

Lecture 3.

Properties of electromagnetic radiation. Polarization. Stokes' parameters.

Main radiation laws. Brightness temperature.

Emission from the ocean and land surfaces.

1. Concepts of extinction (scattering + absorption) and emission.
2. Polarization. Stokes' parameters.
3. Main radiation laws:
 - Blackbody emission and Planck function.
 - Stefan-Boltzmann law.
 - Wien's displacement law.
 - Kirchhoff's law.
4. Brightness temperature.
5. Emission from ocean, sea ice, and land surfaces.

Required reading:

S: 2.3-2.5, Petty: 4, 6

1. Concepts of extinction (scattering + absorption) and emission.

Electromagnetic radiation in the atmosphere interacts with gases, aerosol particles, and cloud particles.

- **Extinction** and **emission** are two main types of the interactions between an electromagnetic radiation field and a medium (e.g., the atmosphere).

General definition:

Extinction is a process that decreases the radiative **intensity**, while **emission** increases it.

NOTE: "same name": **extinction** = **attenuation**

Radiation is **emitted** by **all** bodies that have a temperature above absolute zero ($^{\circ}$ K) (often referred to as **thermal emission**).

- **Extinction** is due to **absorption** and **scattering**.

Absorption is a process that removes the radiative energy from an electromagnetic field and transfers it to other forms of energy.

Scattering is a process that **does not** remove energy from the radiation field, but may redirect it.

NOTE: Scattering can be thought of as **absorption** of radiative energy followed by **re-emission** back to the electromagnetic field with negligible conversion of energy. Thus, scattering can remove radiative energy of a light beam traveling in one direction, but can be a “source” of radiative energy for the light beams traveling in other directions.

- Elastic scattering is the case when the scattered radiation has the same frequency as that of the incident field. Inelastic (Raman) scattering results in scattered light with a frequency different from that of the incident light.

2. Polarization. Stokes parameters.

Electromagnetic radiation travels as **transverse** waves, i.e., waves that vibrate in a direction perpendicular to their direction of propagation. **Polarization** is a phenomenon peculiar to **transverse** waves.

NOTE: Unlike electromagnetic (transverse) waves, sound is a **longitudinal** wave that travels through media by alternatively forcing the molecules of the medium closer together, then spreading them apart.

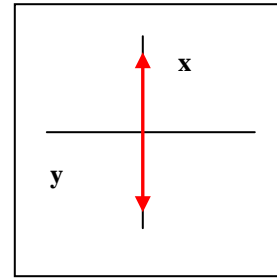
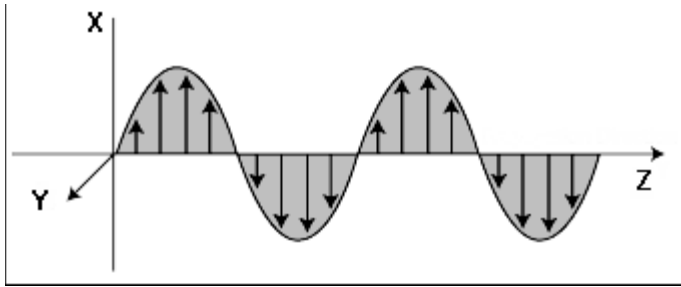
Polarization is the distribution of the electric field in the plane normal to the propagation direction.

Unpolarized radiation (or randomly polarized) is an electromagnetic wave in which the orientation of the electrical vector changes randomly.

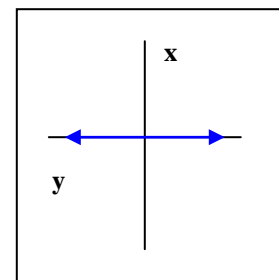
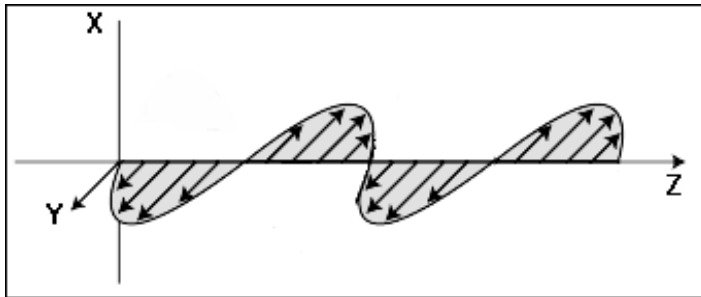
If there is a definite relation of phases between different scatterers => radiation is called **coherent**. If there is no relations in phase shift => light is called **incoherent**

- **Natural light is incoherent.**
- **Natural light is unpolarized.**

Vertically polarized wave is one for which the electric field lies only in the x-z plane.



