Lecture 3.

Blackbody radiation. Main radiation laws.

Sun as an energy source. Solar spectrum and solar constant.

Objectives:

1. Concepts of a blackbody, thermodynamical equilibrium, and local thermodynamical equilibrium.
2. Main radiation laws: Planck function. Stefan-Boltzmann law. Wien's displacement law. Kirchhoff's law.
3. Sun as an energy source.

Required reading:

L02: 1.2, 1.4.3, Appendix A; 2

Additional reading:

Harder, J. W., J. M. Fontenla, P. Pilewskie, E. C. Richard, and T. N. Woods (2009), Trends in solar spectral irradiance variability in the visible and infrared, *Geophys. Res. Lett.*, 36, L07801, doi:10.1029/2008GL036797.

1. Concepts of a blackbody and thermodynamical equilibrium.

Thermodynamical equilibrium describes the state of matter and radiation inside an isolated constant-temperature enclosure.

Blackbody is a perfect absorber (emitter) of radiation.

***Properties of blackbody radiation*:**

- Radiation emitted by a blackbody is isotropic, homogeneous and unpolarized;
- Blackbody radiation at a given wavelength depends only on the temperature;
- Any two blackbodies at the same temperature emit precisely the same radiation;
- A blackbody emits more radiation than any other type of an object at the same temperature;

NOTE: The atmosphere is not strictly in the thermodynamic equilibrium because its temperature and pressure are functions of position. Therefore, it is usually subdivided into small subsystems each of which is effectively isothermal and isobaric referred to as **Local Thermodynamical Equilibrium (LTE).**

2. Main radiation laws.

Planck function B_λ(T) gives the **intensity (or radiance)** emitted by a blackbody having temperature T.

NOTE: Distribution of blackbody radiation as a function of wavelength, known as the Planck law, cannot be predicted using classical physics. This derivation requires quantum mechanics. See L02, Appendix A, for the derivation of the Plank function.

Plank function can be expressed in wavelength, frequency, or wavenumber domains as

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5(\exp(hc/k_B T\lambda) - 1)} \quad [3.1]$$

$$B_{\tilde{\nu}}(T) = \frac{2h\tilde{\nu}^3}{c^2(\exp(h\tilde{\nu}/k_B T) - 1)} \quad [3.2]$$

$$B_\nu(T) = \frac{2h\nu^3 c^2}{\exp(h\nu c/k_B T) - 1} \quad [3.3]$$

where λ is the wavelength; $\tilde{\nu}$ is the frequency; ν is the wavenumber;
h is the Plank's constant; $\boldsymbol{k_B}$ is the Boltzmann's constant ($k_B = 1.3806 \times 10^{-23}$ J K^{-1});
c is the speed of light; and ***T*** is the absolute temperature of a blackbody.

NOTE: The relations between $B_{\tilde{\nu}}(T)$; $B_\nu(T)$ and $B_\lambda(T)$ are derived using that

$B_{\tilde{\nu}}(T)d\tilde{\nu} = B_\nu(T)d\nu = B_\lambda(T)d\lambda$, and that $\lambda = c/\tilde{\nu} = 1/\nu$ =>

$$B_{\tilde{\nu}}(T) = \frac{\lambda^2}{c} B_\lambda(T) \quad \text{and} \quad B_\nu(T) = \lambda^2 B_\lambda(T)$$

