CHAPTER 2

Properties of Radiation

The first order of business in a book about atmospheric radiation is to clarify what radiation *is*, how it behaves at the most fundamental physical level, what conventions are used to classify it according to wavelength and other properties, and how we define the characteristics (e.g. intensity) that appear in quantitative descriptions of radiation and its interactions with the atmosphere. We will start from absolute basics, which requires us to at least touch on some topics in classical physics. Our forays into such matters will be as brief and descriptive as the subsequent material allows, in keeping with the title of this book. Here, and throughout the book, students interested in a more comprehensive treatment should consult the more specialized textbooks and other sources cited at the end of this book.

2.1 The Nature of Electromagnetic Radiation

Electric and Magnetic fields

Everyone has experienced the effects of electric and magnetic fields. A nylon shirt pulled from a dryer may be found to have several socks clinging firmly to it. A magnet may be used to affix a note or photograph to a refrigerator door. In the first case, an electric field induced by excess charge on one item of clothing exerts an attractive force on the excess opposite charge found on the others. In the second, a magnetic field exerts an attractive force on the iron atoms in the door.

Both magnetic and electric fields are detectable at some distance from their source. A plastic balloon that was rubbed on a sweater may attract your hair from a foot or so away. A refrigerator magnet may deflect the needle of a compass from a meter or more away. Taking the magnet away, the needle returns to a position aligned with the magnetic field of the earth's iron core, thousands of kilometers beneath your feet.

The basic laws governing *static* electric and magnetic fields are well known and probably already at least somewhat familiar to you from an earlier physics course. According to Coulomb's Law, the electric field at any point is determined entirely by the distribution of electric charge in the space surrounding that point. According to Faraday's Law, magnetic fields are determined by the distribution of electric *current* (moving electric charge) in the neighborhood.

The latter law is what makes electromagnets possible, without which electric motors and most audio speakers would not exist in their current form. A related law tells us that a changing magnetic field induces an electric field that can drive a current. Thus, an electric motor that is caused to rotate by an external torque (e.g., a steam-driven turbine) can reverse roles and become a generator instead.

Thus, although static magnetic and static electric fields — as illustrated by the examples given at the beginning — may appear to have little or nothing to do with each other, they are in fact intimately connected: a *changing* electric field induces a magnetic field, and a *changing* magnetic field induces an electric field. Quantitatively, this interplay of electric and magnetic fields is embodied both completely and remarkably succinctly in Maxwell's Equations (see Section 2.5). However, one does not need to look at equations themselves to appreciate a few of the implications of this interplay.

Electromagnetic Waves

Imagine a refrigerator magnet resting on the kitchen table. It creates a magnetic field that extends essentially indefinitely (though with ever-weaker strength at increasing distances from the source) in all directions. Pick up the magnet and stick it back on the refrigerator door. Because the magnet has moved (and probably changed its orientation as well), its induced magnetic field has also changed. But a changing magnetic field creates an electric field, which persists as long as the magnetic field continues to change. Once the magnet is stationary again, the magnetic field stops changing and the electric field must disappear. But wait: this means the electric field is undergoing a change, which leads to the reappearance of a magnetic field!

You can see where this is leading. A change in either an electric or magnetic field, however brief, leads to a disturbance that is self-perpetuating. Less obvious to the casual observer is the fact that this electromagnetic disturbance propagates away from the source at a finite speed, just as ripples propagate outward from the point where a pebble strikes the surface of a pond (in the latter case, the interplay is not between magnetic and electric fields but rather between the kinetic and potential energy of the water's surface).

In the case of both electromagnetic waves and ripples on a pond, the disturbances carry energy. In the absence of viscosity, the pond ripples will transport the original energy outward, with no net loss, until they encounter something that can convert some or all of that energy to another form, such as heat or/and sound — e.g., through the breaking of the waves on the muddy bank.

Likewise, in a perfect vacuum, where there is no opportunity to convert the energy carried by electromagnetic waves into another form, such as heat, kinetic, or chemical energy (all of which can only exist in association with matter), the waves propagate indefinitely without net loss, though distributed over an ever-larger volume of space. Furthermore, it has been observed that, unlike pond ripples, electromagnetic waves always travel in a vacuum at an absolutely constant speed. This *speed of light* is approximately 3.0×10^8 m s^{-1}. By convention, it is represented by the symbol c.

The direction that an EM wave travels in a vacuum is always perpendicular to the wave crest, again just like pond ripples. And because c is a constant in a vacuum, the wave can only propagate directly away from the source. Any change of direction would imply a slowing down or speeding up of part of a wave crest at some point during its travels. This *can* happen in the presence of matter, but remember we're still talking about a vacuum here.

Sometimes it is helpful to visualize the propagation of waves in terms of *rays*. A ray is an imaginary line that always crosses wave fronts at right angles. Thus at any point on a ray, the direction of propagation of the wave is along that ray. In the case of pond ripples, all rays would be straight lines originating at the point where the pebble strikes the surface. If you like, you can think of each ray as carrying a unit of energy. Thus, the density of their intersections with a given wave front is a measure of the energy content of the wave at that location. If a given wave front propagates outward from a source without losses, the total number of rays remains constant (implying conservation of energy), but the density of intersections along the wave front decreases with distance from the source. This is consistent with the spreading of the wave's energy over a larger area and thus of its weakening as it gets further and further away.

Again, the bending of rays can only occur in the presence of local changes in wave propagation speed. This phenomenon, known as *refraction*, is the subject of Section 4.2.

Imagine throwing two pebbles into a pond at the same time. Each one produces its own set of ripples that propagate outward from the source. Sooner or later, the ripples from one pebble encounter those from the other. What happens? Do they bounce off of each other? Do they annihilate each other in a flash of heat and gamma rays? Of course not; each passes through the other as if it didn't exist. At each point on the water surface, the height perturbations associated with each set of waves simply add in a linear fashion. Where two crests intersect, the water surface is (temporarily) raised to about twice the height of either crest individually. Where a crest from one wave intersects the trough from another, the two may partially or completely cancel, leaving the water (temporarily) at its original level. This effect is called *interference* — constructive interference in the first case, destructive in the second. It must be emphasized however that nothing is created or destroyed; the ef-

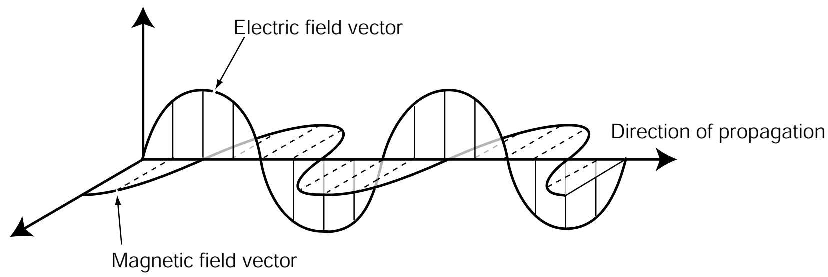

Fig. 2.1: Schematic depiction of the interplay between the electric and magnetic fields in an electromagnetic wave.

fect is purely local at the points where the waves overlap and has no influence on the subsequent propagation of the individual waves.