Horizontally polarized wave is one for which the electric field lies only in the y-z plane.



- Horizontal and vertical polarizations are an example of **linear polarization**.

Mathematical representation of a plane wave propagating in the direction z is

$$E = E_0 \cos(kz - \omega t + \varphi_0) \quad [3.1]$$

where E_0 is the **amplitude**;

k is the propagation (or wave) constant, $k = 2\pi/\lambda$;

ω is the circular frequency, $\omega = kc = 2\pi c/\lambda$; φ_0 is the constant (or initial phase)

$\varphi = (kz - \omega t + \varphi_0)$ is **the phase of the wave**.

Introducing complex variables, Eq.[3.1] can be expressed as

$$E = E_0 \exp(i\varphi) \quad [3.2]$$

NOTE: we use $\exp(\pm i\varphi) = \cos(\varphi) \pm i \sin(\varphi)$

The electric vector \vec{E} may be decomposed into the parallel E_l and perpendicular E_r components as

$$\vec{E} = E_l \vec{l} + E_r \vec{r}$$

We can express E_l and E_r in the form

$$E_l = E_{l0} \cos(kz - \omega t + \varphi_{l0})$$

$$E_r = E_{r0} \cos(kz - \omega t + \varphi_{r0})$$

Then we have

$$E_l / E_{l0} = \cos(\zeta) \cos(\varphi_{l0}) - \sin(\zeta) \sin(\varphi_{l0})$$

$$E_r / E_{r0} = \cos(\zeta) \cos(\varphi_{r0}) - \sin(\zeta) \sin(\varphi_{r0})$$

where $\zeta = kz - \omega t$.

Performing simple mathematical manipulation, we obtain

$$(E_l / E_{l0})^2 + (E_r / E_{r0})^2 - 2(E_l / E_{l0})(E_r / E_{r0}) \cos(\Delta\varphi) = \sin^2(\Delta\varphi) \quad [3.3]$$

where $\Delta\varphi = \varphi_{l0} - \varphi_{r0}$ called the **phase shift**.

Eq.[3.3] defines an ellipse => **elliptically polarized wave**.

If the phase shift $\Delta\varphi = n\pi$ ($n=0, +/-1, +/-2, \dots$), then

$\sin(\Delta\varphi) = 0$ and $\cos(\Delta\varphi) = \pm 1$, and Eq.[3.3] becomes

$$\left(\frac{E_l}{E_{l0}} \pm \frac{E_r}{E_{r0}} \right)^2 = 0 \quad \text{or} \quad E_r = \pm \frac{E_{r0}}{E_{l0}} E_l \quad [3.4]$$

Eq.[3.4] defines straight lines => **linearly polarized wave**

If the phase shift $\Delta\varphi = n\pi/2$ ($n= +/-1, +/-3, \dots$) and $E_{l0} = E_{r0} = E_0$, then

$\sin(\Delta\varphi) = \pm 1$ and $\cos(\Delta\varphi) = 0$, and Eq.[3.3] becomes

$$E_l^2 + E_r^2 = E_0^2 \quad [3.5]$$

Eq.[3.5] defines a circle => **circular polarized wave**

NOTE: The sign of the phase shift gives **handedness**: right-handed and left-handed polarization.

- The state of polarization is completely defined by the four parameters: two amplitudes, the magnitude and the sign of the phase shift (see Eq.[3.3]). Because the phase difference is hard to measure, the alternative description called a **Stokes vector** is often used.

Stokes Vector consists of four parameters (called **Stokes parameters**):

intensity I ,
the degree of polarization Q ,
the plane of polarization U ,
the ellipticity V .

