiation deficit



must include poleward transport of energy

One-dimensional (latitude) energy balance model:

(Budyko 1969; Sellers 1969, Cess 1976)

Atmosphere is only implicit: TOA outgoing longwave flux is parameterized as a function of T_s

Budyko's parameterization is based on monthly mean atmospheric temperature and humidity profiles, and cloud cover observed at 260 stations

$$F_{LW}(x) = a_1 + b_1 T_s(x) - [a_2 + b_2 T(x)]\eta \quad [25.16]$$

where a_i and b_i are the empirical constants based on statistical fitting, $x = \sin(\phi)$ and ϕ is latitude. If cloud cover is taken constant of 0.5 then

$$F_{LW}(x) = (1.55 \text{ W} / \text{m}^2 / \text{K}) T_s(x) - 212 \text{ W} / \text{m}^2 \quad [25.17]$$

NOTE: The approximation for linear relation between OLR and the surface temperature may be argued from the fact that the temperature profiles have more or less the same shape at all latitudes, and that OLR, which depend on temperatures at all levels, may be expressed as a function of surface temperature.

Annual mean TOA solar insolation fit well with

$$S(x) = F_0 / 4[1 - 0.482P_2(x)] \quad [25.18]$$

$P_2(x) = (3x^2-1)/2$ is the second Legendre polynomial.

Thus energy balance equilibrium ($\delta T/\delta t=0$) with diffuse transport:

$$-D \frac{\partial}{\partial x} (1-x^2) \frac{\partial F_{LW}}{\partial x} + F_{LW} = S(x)[1 - r(x)] \quad [25.19]$$

where D is diffusion coefficient for energy transport and $r(x)$ is albedo.

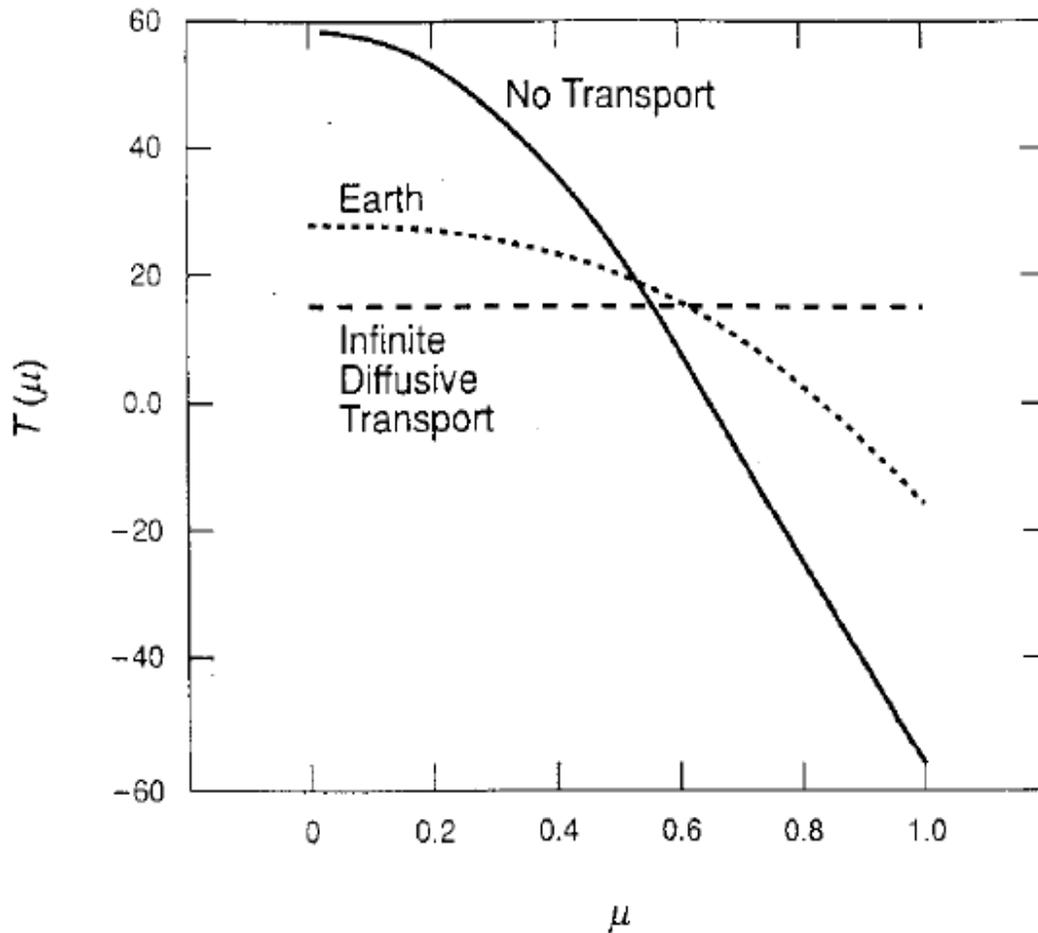


Figure 24.4 Zonally average surface temperature (K) as a function of the sine of the latitude, μ (μ is same as x in [25.16]), observed and for cases of no horizontal heat transport, and infinite horizontal heat transport (North et al., 1981).