Exactly the same principle of linear superposition applies to EM waves. If you shine a flashlight on a wall in a dark room and then turn on the overhead light, the latter adds its illumination to that of the flashlight but otherwise has no influence on the propagation of the flashlight beam.

There is one further analogy to pond ripples that we can use to illustrate an important property of EM waves. Unlike sound waves in air, which entail oscillations of air molecules parallel to the direction of propagation, water waves arise from vertical displacements of the water surface; i.e., *perpendicular, or normal, to the direction of propagation*. Likewise, EM waves are associated with an oscillating electric field vector that is normal to the direction of propagation (Fig. 2.1). Both are therefore *transverse* waves, unlike sound waves, which are *longitudinal*.

Now we finally come to a couple of points where the analogy to pond ripples starts to break down:

- Unlike sound waves, water waves, and all other everyday kinds of waves, EM waves require no material medium in which to propagate. You can't have pond ripples without a pond; you can't have sound without air (or some other medium). But EM waves are quite at home, and indeed easiest to understand, in a perfect vacuum. It is when matter, such as the atmosphere and its various resident particles, gets into the picture that the propagation of EM waves become considerably more complex and interesting.

- Whereas pond ripples are confined to the water surface and therefore propagate in only two dimensions, EM waves propagate in three dimensional space, like sound waves (imagine if you could only hear your stereo when you put your ear on the floor!).

- Whereas the transverse oscillations of water particles in pond ripples are constrained to be vertical, there is no similar constraint on the electric field oscillations in EM waves. The orientation of the electric field vector may lie in any direction, as long as it is in the plane normal to the direction of propagation. Occasionally it is necessary to pay attention to this orientation, in which case we refer to the *polarization* of the wave as vertical, horizontal, or some other direction. A more detailed discussion of polarization will be taken up in Section 2.3

2.2 Frequency

Up until now, we have imagined an arbitrary EM disturbance and given no thought to its detailed dependence in time. In principle, we could assume any kind of EM disturbance we like — a lightning discharge, a refrigerator magnet dropping to the kitchen floor, the radiation emitted by a radio tower, or a supernova explosion in deep space. In each case, Maxwell's equations would describe the propagation of the resulting EM disturbance equally well.

Let's consider a special case, however. Imagine that we take our magnet and place it on a steadily rotating turntable. The fluctuations in the magnetic field (and in the associated electric field) are now periodic. The frequency of the fluctuations measured at a distance by a stationary detector will be the same as the frequency of rotation ν of the turntable. But recall that the periodic disturbance propagates outward not instantaneously but at the fixed speed of light c. The distance λ that the fluctuation propagates during one cycle of the turntable is called the *wavelength* and is given by

$$\boxed{\lambda = \frac{c}{\nu}\,.} \tag{2.1}$$

In the above thought experiment, ν is extremely low (order 1 $\sec^{-1}$) and the wavelength is therefore extremely large (order 10^5 km).

In nature, electromagnetic waves can exist with an enormous range of frequencies, from a few cycles per second or even far less to more than 10^{26} cyles per second in the case of extremely energetic gamma waves produced by nuclear reactions. According to (2.1), EM wavelengths can thus range from hundreds of thousands of kilometers or more to less than the diameter of an atomic nucleus.

Because it is a common point of confusion, it bears emphasizing that the wavelength is *not* a measure of how *far* an EM wave can propagate. In a vacuum, that distance is always infinite, regardless of wavelength. In a medium such as water or air, wavelength *does* matter, but in a rather indirect and highly complex way.

Problem 2.1:

(a) Visible light has a wavelength of approximately 0.5 μm. What is its frequency in Hz?

(b) Weather radars typically transmit EM radiation with a frequency of approximately 3 GHz (GHz = "Gigahertz" = 10^9 Hz). What is its wavelength in centimeters?

(c) In the U.S., standard AC electrical current has a frequency of 60 Hz. Most machinery and appliances that use this current, as well as the power lines that transport the electric power, emit radiation with this frequency. What is its wavelength in km?

Problem 2.2:

Whenever we talk about a single frequency ν characterizing an electromagnetic wave, we are tacitly assuming that the source and the detector are stationary relative to one another, in which case ν is indeed the same for both. However, if the distance between the two is changing with velocity v (positive v implying increasing separation), then the frequency of radiation ν_1 emitted by the source will be different than the frequency ν_2 observed by the detector. In particular, the frequency shift $\Delta\nu = \nu_1 - \nu_2$ is approximately proportional to v, a phenomenon known as *Doppler shift*.

(a) Derive the precise relationship between $\Delta\nu$ and v, by considering the time Δt elapsed between two successive wave crests reaching the detector with speed c.

(b) For the case that $v \ll c$, show that your solution to (a) simplifies to a proportionality between $\Delta\nu$ and v.

2.2.1 Frequency Decomposition

The above discussion of periodic waves is interesting, but what does it have to do with real EM radiation? After all, the EM disturbance that arises from a lightning discharge, or from dropping a magnet on the floor, is clearly not a steadily oscillating signal but more likely a short, chaotic pulse! What sense does it make to speak of a specific frequency or wavelength in these cases?

The answer is that any arbitrary EM fluctuation, short or long, can be thought of as a *composite* of a number (potentially infinite) of different "pure" periodic fluctuations. Specifically, any continuous function of time $f(t)$ can be expressed as a sum of pure sine functions as follows:

$$f(t) = \int_0^\infty \alpha(\omega) \sin[\omega t + \phi(\omega)] \, d\omega \tag{2.2}$$

where $\alpha(\omega)$ is the amplitude of the sine function contribution for each specific value of the angular frequency ω and $\phi(\omega)$ gives the corresponding phase. If $f(t)$ itself is already a pure sine function $\sin(\omega_0 t + \phi_0)$, then of course $\alpha = 0$ for all values of ω except ω_0. For more general functions $f(t)$, $\alpha(\omega)$ and $\phi(\omega)$ may be quite complicated. It is beyond the scope of this book to explain *how* we find $\alpha(\omega)$ etc. for any given $f(t)$; it is only important to recognize that it can, in principle, always be done.[1]

[1] This so-called *Fourier decomposition* is extremely useful throughout the physical and engineering sciences, including other areas of atmospheric dynamics and climatology, so if you haven't seen anything like this before, it is certainly worth taking the time to read up on it.