Notation

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} \quad \text{or} \quad \{I, Q, U, V\}$$

- **Stokes parameters** are defined via the intensities which can be measured:

I = total intensity

$Q = I_0 - I_{90}$ = differences in intensities between horizontal and vertical linearly polarized components;

$U = I_{+45} - I_{-45}$ = differences in intensities between linearly polarized components oriented at $+45^\circ$ and -45°

$V = I_{rcr} - I_{lcr}$ = *differences in intensities between right and left circular polarized components.*

- **Stokes parameters** can be expressed via the amplitudes and the phase shift of the parallel and perpendicular components of the electric field vector

$$\begin{aligned} I &= E_{ro}^2 + E_{lo}^2 \\ Q &= E_{ro}^2 - E_{lo}^2 \end{aligned} \quad [3.6]$$

$$U = 2E_{ro}E_{lo} \cos(\Delta\varphi)$$

$$V = 2E_{ro}E_{lo} \sin(\Delta\varphi)$$

Example: Stokes parameters for the vertical polarization:

For this case $E_l = 0$

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} E_{ro}^2 \\ E_{ro}^2 \\ 0 \\ 0 \end{pmatrix} = E_{ro}^2 \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

For **unpolarized** light:

$$Q = U = V = 0 \quad [3.7]$$

The **degree of polarization P** of a light beam is defined as

$$P = (Q^2 + U^2 + V^2)^{1/2} / I \quad [3.8]$$

The **degree of linear polarization LP** of a light beam is defined by neglecting U and V

$$LP = \frac{Q}{I} \quad [3.9]$$

NOTE: Measurements of polarization are actively used in remote sensing in the solar and microwave regions.

Polarization in the microwave – mainly due to reflection from the surface.

Polarization in the solar – reflection from the surface and scattering by molecules and particulates.

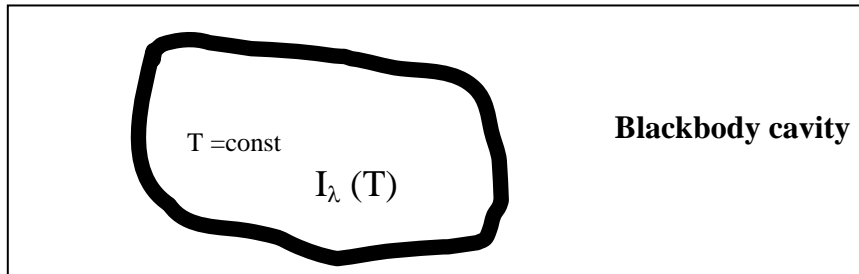
Active remote sensing (e.g., radar) commonly uses polarized radiation.

3. Main radiation laws.

Blackbody is a body that absorbs all radiation incident upon it.

Thermodynamical equilibrium describes the state of matter and radiation inside an isolated constant-temperature enclosure.

Blackbody radiation is the radiative field inside a cavity in thermodynamic equilibrium.



NOTE: A blackbody cavity is an important element in the design of radiometers. Cavities are used to provide a well-defined source for calibration of radiometers. Another use of a cavity is to measure the radiation that flows into the cavity (e.g., to measure the radiation of sun).

Properties of blackbody radiation:

- Radiation emitted by a blackbody is isotropic, homogeneous and unpolarized;
- Blackbody radiation at a given wavelength depends only on the temperature;
- Any two blackbodies at the same temperature emit precisely the same radiation;
- A blackbody emits more radiation than any other type of an object at the same temperature;

NOTE: The atmosphere is not strictly in the thermodynamic equilibrium because its temperature and pressure are functions of position. Therefore, it is usually subdivided into small subsystems each of which is effectively isothermal and isobaric referred to as **Local Thermodynamical Equilibrium (LTE)**.

- A concept of **LTE** plays a fundamental role in atmospheric studies: e.g., the main radiation laws discussed below, which are strictly speaking valid in **thermodynamical equilibrium**, can be applied to an atmospheric air parcel in **LTE**.

➤ **Blackbody Emission: Planck function.**

Planck function, $B_\lambda(T)$, gives the **intensity (or radiance)** emitted by a blackbody having a given temperature.

- **Planck function** can be expressed in wavelength, frequency, or wavenumber domains as

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5 (\exp(hc / k_B T \lambda) - 1)} \quad [3.10]$$

$$B_{\tilde{\nu}}(T) = \frac{2h\tilde{\nu}^3}{c^2 (\exp(h\tilde{\nu} / k_B T) - 1)} \quad [3.11]$$

$$B_\nu(T) = \frac{2h\nu^3 c^2}{\exp(h\nu c / k_B T) - 1} \quad [3.12]$$

where λ is the wavelength; $\tilde{\nu}$ is the frequency; ν is the wavenumber;

h is the Planck's constant; k_B is the Boltzmann's constant ($k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$);

c is the velocity of light; and T is the absolute temperature (in K) of a blackbody.