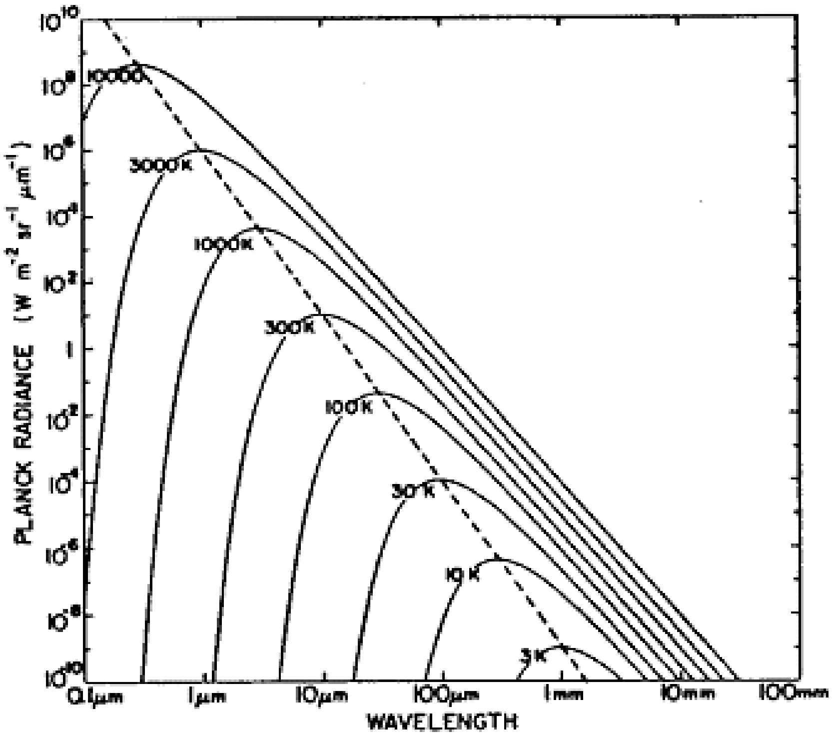


Figure 3.1 Planck function on log-log plot for several temperatures.

Asymptotic behavior of Planck function:

- **If λ $\rightarrow \infty$ (or $\tilde{\nu}$ -> 0) (known as Rayleigh –Jeans distributions):**

$$B_\lambda(T) = \frac{2k_B Tc}{\lambda^4} \quad [3.4]$$

$$B_{\tilde{\nu}} = \frac{2k_B T\tilde{\nu}^2}{c^2} \quad [3.5]$$

NOTE: This longwave limit has a direct application to passive **microwave** remote sensing.

- **If λ -> 0 (or $\tilde{\nu}$ $\rightarrow \infty$):**

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\exp(-hc/\lambda k_B T) \quad [3.6]$$

$$B_{\tilde{\nu}} = \frac{2h\tilde{\nu}^3}{c^2}\exp(-h\tilde{\nu}/k_B T) \quad [3.7]$$

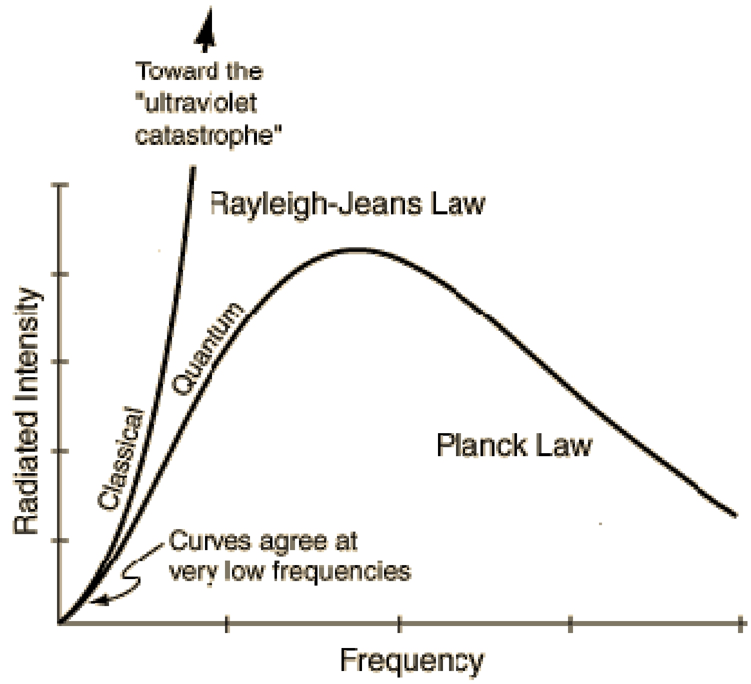


Figure 3.2 Plank function and its asymptotic behavior.