Now recall what I pointed out earlier: individual EM disturbances propagate *completely independently* of one another. They may intersect, or even travel together, but the presence of one does not *influence* the other. This principle applies equally well to the pure sine wave components of a Fourier decomposition. Thus, not only can you regard any arbitrary electromagnetic disturbance as a mixture of pure sine waves of differing angular frequencies ω, but *you can then track the propagation of each frequency component completely separately from all the others.*

The implications of this observation are profound for atmospheric radiative transfer. The most basic path to understanding and/or modeling the interaction of EM radiation with clouds, water vapor, oxygen, carbon dioxide, etc., is to consider one frequency at a time and then, if required, to sum the results over all relevant frequencies. There are shortcuts that sometimes allow you to forget you are doing this, but that is nevertheless what is going on under the surface.

2.2.2 Broadband vs Monochromatic Radiation

Now is a good time to introduce a few more definitions:

- EM radiation composed entirely of a single frequency is termed *monochromatic* ("one color").
- Radiation that consists of a mixture of a wide range of frequencies is called *broadband* radiation.

As already noted, the transport of broadband radiation in the atmosphere can always be understood in terms of the transport of the individual constituent frequencies. Therefore, we will tend to focus at least initially on the monochromatic radiation.

It is quite common to have to deal with radiation that is not strictly monochromatic but is nevertheless confined to an extremely narrow range of frequencies. Such radiation is often called monochromatic, even though *quasi-monochromatic* would be a more precise term. Another way to distinguish between the two cases is via the terms *coherent* and *incoherent*:

- *Coherent* radiation is what you get from a single oscillator, like the magnet on a turntable described earlier, or from a group of oscillators that are, for whatever reason, in perfect synchronization with one another. Imagine a stadium full of people doing "the wave" at a football game, or an audience clapping in unison for an encore at the end of a rock concert. Microwave ovens, radars, lasers, and radio towers all produce coherent radiation. Note that these are all artificial sources. As an atmospheric scientist, you are most likely to encounter coherent radiation in the context of the artificial sources used in remote sensing, such as radar and lidar.

- *Incoherent* radiation is what you get from a set of independent oscillators that may have nearly the same frequency (quasi-monochromatic) but are not phase-locked to one another. Imagine a large audience applauding at the end of a speech. There is no synchronization of the claps of different individuals, even if everyone is clapping with about the same frequency. Natural radiation in the lower atmosphere is, for all practical purposes, incoherent.

2.3 Polarization

As mentioned earlier, *the orientation of the oscillating electric field vector in an EM wave can be any direction that is perpendicular to the direction of propagation.* In some radiative transfer applications (especially remote sensing) it is sometimes important to keep track of that orientation and how it evolves over the course of a complete cycle.

In coherent radiation, there is a unique, repeating pattern to the oscillating electric field vector when viewed along the direction of propagation. There are several basic possibilities for this pattern:

1. It may vibrate back and forth in a fixed plane, like a pendulum or plucked guitar string. This is called *linear polarization* [Fig. 2.2 (a)–(c)].

2. It may oscillate in spiral fashion about the direction of propagation, either clockwise or counterclockwise, for *circular polarization* [Fig. 2.2 (f)].

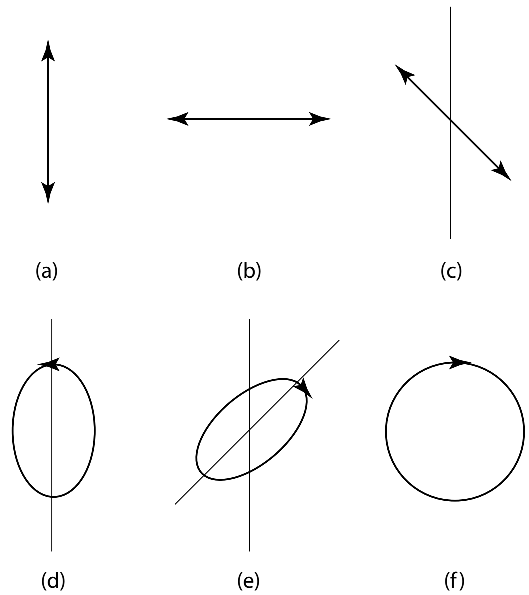

Fig. 2.2: Examples of different types of polarization, depicted in terms of the vibration of the electric field vector in a fixed plane perpendicular to the direction of propagation. (a) vertical linear, (b) horizontal linear, (c) linear at 45°, (d) elliptical counterclockwise, with vertical major axis, (e) elliptical clockwise, with the major axis oriented at a 45°angle, (f) circular, clockwise. Note that an infinity of combinations of orientation, ellipticity, and sense of rotation are also possible.

3. *Elliptical polarization* is essentially a hybrid of the first two. Note that elliptical polarization can be viewed as including both linear and circular polarization as limiting cases [Fig. 2.2 (d)–(e)].

Standard weather radar equipment typically transmits coherent radiation with linear polarization (either vertical or horizontal) and then measures the backscattered radiation with the same polarization. More sophisticated radars may transmit in one polarization but then separately measure the returned radiation in both vertical and horizontal polarizations in order to gain additional information about the targets.

In incoherent radiation, a systematic tendency toward one type of polarization may or may not be discernible. Therefore, in addition to the above *types* of polarization, one must specify the *de-*

gree of polarization. As a general rule, natural emissions of radiation in the atmosphere are completely unpolarized, but the radiation may become partially or completely polarized in the course of its interactions with particles and/or the surface. In particular, it will be shown later that a smooth surface, like calm water, preferentially reflects radiation having horizontal linear polarization. This is the phenomenon that motivated the invention of polarized sunglasses, which block the reflected horizontally polarized radiation while transmitting the rest. It is also a phenomenon of great practical importance for satellite remote sensing in the microwave band.

The purpose of this section was just to introduce you to the existence and qualitative nature of polarization. Having done that, I'll point out that polarization is often disregarded in radiative transfer calculations, especially at the level targeted by this book. Where polarization cannot be ignored, one can often gain at least qualitative insight into its role without getting too deep into the relevant mathematics. But since some of you may eventually require a more complete and quantitative understanding of polarization, I will introduce some elements of the mathematical treatment of polarization at appropriate points along the way.

2.4 Energy

Barely mentioned so far, but central to our interest in atmospheric radiation, is the fact that EM radiation transports energy. Just as gravity waves on the surface of the ocean efficiently transport energy from a North Pacific storm to the sunny beaches of California, where that energy is violently deposited on the bodies of inattentive bathers, EM radiation transports vast quantities of energy from the thermonuclear furnace of the sun to the vinyl seat cover in your parked car.