NOTE: The relations between $B_{\tilde{\nu}}(T)$; $B_\nu(T)$ and $B_\lambda(T)$ are derived using that

$$I_{\tilde{\nu}} d\tilde{\nu} = I_\nu d\nu = I_\lambda d\lambda, \text{ and that } \lambda = c / \tilde{\nu} = 1 / \nu$$



$$B_{\tilde{\nu}}(T) = \frac{\lambda^2}{c} B_\lambda(T) \quad \text{and} \quad B_\nu(T) = \lambda^2 B_\lambda(T)$$

Asymptotic behavior of Planck function:

- If $\lambda \rightarrow \infty$ (or $\tilde{\nu} \rightarrow 0$) (known as Rayleigh –Jeans distributions):

$$B_\lambda(T) = \frac{2k_B c}{\lambda^4} T \quad [3.13a]$$

$$B_{\tilde{\nu}}(T) = \frac{2k_B \tilde{\nu}^2}{c^2} T \quad [3.13b]$$

NOTE:

- ✓ **Rayleigh –Jeans distributions** has a direct application to passive **microwave** remote sensing.
- ✓ For large wavelengths, the emission is directly proportional to T.

• If $\lambda \rightarrow 0$ (or $\tilde{\nu} \rightarrow \infty$):

$$B_{\lambda}(T) = \frac{2hc^2}{\lambda^5} \exp(-hc / \lambda k_B T) \quad [3.14a]$$

$$B_{\tilde{\nu}} = \frac{2h\tilde{\nu}^3}{c^2} \exp(-h\tilde{\nu} / k_B T) \quad [3.14b]$$

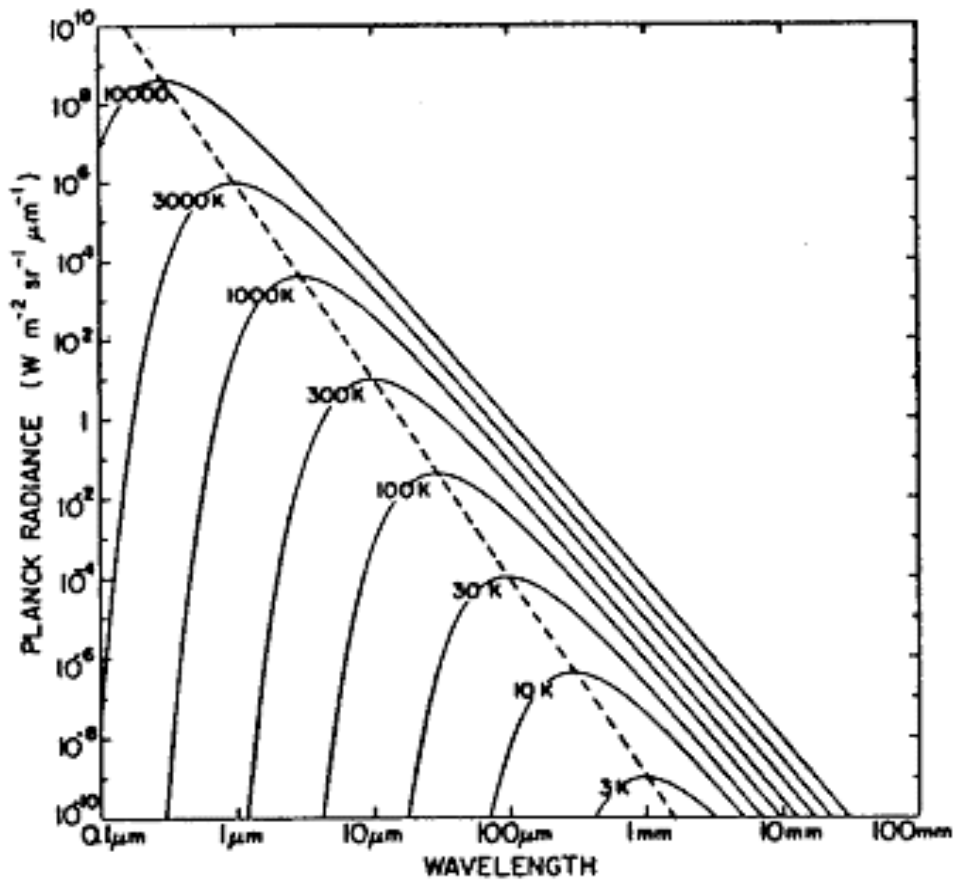


Figure 3.1 Planck function on log-log plot for several temperatures.