Stefan-Boltzmann law:

The total power (energy per unit time) emitted by a **blackbody,** per unit surface area of the **blackbody**, varies as the fourth power of the temperature.

$$\mathbf{F = \pi B(T) = \sigma_b T^4} \qquad [3.8]$$

where σ_b is the *Stefan-Boltzmann constant* (σ_b = 5.671x10^{-8} W m^{-2} K^{-4}),

F is the radiant flux [W m^{-2}], and **T** is blackbody temperature [K]; and

$$B(T) = \int_0^{\infty} B_\lambda(T)\, d\lambda$$

Wien's displacement law:

The wavelength at which the blackbody emission spectrum is most intense varies inversely with the blackbody's temperature. The constant of proportionality is Wien's constant (2897 K μm):

$$\boxed{\lambda_m = 2897 / T} \qquad [3.9]$$

where λ_m is the wavelength (in micrometers, μm) at which the peak emission intensity occurs, and T is the temperature of the blackbody (in degrees Kelvin, K).

NOTE: This law is derived from $dB_\lambda/d\lambda = 0$

NOTE: Easy to remember statement of the Wien's displacement law:

the hotter the object the shorter the wavelengths of the maximum intensity emitted

Kirchhoff's law:

The emissivity, ε_λ, of a medium is equal to the absorptivity, A_λ, of this medium under thermodynamic equilibrium:

$$\boxed{\varepsilon_\lambda = A_\lambda} \qquad [3.10]$$

where ε_λ is defined as the ratio of the emitted intensity to the Planck function;

A_λ is defined as the ratio of the absorbed intensity to the Planck function.

For a **blackbody**: $\varepsilon_\lambda = A_\lambda = 1$ For a **non-blackbody**: $\varepsilon_\lambda = A_\lambda < 1$

For a **gray body** (i.e., no dependency on the wavelength): $\varepsilon = A < 1$

NOTE: The **Kirchhoff's law** applies to gases, liquids and solids if they in TE or LTE.

- For atmospheric radiation transfer applications, one needs to distinguish between the **emissivity of the surface** (e.g., various types of lands, ice, etc.) and the **emissivity of an atmospheric volume** (consisting of gases, aerosols, and/or clouds).

- **<u>Emissivity of an atmospheric volume:</u>**

Absorption and thermal emission of the atmosphere volume <u>is **isotropic**</u>.
Kirchhoff's law applied to volume thermal emission gives

$$j_{\lambda,thermal} = \beta_{a,\lambda} B_\lambda(T) \qquad [3.11]$$

where $\beta_{a,\lambda}$ is the absorption coefficient of the atmospheric volume and

j_λ is the **thermal emission coefficient** which relates to **the source function** J_λ (introduced in Lecture 2) as $J_\lambda = (j_{\lambda,\,thermal} + j_{\lambda,\,scattering}) / \beta_{e,\lambda}$

and $\beta_{e,\lambda}$ is the extinction coefficient of the atmospheric volume.

Recall the elementary solution of the radiative transfer equation (Eq.[2.13], Lecture 2):

$$I_\lambda(s_1) = I_\lambda(0)\exp(-\tau_\lambda(s_1;0)) + \int_0^{s_1} \exp(-\tau_\lambda(s_1;s)) J_\lambda \beta_{e,\lambda} ds$$

For a **non-scattering medium in the thermodynamical equilibrium:**

$J_\lambda = B_\lambda(T)$, where $B_\lambda(T)$ is Plank's function.

Also, for the non-scattering medium, we have $\beta_{e,\lambda} = \beta_{a,\lambda} = k_{a,\lambda}\rho$, where $k_{a,\lambda}$ is the mass absorption coefficient and ρ is the density (see Lecture 2). Thus, the **solution** of the **equation radiative transfer** for this case can be expressed as