In view of this fact, it would seem natural to characterize radiation in terms of its energy content, using the standard SI units of joules (J). But because natural radiation is not pulsed but rather continuous, it actually makes more sense to speak of the *rate* of energy transfer, or *power*, in watts (W = J/sec). Furthermore, radiation doesn't transport its energy through a single point but rather is distributed over an area. Therefore, the most natural way to describe the transport of energy by radiation at a location is in terms of its *flux density* F (commonly, though somewhat inaccurately, shortened to just *flux* in W/m^2. Other names you may see for the exact same quantity are 1) *irradiance*, and 2) radiant *exitance*. The term "irradiance" is often preferred when referring to the flux of radiation *incident on* a plane or surface, while the term "exitance" is usually applied to the flux of radiation *emerging from* a surface.

The magnitude of the flux depends on the orientation of the reference surface. If the radiation is due to a single source, such as the sun, one might measure the flux through an imaginary surface normal to the direction of propagation. On the other hand, one is often interested in the flux density of sunlight (or other radiation) obliquely incident on a horizontal surface. We will refine these ideas later in this chapter.

Problem 2.3: The atmospheric *boundary layer* is that region near the surface that is "well-mixed" by mechanical and/or convective turbulence originating at the surface. Its thickness may range from a few meters to several kilometers. Heat added by conduction from the surface is typically distributed quickly throughout the boundary layer.

At a certain location in the tropics, the sun rises at 06 Local Solar Time (LST), is directly overhead at noon, and sets at 18 LST (=6 PM). Assume that, during the twelve hours that the sun is up, the net flux of solar energy absorbed by dry vegetation and immediately transferred to the overlying air is $F(t) = F_0 \cos[\pi(t-12)/12]$, where t is the time of day (in hours), and $F_0 = 500\ \text{W m}^{-2}$.

(a) Ignoring other heating and cooling terms, compute the total solar energy (in J/m^2) added to the boundary layer over one 24-hour period.

(b) If the boundary layer depth $\Delta Z = 1$ km, its average air density is $\rho_a = 1\ \text{kg/m}^3$, and the heat capacity at constant pressure is $c_p = 1004$ J/(kg K), compute the temperature increase ΔT implied by your answer to (a).

(c) If, instead, the boundary layer depth started out at sunrise only 10 m deep and remained at that depth throughout the day, what would be the corresponding change of temperature? Why is it much more likely that the boundary layer would deepen quickly after sunrise?

2.5 A Mathematical Description of EM Waves†

Maxwell's Equations

We have been able to come quite far in describing the behavior of EM radiation without resorting to equations. Having laid the groundwork by defining terms and building mental models of electromagnetic waves, we can now tear through a more rigorous mathematical treatment with scarcely a pause for breath.

Just as the Navier-Stokes equation, the hydrostatic approximation, and the Ideal Gas Law serve as the underpinnings for virtually all of atmospheric dynamics, the so-called Maxwell equations govern classical electrodynamics. If one assumes SI units for all variables, these equations take the following form:

$$\nabla \cdot \vec{\mathbf{D}} = \rho, \tag{2.3}$$

$$\nabla \cdot \vec{\mathbf{B}} = 0, \tag{2.4}$$

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}, \tag{2.5}$$

$$\nabla \times \vec{\mathbf{H}} = \vec{\mathbf{J}} + \frac{\partial \vec{\mathbf{D}}}{\partial t}, \tag{2.6}$$

where $\vec{\mathbf{D}}$ is the *electric displacement*, $\vec{\mathbf{E}}$ is the *electric field*, $\vec{\mathbf{B}}$ is the *magnetic induction*, $\vec{\mathbf{H}}$ is the *magnetic field*, ρ is the density of electric charge, and $\vec{\mathbf{J}}$ is the electric current vector.

If we further assume (quite reasonably, for most materials) that we are dealing with a macroscopic, homogeneous medium whose electrical and magnetic properties are (i) directionally isotropic and (ii) linear with respect to $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$, then

$$\vec{\mathbf{D}} = \varepsilon_0(1+\chi)\vec{\mathbf{E}}, \tag{2.7}$$

where ε_0 is the *permittivity* of free space and χ is the electric *susceptibility* of the medium, which describes the degree to which the material becomes electrically polarized under the influence of an external field. Also,

$$\vec{\mathbf{B}} = \mu\vec{\mathbf{H}}, \tag{2.8}$$

where μ is the magnetic *permeability* of the medium, and

$$\vec{\mathbf{J}} = \sigma\vec{\mathbf{E}}, \tag{2.9}$$

where σ is the *conductivity.* Finally, we can usually assume that the charge density ρ is effectively zero.

With the above assumptions, Maxwell's equations reduce to

$$\nabla \cdot \vec{\mathbf{E}} = 0, \tag{2.10}$$

$$\nabla \cdot \vec{\mathbf{H}} = 0, \tag{2.11}$$

$$\nabla \times \vec{\mathbf{E}} = -\mu \frac{\partial \vec{\mathbf{H}}}{\partial t}, \tag{2.12}$$

$$\nabla \times \vec{\mathbf{H}} = \sigma \vec{\mathbf{E}} + \varepsilon_0 (1 + \chi) \frac{\partial \vec{\mathbf{E}}}{\partial t}. \tag{2.13}$$

Note that only two field variables, $\vec{\mathbf{E}}$ and $\vec{\mathbf{H}}$, appear in the above four equations. The (complex) parameters σ, χ, and μ then determine the relationship between $\vec{\mathbf{E}}$ and $\vec{\mathbf{H}}$ in any given medium.

Time-Harmonic Solution

As discussed in Section 2.2.1, we can always decompose an arbitrary EM disturbance into its various frequency components and consider each frequency separately from the others. In keeping with this approach, we will consider a general time-harmonic electric field of the form

$$\vec{\mathbf{E}}_c(\vec{\mathbf{x}}, t) = \vec{\mathbf{C}}(\vec{\mathbf{x}}) \exp(-i\omega t) \tag{2.14}$$

where $\vec{\mathbf{C}} = \vec{\mathbf{A}} + i\vec{\mathbf{B}}$ is a complex vector field, $\vec{\mathbf{x}}$ is the position vector, and $\omega = 2\pi\nu$ is the angular frequency (radians per second). An analogous representation can be written for the complex magnetic field $\vec{\mathbf{H}}_c$.

Note that the use of complex quantities is strictly a notational convenience; in physical problems, we are almost always interested in just the real part of whatever solutions we derive; e.g.,

$$\vec{\mathbf{E}} = \mathrm{Re}\{\vec{\mathbf{E}}_c\} = \vec{\mathbf{A}} \cos \omega t + \vec{\mathbf{B}} \sin \omega t \tag{2.15}$$

Substituting (2.14) into (2.10)–(2.13), we obtain the following relationships:

$$\nabla \cdot (\varepsilon \vec{\mathbf{E}}_c) = 0, \tag{2.16}$$

$$\nabla \times \vec{\mathbf{E}}_c = i\omega\mu \vec{\mathbf{H}}_c, \tag{2.17}$$

$$\nabla \cdot \vec{\mathbf{H}}_c = 0, \tag{2.18}$$

$$\nabla \times \vec{\mathbf{H}}_c = -i\omega\varepsilon\vec{\mathbf{E}}_c, \tag{2.19}$$

where the complex permittivity of the medium is defined as

$$\varepsilon = \varepsilon_0(1+\chi) + i\frac{\sigma}{\omega}. \tag{2.20}$$

Except for ε_0, which is a fundamental, real-valued physical constant, all other parameters depend on the medium under consideration as well as on the frequency ω.