➤ **Stefan-Boltzmann law.**

The **Stefan-Boltzmann law** states that the radiative flux emitted by a **blackbody**, per unit surface area of the **blackbody**, varies as the fourth power of the temperature.

$$\mathbf{F} = \pi \mathbf{B}(\mathbf{T}) = \sigma_b \mathbf{T}^4 \quad [3.15]$$

where σ_b is the *Stefan-Boltzmann constant* ($\sigma_b = 5.671 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$),

\mathbf{F} is energy flux [W m^{-2}], and \mathbf{T} is blackbody temperature (in degrees Kelvin, K);

and $\mathbf{B}(\mathbf{T}) = \int_0^{\infty} B_{\lambda}(T) d\lambda$

➤ **Wien's displacement law.**

The **Wien's displacement law** states that the wavelength at which the blackbody emission spectrum is most intense varies inversely with the blackbody's temperature. The constant of proportionality is Wien's constant (2897 K μm):

$$\lambda_m = 2897 / \mathbf{T} \quad [3.16]$$

where λ_m is the wavelength (in micrometers, μm) at which the peak emission intensity occurs, and \mathbf{T} is the temperature of the blackbody (in degrees Kelvin, K).

NOTE: This law is simply derived from $dB_{\lambda}/d\lambda = 0$.

NOTE: Easy to remember statement of the Wien's displacement law:

the hotter the object the shorter the wavelengths of the maximum intensity emitted

➤ **Kirchhoff's law.**

The **Kirchhoff's law** states that the emissivity, ϵ_{λ} , of a medium is equal to the absorptivity, A_{λ} , of this medium under thermodynamic equilibrium:

$$\epsilon_{\lambda} = A_{\lambda} \quad [3.17]$$

where ϵ_{λ} is defined as the ratio of the emitting intensity to the Planck function;

A_{λ} is defined as the ratio of the absorbed intensity to the Planck function.

For a **blackbody**: $\epsilon_{\lambda} = A_{\lambda} = 1$

For a **gray body** (i.e., no dependency on the wavelength): $\epsilon = A < 1$

For a **non-blackbody**: $\epsilon_{\lambda} = A_{\lambda} < 1$

NOTE: Kirchhoff's law applies to gases, liquids and solids if they in TE or LTE.

NOTE: In remote sensing applications, one needs to distinguish between the **emissivity of the surface** (e.g., various types of lands, ice, ocean etc.) and the **emissivity of an atmospheric volume** (consisting of gases, aerosols, and/or clouds).

4. Brightness temperature.

Brightness temperature, T_b , is defined as the temperature of a blackbody that emits the same intensity as measured at a given wavelength (or frequency or wavenumber).

Brightness temperature is found by inverting the Planck function.

For instance, from Eq.[3.10]:

$$T_b = \frac{C_2}{\lambda \ln\left[1 + \frac{C_1}{\lambda^5 I_{\lambda}}\right]} \quad [3.18]$$

where I_{λ} is the measured intensity, and

$$C_1 = 1.1911 \times 10^8 \text{ W m}^{-2} \text{ sr}^{-1} \mu\text{m}^4; \quad C_2 = 1.4388 \times 10^4 \text{ K } \mu\text{m}$$

- For a blackbody: brightness temperature = kinetic temperature ($T_b = T$)
- For natural materials over the broad spectrum: $T_b^4 = \epsilon T^4$ (ϵ is the broadband emissivity)