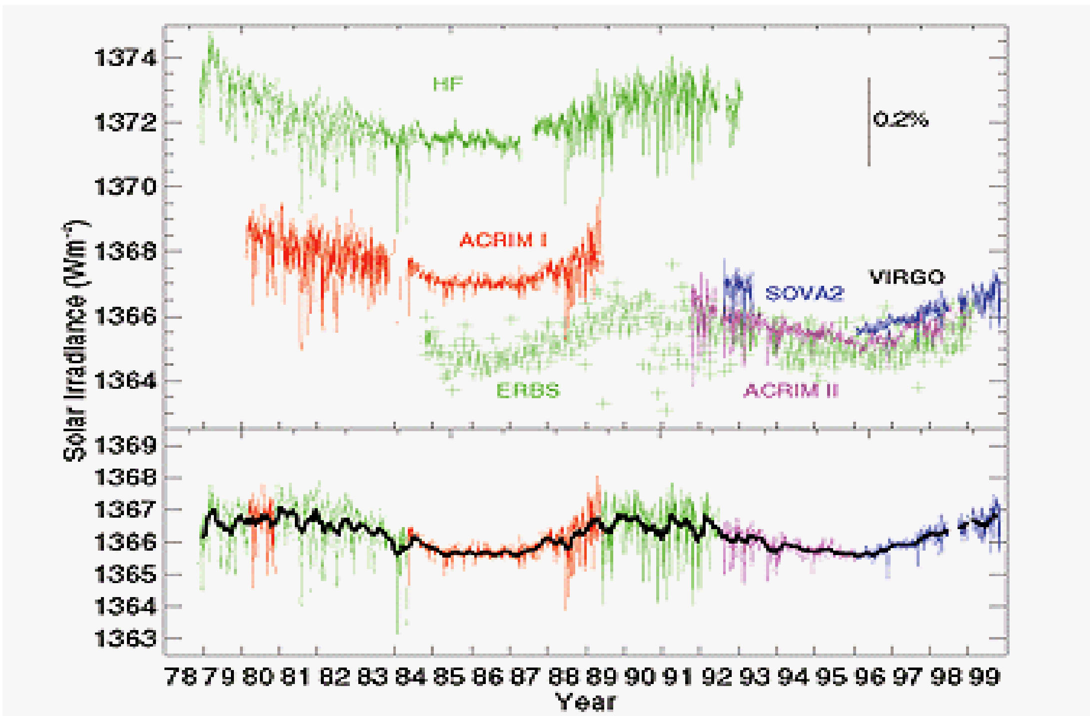
$$\boxed{I_\lambda(s_1) = I_\lambda(0)\exp(-\tau_\lambda(s_1;0)) + \int_0^{s_1} \exp(-\tau_\lambda(s_1;s)) B_\lambda(T(s)) k_{a,\lambda} \rho ds} \qquad [3.12]$$

NOTE: The optical depth in Eq.[3.12] is due to absorption only, so

$$\tau_\lambda(s_1;s) = \int_s^{s_1} \beta_{a,\lambda}(s) ds = \int_s^{s_1} k_{a,\lambda} \rho ds$$

4. Sun as an energy source.

Solar constant is total radiation emitted by the Sun reaching the top of the Earth's atmosphere. Not a constant, but it varies as a function of several parameters (including sun activity, sun spots, distance between Sun and Earth).



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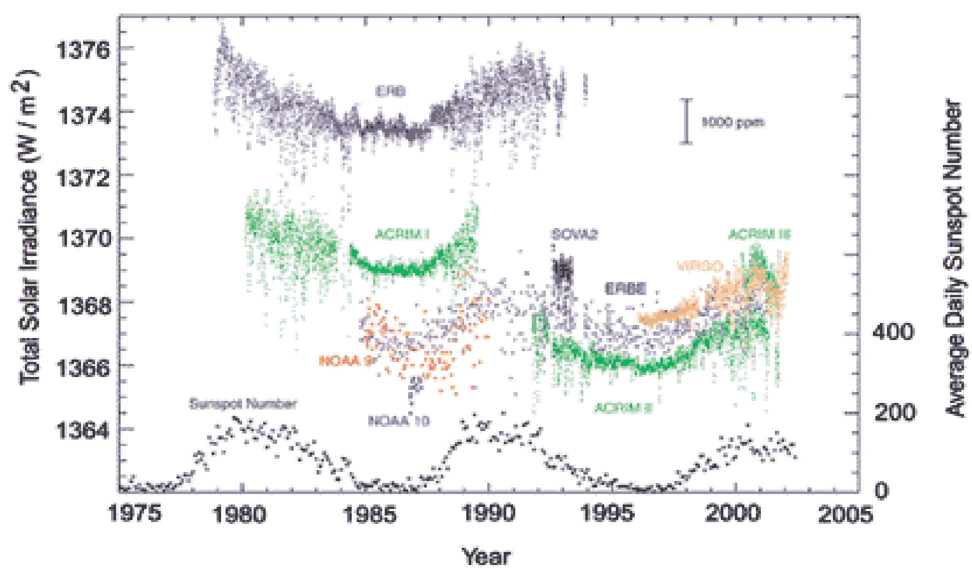