Problem 2.4: Verify that (2.16)–(2.19) follow from (2.3)–(2.9) and (2.14).

Solution for a Plane Wave

Any harmonic electromagnetic field satisfying the above equations is physically realizable. However, we will restrict our attention to solutions describing a *plane wave*. Such solutions have the form

$$\vec{\mathbf{E}}_c = \vec{\mathbf{E}}_0 \exp(i\vec{\mathbf{k}} \cdot \vec{\mathbf{x}} - i\omega t), \qquad \vec{\mathbf{H}}_c = \vec{\mathbf{H}}_0 \exp(i\vec{\mathbf{k}} \cdot \vec{\mathbf{x}} - i\omega t), \tag{2.21}$$

where $\vec{\mathbf{E}}_0$ and $\vec{\mathbf{H}}_0$ are constant (complex) vectors, and $\vec{\mathbf{k}} = \vec{\mathbf{k}}' + i\vec{\mathbf{k}}''$ is a complex *wave vector*. Thus

$$\vec{\mathbf{E}}_c = \vec{\mathbf{E}}_0 \exp(-\vec{\mathbf{k}}'' \cdot \vec{\mathbf{x}}) \exp[i(\vec{\mathbf{k}}' \cdot \vec{\mathbf{x}} - \omega t)], \tag{2.22}$$

$$\vec{\mathbf{H}}_c = \vec{\mathbf{H}}_0 \exp(-\vec{\mathbf{k}}'' \cdot \vec{\mathbf{x}}) \exp[i(\vec{\mathbf{k}}' \cdot \vec{\mathbf{x}} - \omega t)]. \tag{2.23}$$

These relationships imply that the vector $\vec{\mathbf{k}}'$ is normal to planes of constant phase (and thus indicates the direction of propagation of the wave crests), while $\vec{\mathbf{k}}''$ is normal to planes of constant amplitude. The two are not necessarily parallel. When they are, or when $\vec{\mathbf{k}}''$ is zero, the wave is called *homogeneous.*

The term $\vec{\mathbf{E}}_0 \exp(-\vec{\mathbf{k}}'' \cdot \vec{\mathbf{x}})$ gives the *amplitude* of the electric wave at location $\vec{\mathbf{x}}$. If $\vec{\mathbf{k}}''$ is zero, then the medium is nonabsorbing, because the amplitude is constant. The quantity $\phi \equiv \vec{\mathbf{k}}' \cdot \vec{\mathbf{x}} - \omega t$ gives the *phase*. The *phase speed* of the wave is given by

$$v = \frac{\omega}{|\vec{\mathbf{k}}'|} \tag{2.24}$$

Substituting (2.21) into (2.16)–(2.19) yields

$$\vec{\mathbf{k}} \cdot \vec{\mathbf{E}}_0 = 0, \tag{2.25}$$

$$\vec{\mathbf{k}} \cdot \vec{\mathbf{H}}_0 = 0, \tag{2.26}$$

$$\vec{\mathbf{k}} \times \vec{\mathbf{E}}_0 = \omega\mu\vec{\mathbf{H}}_0, \tag{2.27}$$

$$\vec{\mathbf{k}} \times \vec{\mathbf{H}}_0 = -\omega\varepsilon\vec{\mathbf{E}}_0. \tag{2.28}$$

Problem 2.5: Verify that (2.25) follows from (2.21) and (2.16) by (a) expanding (2.21) in terms of the individual components of the vectors $\vec{\mathbf{E}}_0$, $\vec{\mathbf{k}}$, $\vec{\mathbf{x}}$ and (b) substituting this expression into (2.16) and applying the divergence operator $(\nabla \cdot)$. The remaining equations (2.26)–(2.28) are derived in an analogous way.

If we now take the vector product of $\vec{\mathbf{k}}$ with both sides of (2.27),

$$\vec{\mathbf{k}} \times (\vec{\mathbf{k}} \times \vec{\mathbf{E}}_0) = \omega\mu\vec{\mathbf{k}} \times \vec{\mathbf{H}}_0 = -\varepsilon\mu\omega^2\vec{\mathbf{E}}_0, \tag{2.29}$$

and use the vector identity

$$\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = \vec{\mathbf{b}}(\vec{\mathbf{a}} \cdot \vec{\mathbf{c}}) - \vec{\mathbf{c}}(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}), \tag{2.30}$$

we see from (2.25) that the first term on the right is zero, thus

$$\vec{\mathbf{k}} \cdot \vec{\mathbf{k}} = \varepsilon\mu\omega^2. \tag{2.31}$$

In the case of a homogeneous wave, the above simplifies to

$$(|\vec{\mathbf{k}}'| + i|\vec{\mathbf{k}}''|)^2 = \varepsilon\mu\omega^2 \tag{2.32}$$

or

$$|\vec{\mathbf{k}}'| + i|\vec{\mathbf{k}}''| = \omega\sqrt{\varepsilon\mu}. \tag{2.33}$$

Phase Speed

In a vacuum, $\vec{\mathbf{k}}'' = 0$, the permittivity of free space $\varepsilon \equiv \varepsilon_0 = 8.854 \times 10^{-12}$ F m^{-1}, and the magnetic permeability $\mu \equiv \mu_0 = 1.257 \times 10^{-6}$ N A^{-2}. From (2.24), we have the following phase speed in a vacuum:

$$c \equiv 1/\sqrt{\varepsilon_0 \mu_0}. \tag{2.34}$$

If we substitute the above numerical values of ε_0 and μ_0 into this expression, we obtain the speed of light in a vacuum $c = 2.998 \times 10^8$ m s^{-1}.

In a nonvacuum, we can write

$$|\vec{\mathbf{k}}'| + i|\vec{\mathbf{k}}''| = \omega \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} \sqrt{\varepsilon_0\mu_0} = \frac{\omega N}{c}, \tag{2.35}$$

where the complex *index of refraction* N is given by

$$\boxed{N \equiv \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} = \frac{c}{c'}}\,, \tag{2.36}$$

with $c' \equiv 1/\sqrt{\varepsilon\mu}$. If N happens to be pure real (i.e., if $\vec{\mathbf{k}}'' = 0$ and the medium is therefore nonabsorbing), then c' may be interpreted as the phase speed of the wave within the medium. Even in absorbing media, this is still a reasonable, though approximate, way to view c'.