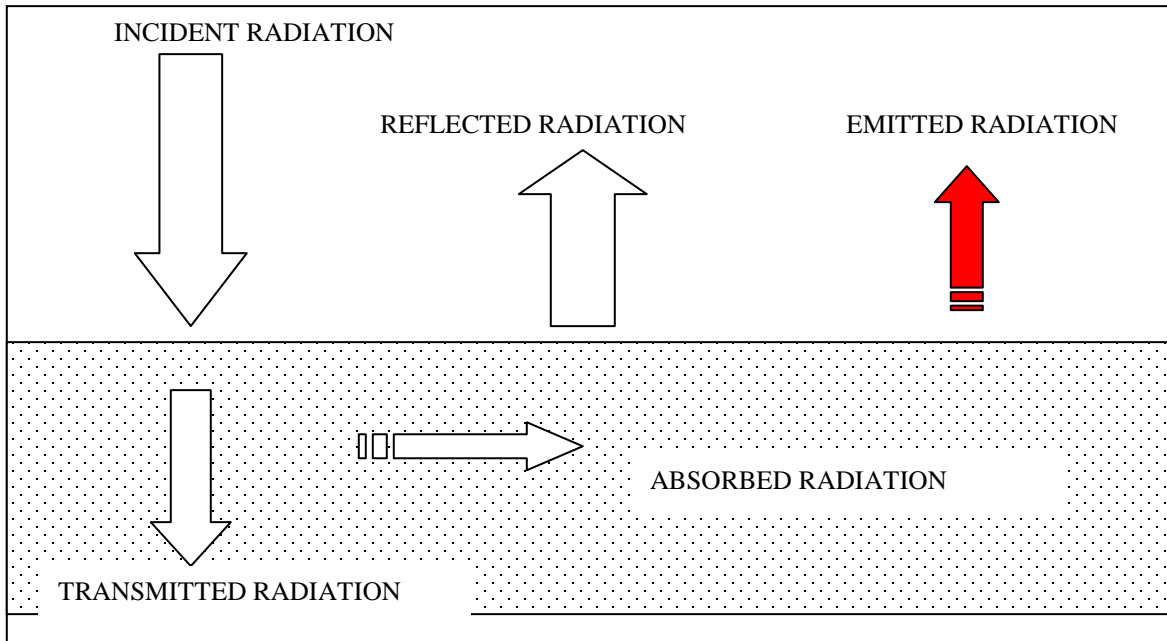
NOTE: In the microwave region, the Rayleigh –Jeans distributions gives $T_b = \epsilon_{\lambda} T$.

However, ϵ_{λ} is a complex function of several parameters (see below).

5. Emission from ocean and land surfaces.

The ocean and land surfaces can modify the atmospheric radiation field by

- reflecting a portion of the incident radiation back into the atmosphere;
- transmitting some incident radiation;
- absorbing a portion of incident radiation (Kirchhoff's law);
- emitting the thermal radiation (Kirchhoff's law);



Conservation of energy requires that monochromatic radiation incident upon any surface, I_i , is either reflected, I_r , absorbed, I_a , or transmitted, I_t . Thus

$$I_i = I_r + I_a + I_t \quad [3.19]$$

$$1 = I_r / I_i + I_a / I_i + I_t / I_i = R + A + T \quad [3.20]$$

where T is the transmission, A is the absorption, and R is the reflection of the surface.

In general, T, A, and R are functions of wavelength:

$$R_\lambda + A_\lambda + T_\lambda = 1 \quad [3.21]$$

Blackbody surfaces (no reflection) and surfaces in LTE (from Kirchhoff's law):

$$A_\lambda = \epsilon_\lambda \quad [3.22]$$

Opaque surfaces (no transmission):

$$R_{\lambda} + A_{\lambda} = 1 \quad [3.23]$$

Thus for the opaque surfaces $\epsilon_{\lambda} = 1 - R_{\lambda}$ [3.24]

Emission from the ocean and land surfaces:

- In general, emissivity depends on the direction of emission, surface temperature, wavelength and some physical properties of the surface
- In the thermal IR ($4\mu\text{m} < \lambda < 100\mu\text{m}$), nearly all surfaces are efficient emitters with the emissivity > 0.8 and their emissivity does not depend on the direction. Therefore, the intensity emitted from a unit surface area at a given wavelength is

$$I_{\lambda} = \epsilon_{\lambda} B_{\lambda}(T_s)$$

- In the shortwave region ($0.1 \mu\text{m} < \lambda < 4 \mu\text{m}$), emissivity is negligibly small.
- In microwave ($0.1 \text{ cm} < \lambda < 100 \text{ cm}$), emissivity depends on the type and state of the surface.

NOTE: Differences in the emissivity of ice vs. water provide the basis for microwave remote sensing of sea-ice (see Computer Modeling Lab 1)

Table 3.1 Example of emissivities of natural surfaces in the IR window (10 to 12 μm).

Surface	Emissivity
Water	0.993-0.998
Ice	0.98
Green grass	0.975-0.986
Sand	0.949-0.962
Snow	0.969-0.997
Granite	0.898

NOTE: Many natural surfaces have high emissivity in the IR window and hence low (negligible) reflectivity in this spectral region.