B

Figure 3.4 A) Measurements of solar constant from five independent space-based radiometers since 1978 (*top*) have been combined to produce the composite solar irradiance (*bottom*) over two decades. They show that the Sun's output fluctuates during each 11-year sunspot cycle, changing by about 0.1 percent between maximums (1980 and 1990) and minimums (1987 and 1997) in

magnetic activity. The larger number of sunspots near the peak in the 11-year cycle is accompanied by a rise in magnetic activity that creates an increase in solar radiation. The capital letters are acronyms for the different satellite radiometers (see L02, capture for figure 2.12)
B) Updated measurements (from http://lasp.colorado.edu/sorce/science/introduction.htm)

The **spectrum** of solar radiation is well approximated by the emission of a blackbody with temperature of about 5800K

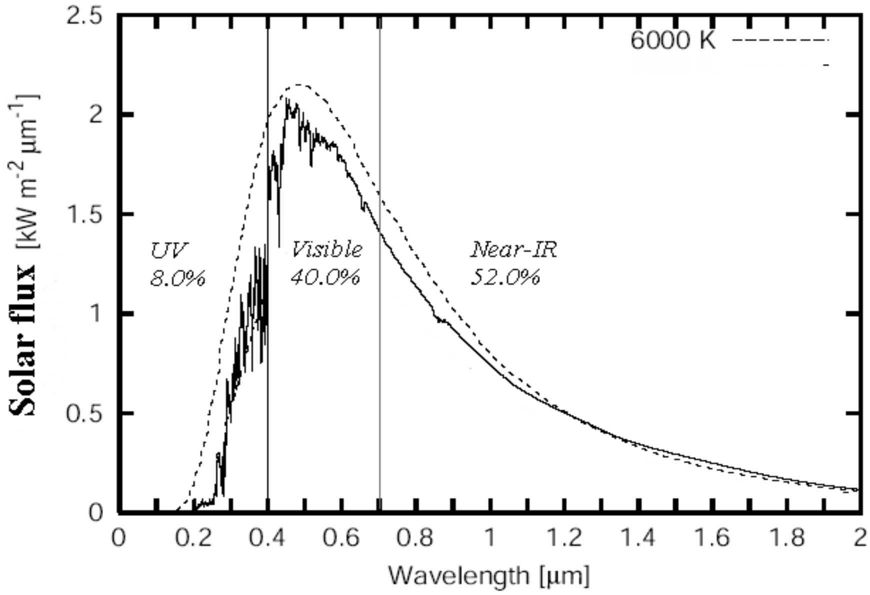

Figure 3.4 The spectrum of solar radiation (at the top of the atmosphere) and a blackbody with T=6000 K.

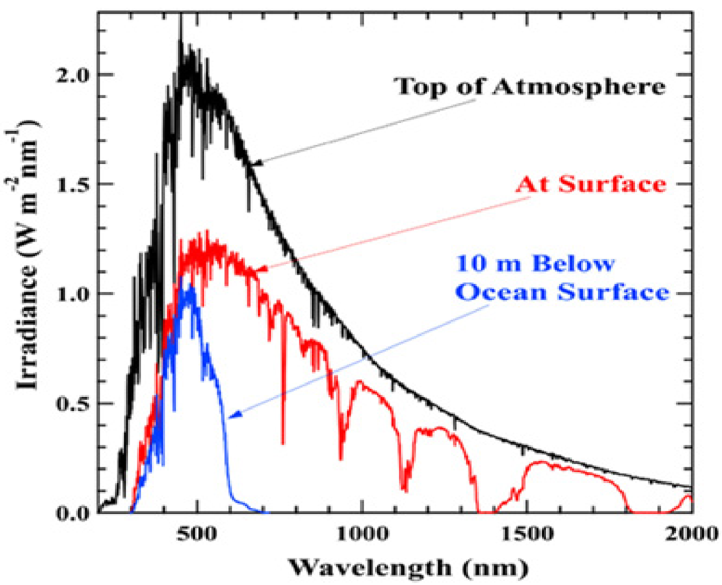

Figure 3.5 Solar radiation at the top of the atmosphere, at the surface (for representative atmospheric conditions) and 10 m under the ocean surface.