For most physical media, $N > 1$, which implies a reduced speed of light relative to that in a vacuum. It is important to keep in mind that N is not only a property of a particular medium but also generally a strong function of frequency.

Absorption

Earlier, it was mentioned that the term $\vec{\mathbf{E}}_0 \exp(-\vec{\mathbf{k}}'' \cdot \vec{\mathbf{x}})$ gives the *amplitude* of the electric wave at location $\vec{\mathbf{x}}$. It turns out that the *flux density* F (power per unit area) transported by an EM wave is proportional to the *square* of this amplitude. Thus, for a plane wave with initial flux density F_0 at $\vec{\mathbf{x}} = 0$,

$$F = F_0[\exp(-\vec{\mathbf{k}}'' \cdot \vec{\mathbf{x}})]^2 = F_0 \exp(-2\vec{\mathbf{k}}'' \cdot \vec{\mathbf{x}}). \tag{2.37}$$

If we now consider a plane wave propagating in the x direction, and note from (2.35) that

$$|\vec{\mathbf{k}}''| = \frac{\omega}{c}\text{Im}\{N\} = \frac{\omega n_i}{c} = \frac{2\pi\nu n_i}{c} \tag{2.38}$$

where ν is the frequency in Hz, we have

$$F = F_0 e^{-\beta_a x}, \tag{2.39}$$

where the *absorption coefficient* β_a is defined as

$$\boxed{\beta_a = \frac{4\pi\nu n_i}{c} = \frac{4\pi n_i}{\lambda},} \tag{2.40}$$

with λ the wavelength of the radiation in a vacuum. In summary, the quantity $1/\beta_a$ gives the distance required for the wave's energy to be attenuated to $e^{-1} \approx 37\%$ of its original value.

Problem 2.6: Within a certain material, an EM wave with $\lambda = 1\ \mu$m is attenuated to 10% of its original intensity after propagating 10 cm. Determine the imaginary part of the index of refraction n_i.

Problem 2.7: For red light ($\lambda = 0.64\ \mu$m), n_i in pure water is approximately 1.3×10^{-8}; for blue light ($\lambda = 0.48\ \mu$m), $n_i \approx 1.0 \times 10^{-9}$. The deep end of a typical home swimming pool is approximately 2.5 m deep. Compute the fraction of each wavelength that survives the two-way trip to the bottom of the pool and back, when illuminated (and viewed) from directly above. In light of your findings (and in view of the appearance of most swimming pools as seen from the air), comment on the common assumption that water is "colorless."

2.6 Quantum Properties of Radiation

Now that I have succeeded in indoctrinating you with the wave-based mental model of EM radiation, including even a neat mathematical expression for a plane wave, I will demolish your newly won confidence by asserting that you should view EM radiation not as waves but as particles! At least some of the time.

It may surprise you to learn that Albert Einstein won his Nobel Prize not for his famous theory of relativity but rather for his explanation in 1905 of the *photoelectric effect*. This effect refers to the phenomenon by which electrons are jarred loose from a material surface exposed to light in a vacuum. A previously mysterious aspect of the photoelectric effect had been the observation that incident light with wavelengths longer than a certain threshold, no matter how intense, would not generate free electrons, whereas light with shorter wavelengths readily dislodges the electrons and continues to do so, sporadically at least, no matter how weak the illumination.

The essence of Einstein's explanation was that light falls on a surface not as a smoothly continuous flux of wave energy, but rather as a staccato hail of little discrete packets of energy, called *photons*. The energy content E of each individual photon, and therefore that photon's ability to knock electrons loose from the surface, is determined solely by the frequency or wavelength of the radiation via the relationship

$$\boxed{E = h\nu,} \tag{2.41}$$

where ν is the frequency and $h = 6.626 \times 10^{-34}$ J s is Planck's constant.

Moreover, you can't have just part of a photon; therefore very low intensity light deposits discrete packets of energy on a surface in a manner analogous to the occasional random splashes of fat raindrops on your windshield at the early onset of a rain shower. Thus, if monochromatic radiation of wavelength λ deposits F watts per unit area on a surface, then this corresponds to

$$\mathcal{N} = \frac{F}{h\nu} = \frac{F\lambda}{hc} \tag{2.42}$$

photons per unit area per unit time. For fluxes of the magnitude typically encountered in the atmosphere, $\mathcal{N}$ is rather large, and it is therefore hard to distinguish the effects of discrete particles, just as it is hard to distinguish the contributions of each individual raindrop to your growing wetness when you get caught out in a heavy downpour.

Problem 2.8: Only radiation with wavelengths smaller than 0.2424 μm is capable of dissociating molecular oxygen into atomic oxygen, according to the reaction

$$O_2 + \text{photon} \rightarrow O + O$$

Based on this information, how much energy is apparently required to break the molecular bond of a single molecule of O_2?

Problem 2.9: A small light source emits 1 W of radiation uniformly in all directions. The wavelength of the light is 0.5 μm.

(a) How many photons per second are emitted by the light source?

(b) If the light source were on the moon and were viewed by a telescope on Earth having a 20 cm diameter circular aperture, how many photons per second would the telescope collect? Ignore atmospheric attenuation. Assume a distance $D = 3.84 \times 10^5$ km between the moon and the earth.

The above *quantum* description of EM radiation is completely at odds with the previous *wave* description. They cannot both be true. And yet they are! Do not trouble yourself by trying to mentally reconcile the two models — countless smart people have tried throughout the twentieth century and all have failed to successfully explain this paradox in terms most people can visualize.

The important things for you to know are: 1) when radiation must be viewed as waves, 2) when it must be viewed as a shower of particles, and 3) when it doesn't matter.

As a general rule of thumb, the wave nature of radiation matters when computing the *scattering and reflection* properties of atmospheric particles (air molecules, aerosols, cloud droplets, raindrops) and surfaces. By contrast, the quantized nature of radiation, and thus (2.41), comes to the forefront when considering *absorption and emission* of radiation by individual atoms and molecules, including photochemical reactions. Finally, for calculations of large-scale transport of radiation in the atmosphere, the effects of both types of interactions will have already been deeply buried in some generic extinction and scattering coefficients and can be conveniently put out of your mind altogether.

2.7 Flux and Intensity

I have already briefly introduced the concept of *flux density* (or *flux* for short[2]) as a measure of the total energy per unit time (or power) per unit area transported by EM radiation through a plane (or deposited on a surface). It is now time to extend our understanding of this property and to introduce a closely related quantity, called the *radiant intensity* , or just *intensity* for short. In some books, the term *radiance* is substituted for intensity.

Flux and intensity are the two measures of the strength of an EM radiation field that are central to most problems in atmospheric science. The two are intimately related, as we shall see shortly.