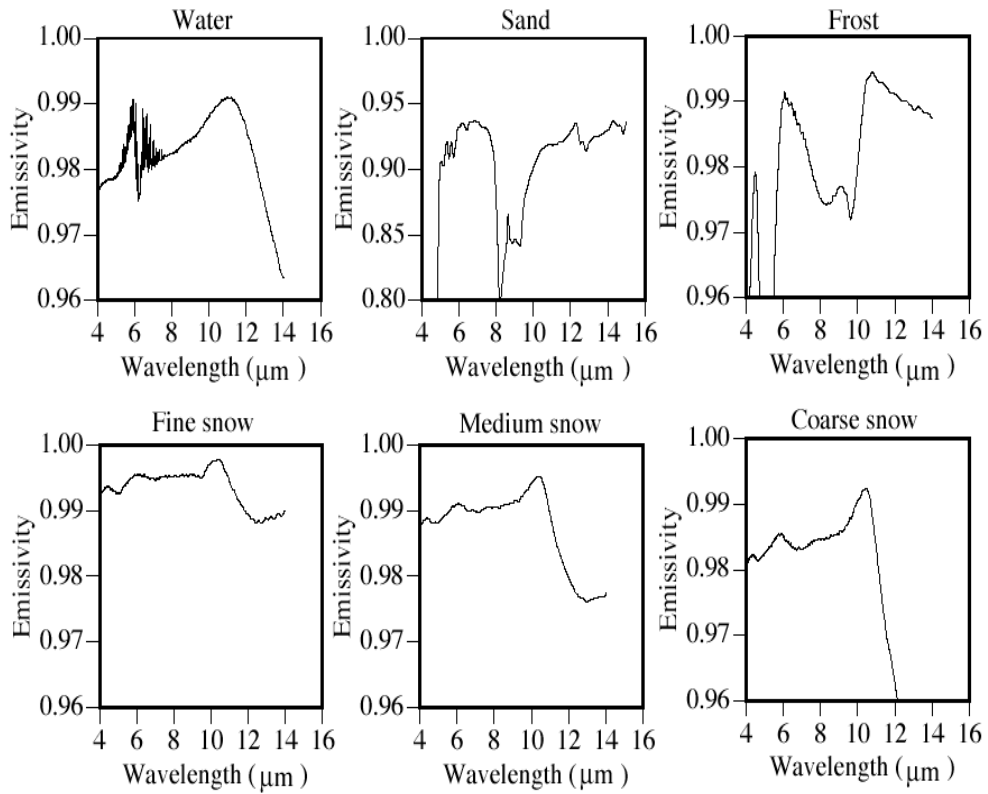


Figure 3.2 Examples of IR spectral emissivity of some surfaces.