NOTE: with no meridional transport, the poles are way too cold!

Appendix. Derivation of the Eddington gray radiative equilibrium.

Assumptions:

- 6) Radiative equilibrium: $\frac{dF_{net}}{dz} = 0$
- 7) Gray atmosphere in longwave
- 8) No scattering and black surface in longwave
- 9) No solar absorption in the atmosphere
- 10) Eddington approximation: $I(\mu) = I_0 + I_1\mu$

Since the atmosphere is gray (all wavelength are equivalent), one can write the wavelength integrated thermal emission radiative transfer equation (no scattering)

$$\mu \frac{dI}{d\tau} = I - B$$

where I is the integrated radiance ($\text{W m}^{-2} \text{sr}^{-1}$), τ increases downward, and $\mu > 0$ in the upward direction. Note that deriving the variation of B with the optical depth τ is equivalent to determining the temperature profiles since the blackbody emission is a function of temperature only.

Using the Eddington approximation, the net flux (positive upward) becomes

$$F_{net} = 2\pi \int_{-1}^1 I\mu d\mu = \frac{4\pi}{3} I_1$$

The radiative equilibrium assumption implies that F_{net} (and I_1) is constant with optical depth.

Integrating the above radiative transfer equation over $d\mu$ gives

$$2\pi \frac{d}{d\tau} \int_{-1}^1 I\mu d\mu = 2\pi \int_{-1}^1 Id\mu - 2\pi \int_{-1}^1 Bd\mu$$
$$\frac{dF_{net}}{d\tau} = 4\pi I_0 - 4\pi B$$

Under the radiative equilibrium assumption, we have

$$I_0 = B$$

Integrating the radiative transfer equation over $\mu d\mu$ gives

$$2\pi \frac{d}{d\tau} \int_{-1}^1 I \mu^2 d\mu = 2\pi \int_{-1}^1 I \mu d\mu - 2\pi \int_{-1}^1 B \mu d\mu$$

Since B is isotropic the last term drops out leaving

$$\frac{4\pi}{3} \frac{dI_0}{d\tau} = F_{net} = \frac{4\pi}{3} I_1$$

$$\frac{dB}{d\tau} = I_1$$

Thus, the solution for B is simply a linear function of optical depth:

$$B(\tau) = B(0) + I_1 \tau$$

Constants B(0) and I_1 need to be determined from the boundary conditions.

Top of the atmosphere:

First boundary condition: no thermal downwelling flux

$$F^\downarrow(0) = 2\pi \int_{-1}^0 I \mu d\mu = \pi B(0) - \frac{2\pi}{3} I_1 = 0$$

so we have

$$I_1 = \frac{3}{2} B(0) \quad \text{or} \quad F_{net} = 2\pi B(0)$$

Second boundary condition: upwelling longwave flux is equal to the absorbed solar flux F_{sun} :

$$F^\uparrow(0) = 2\pi \int_0^1 I \mu d\mu = \pi B(0) + \frac{2\pi}{3} I_1 = F_{sun}$$

(Recall that the absorbed solar flux F_{sun} is $F_{sun} = (1 - \bar{r}) F_0 / 4$)

Putting in $I_1 = \frac{3}{2} B(0)$ gives

$$F_{sun} = 2\pi B(0) = F_{net}$$

So now we have the B(0) and I_1 and thus the atmosphere Planck function profile is determined

$$B(\tau) = \frac{F_{sun}}{2\pi} \left(1 + \frac{3}{2} \tau\right)$$

The final step is to apply the boundary condition at the surface to obtain the surface temperature T_s . This boundary condition is that the emitted flux by the surface equals to the sum of the downwelling shortwave and longwave flux at the black surface:

$$F_{sun} + F^\downarrow(\tau^*) = \pi B_s$$

where $F^\downarrow(\tau^*) = \pi B(\tau^*) - \frac{2\pi}{3} I_1$

Using $F_{sun} = \frac{4\pi}{3} I_1$ gives the emission from the surface

$$B_s = B(\tau^*) + \frac{F_{sun}}{2\pi}$$

which is discontinuous with the atmospheric emission.

The previous results can be expressed in terms of temperature by

$$T^4(\tau) = T_e^4 \left(\frac{1}{2} + \frac{3}{4} \tau \right)$$

where $\sigma T_e^4 = F_{sun}$

$$T_{top}^4 = \frac{1}{2} T_e^4$$

$$T_s^4 = T_e^4 \left(1 + \frac{3}{4} \tau^* \right)$$

For $F_0 = 1366 \text{ W/m}^2$ and $\bar{r} = 0.3$:

$T_e = 255 \text{ K}$ and a “top” temperature $T_t = 214 \text{ K}$

Assuming a global averaged surface air temperature of $T(\tau^*) = 288 \text{ K}$ gives a gray body optical depth of $\tau^* = 1.5$, and a surface skin temperature of $T_s = 308 \text{ K}$