2.7.1 Flux

Recall first of all that the flux F refers to the rate at which radiation is incident on, or passes through, a flat surface. Without further qualification, flux is expressed in units of watts per square meter. The surface may be real (e.g., the ground, or the top of a cloud layer) or it may be imaginary (e.g., an arbitrary level in the atmosphere). Often, but not always, it is taken to be horizontal. Other times it may be assumed to be perpendicular to a single source of radiation, such as the sun.

Note that a flux of natural (incoherent) radiation expressed sim-

[2]Strictly speaking, an energy *flux* has units of W, whereas the *flux density* is the flux *per unit area* and therefore has units of W m^{-2}. Meteorologists are almost invariably concerned with quantities that are expressed per unit area, volume, mass, etc. Therefore, when a meteorologist says “flux,” it is generally understood that she *means* “flux per unit area” or flux density.

ply in $\mathrm{W/m^2}$ *must* be a broadband quantity.[3] That is, it includes energy contributions from all wavelengths between some specified (or implied) limits λ_1 and λ_2. Those limits might encompass all possible wavelengths (i.e., $\lambda_1 = 0$, $\lambda_2 = \infty$), or they might define a somewhat narrower range. However, the range cannot be zero ($\lambda_1 = \lambda_2$), because the power contained in that range would then also be zero!

It is, however, possible to define a *monochromatic flux* (also known as *spectral* flux) F_λ as follows:

$$F_\lambda = \lim_{\Delta\lambda \to 0} \frac{F(\lambda, \lambda + \Delta\lambda)}{\Delta\lambda}, \tag{2.43}$$

where $F(\lambda, \lambda + \Delta\lambda)$ is the flux in $\mathrm{W/m^2}$ contributed by radiation with wavelengths between λ and $\lambda + \Delta\lambda$. The dimensions of the monochromatic flux are not just power per unit area but rather *power per unit area per unit wavelength*. Typical units would thus be $\mathrm{W\ m^{-2}\ \mu m^{-1}}$.

Having defined the monochromatic (or spectral) flux as above, you get the broadband flux over some extended range of wavelength $[\lambda_1, \lambda_2]$ by integrating over the appropriate range of wavelength:

$$\boxed{F(\lambda_1, \lambda_2) = \int_{\lambda_1}^{\lambda_2} F_\lambda \, d\lambda \; .} \tag{2.44}$$

Problem 2.10: The total radiation flux incident on a surface due to wavelengths between 0.3 μm and 1.0 μm is 200 $\mathrm{W\ m^{-2}}$. (a) What is the average *spectral* flux within this interval? Give your answer in units of $\mathrm{W\ m^{-2}\ \mu m^{-1}}$. (b) If the spectral flux is constant with wavelength, then what is the total flux contributed by wavelengths just between 0.4 μm and 0.5 μm? (c) What is the total flux (in $\mathrm{W\ m^{-2}}$) contributed by radiation of exactly 0.5 μm wavelength?

Let's illustrate the concept of flux with a concrete example. If you mark out an area on flat ground out in the open, the amount

[3] This is not necessarily true for artificial coherent radiation, for which finite power may be associated with exactly one wavelength, rather than being distributed over a range of wavelengths.

of daylight falling on it can be measured in watts per square meter. This is the *incident flux* of solar radiation, and it determines how much total radiation from the sun is available to be absorbed. If a cloud passes in front of the sun, the incident flux temporarily decreases because the transmission of radiation from the sun to the surface is partially blocked. As the afternoon wears on, the incident flux steadily decreases, this time because the light from the sun strikes the surface at an increasingly oblique angle, spreading the same energy over a larger and larger area.

An important point to remember is that the flux makes no distinction concerning where the radiation is coming *from*. 100 W m^{-2} incident on our front lawn is 100 W m^{-2}, regardless of whether it is coming from all directions more or less equally on an overcast day or primarily one particular direction (that of the sun) on a crystal-clear afternoon. *In order to completely characterize the radiation field at a given location, we must know not only the flux but also the direction(s) from which the radiation is coming and thus also in which direction(s) it is going.* This directional information is embodied in the radiant *intensity*.

2.7.2 Intensity

The radiant *intensity* I tells you in detail both the strength and direction of various sources contributing to the incident flux on a surface. For visible radiation, intensity corresponds roughly to the "brightness" your eyes see looking backward along a ray of incoming radiation. Thus, if you lie flat on your back and look up at the sky, you can visually map out which regions of the sky are associated with high radiant intensity and therefore contribute most strongly to the total incident solar flux at your location. The sun itself, if not blocked by a cloud, is seen as a very localized high intensity source, whereas the clear sky is a relatively uniform source of rather low intensity radiation. Isolated clouds, as seen from your vantage point, may be either brighter or darker than the clear sky, depending on how thick they are and from what angle they are viewed, relative to the sun.

Consider again what happens if a small cumulus cloud passes between you and the sun, casting you and your marked out area of ground into shadow. You can now look directly toward the sun without hurting your eyes. From this you can conclude that the intensity of radiation from that direction has been greatly reduced. But the contributions from the rest of the sky are unaffected. The total incident shortwave *flux* is therefore reduced but not eliminated. Although you are now in the shadow of the cloud, it is by no means pitch dark.

Clearly, there is a very close relationship between flux and intensity. In words, the flux incident on a surface is obtained by integrating the contributions of intensity from all possible directions visible from that surface. It is time to delve into the mathematics of this relationship. We will begin by setting up the necessary machinery.

Spherical Polar Coordinates

In any discussion of radiation, direction plays an all-important role. We therefore need to adopt a convention for describing directions. There is no single "right" choice, but one that is very convenient in atmospheric radiation is based on spherical polar coordinates. Fig. 2.3 depicts the geometry.

In this system, the *zenith* angle θ measures the angle from some reference direction, usually the local vertical. Thus, directly overhead usually corresponds to $\theta = 0$. In this case, the horizon corresponds to $\theta = \pi/2$ radians, or 90°. $\pi/2 < \theta < \pi$ corresponds to directions below the horizon, with $\theta = \pi$ being straight down ("nadir").

The *azimuthal* angle ϕ measures the angle counterclockwise from a reference point on the horizon, so that $0 < \phi < 2\pi$. It is not usually terribly important which point of the compass is used as the reference in any given application, and it is often chosen to be whatever is most natural for the problem at hand, such as the direction of the sun.