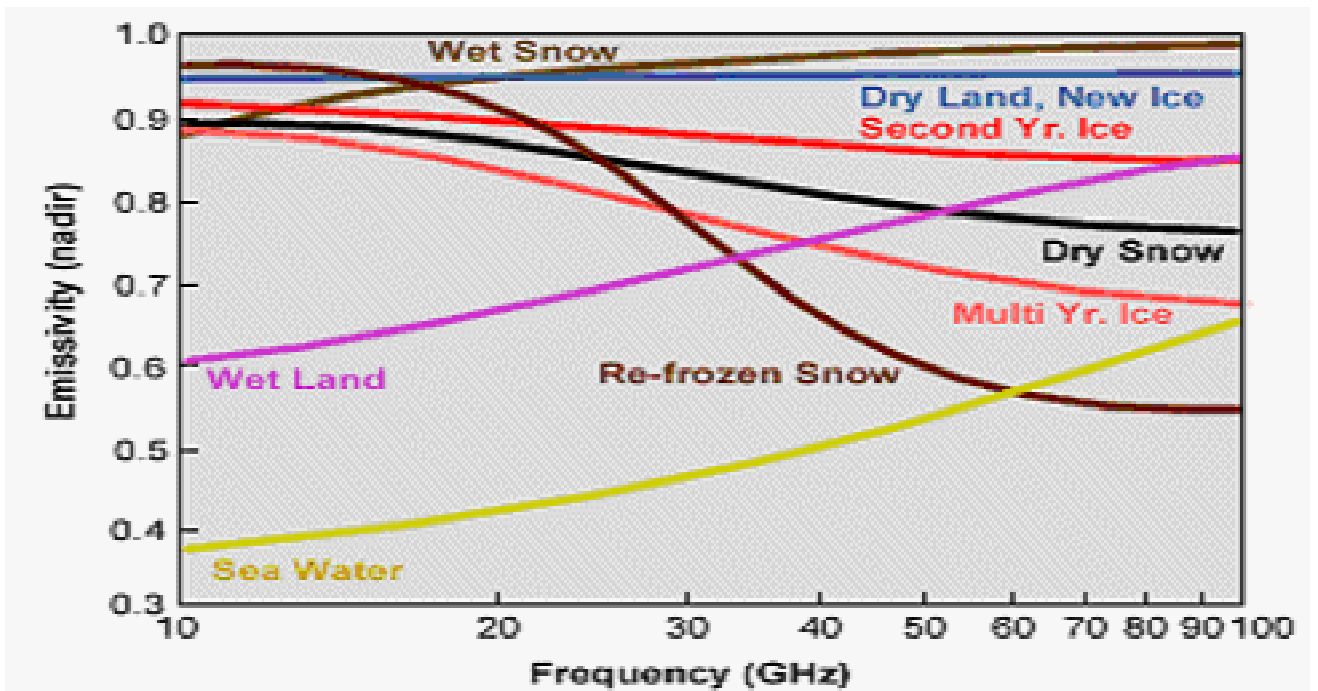


Figure 3.3 Examples of spectral emissivity (at nadir) of some surfaces in the microwave.

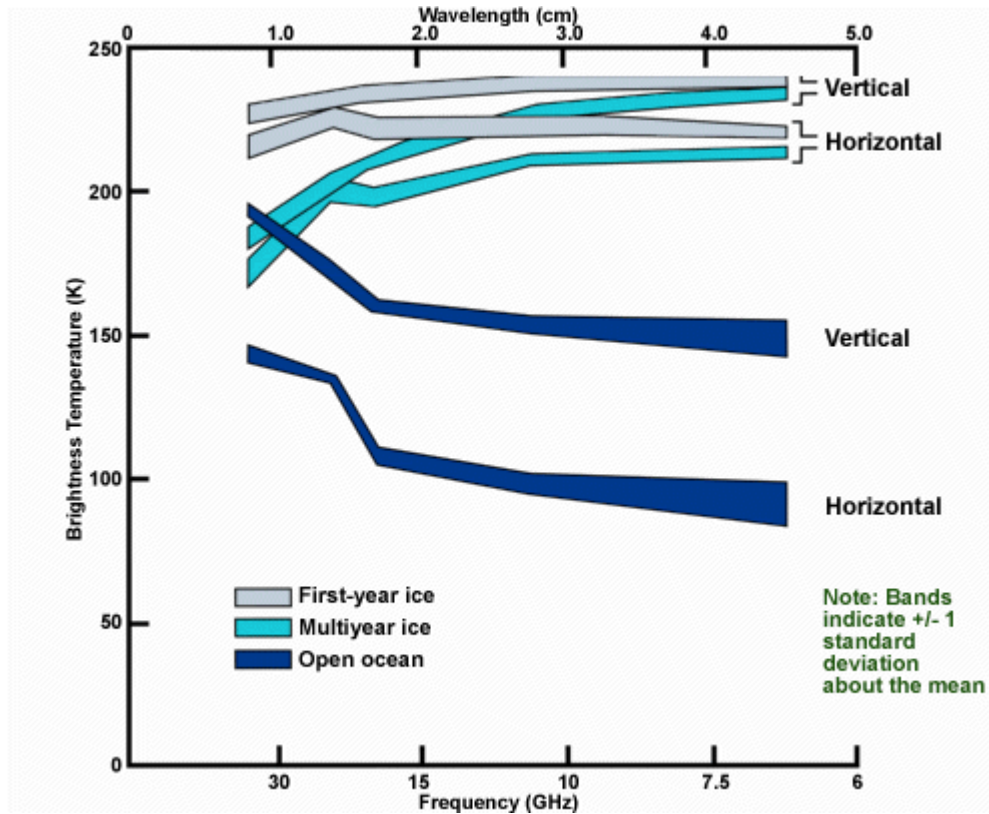


Figure 3.4 Microwave brightness temperature (at vertical and horizontal polarizations) as a function of frequency (wavelength) of first year sea-ice, multiyear sea-ice, and sea water as observed by the Nimbus-7 SMMR (Scanning Multichannel Microwave Radiometer) (Cavalieri et al. 1984).

NOTE: The behavior of microwave BT illustrates the basis for retrieving the coverage and age of sea ice with passive microwave remote sensing: both the slope of spectral BT and polarization can be useful to differentiate between water and sea ice of different age.