Any possible direction above or below the horizon may thus be described via the two angles θ and ϕ. Sometimes directions may be expressed abstractly in terms of a unit vector $\hat{\mathbf{\Omega}}$, in which case no particular coordinate system is implied. Thus the same direction $\hat{\mathbf{\Omega}}$ could be represented by $[\sqrt{2}/2, 0, \sqrt{2}/2]$ in (x, y, z)-coordinates, by $[\pi/4, 0]$ in (θ, ϕ))-coordinates when $\theta = 0$ defines the vertical

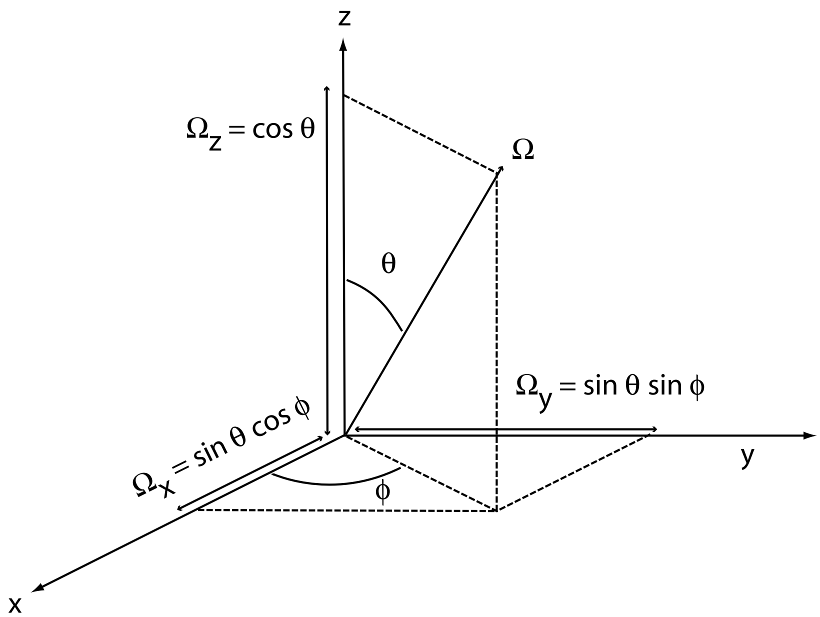

Fig. 2.3: The relationship between Cartesian and spherical coordinates.

direction, or even by $[0, 0]$ if $\theta = 0$ should for some reason be chosen to coincide with the direction of the sun at a time when the latter is 45° above the horizon.

Solid Angle

Another essential concept is that of *solid angle*. Surprisingly many new students of atmospheric radiation find this concept confusing, presumably because they haven't had occasion to consciously use it before, unlike the angles we have been measuring with protractors since third grade. But it's really very simple: *solid angle is to "regular" angle as area is to length.* You can think of solid angle as something you might measure in "square radians" or "square degrees", except the actual unit used is called the *steradian*. We will give a precise definition of this unit later.

In absolute terms, the sun has a certain diameter in kilometers and a certain cross-sectional area in km^2. But absolute dimensions are often of secondary importance in radiative transfer, compared with *angular* dimensions, which describe how big an object or source of radiation *looks* from a particular vantage point. Thus, what matters for solar radiation reaching the earth is that the sun's disk has a particular angular diameter in units of degrees or radians, and it also subtends (or presents) a certain solid angle in units of steradians. Solid angle is thus a measure of how much of your visual field of view is occupied by an object. For example, the sun subtends a much larger solid angle as viewed from the planet Mercury than it does from Earth. Also, from our perspective here on Earth, the full moon subtends nearly the same solid angle as the Sun, even though the latter body is much larger in absolute terms. A half moon, of course, subtends half the solid angle of the full moon.

Definition of Steradian

Now that you understand what solid angle *is*, you can appreciate a simple definition of the unit *steradian* (abbreviation sr). Imagine you are at the center of a sphere of unit radius — it doesn't matter whether the unit is a kilometer, a mile, a furlong or what have you. The total surface area of the sphere is 4π square units. *Likewise, the combined solid angle represented by every direction you can possibly look is* 4π *steradians*. The surface area of just one half of the sphere is 2π units squared. *Likewise, the entire sky above the horizon* (or "celestial dome") *subtends a solid angle of* 2π *steradians, as does (separately) the lower hemisphere of your field of vision, representing everything below the horizon.*

Conceptually, you can determine the solid angle subtended by any object by tracing its outline on your unit sphere and then measuring the actual surface area of the tracing. (Note the analogy to radians as a measure of arc length on the unit circle.) This is generally not a practical approach, however, so we instead invoke our polar coordinate system so as to be able to define an infinitesimal increment of solid angle as follows:

$$\boxed{d\omega = \sin\theta \; d\theta \; d\phi \; .} \qquad (2.45)$$

This relationship is depicted schematically in Fig. 2.4.

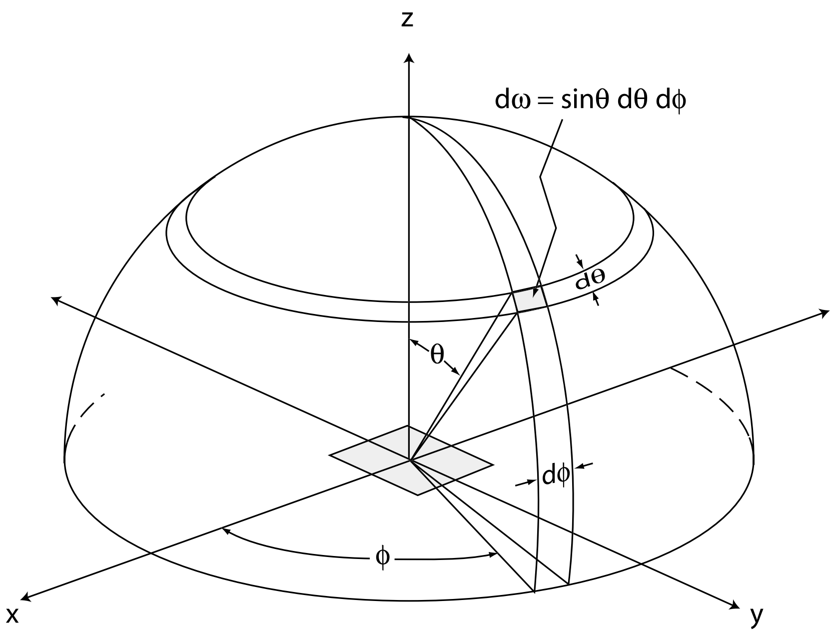

Fig. 2.4: The relationship between solid angle and polar coordinates.

In other words, if you paint an infinitesimal rectangle on the surface of your unit sphere, and it has angular dimensions $d\theta$ (zenith angle) and $d\phi$ (azimuth angle) and is positioned at θ, then the above expression gives you the increment of solid angle subtended. Why does $\sin\theta$ appear in there? Simple: for the same reason that a 1° latitude by 1° longitude box encompasses far less real estate near the North Pole than near the equator, because of the convergence of lines of equal ϕ to a point.

Now let's demonstrate that we can recover the expected value of 4π steradians for the entire sphere:

$$\int_{4\pi} d\omega = \int_0^{2\pi}\int_0^{\pi} \sin\theta \; d\theta \; d\phi = 2\pi \int_0^{\pi} \sin\theta \; d\theta = 4\pi. \qquad (2.46)$$

Note that the notation used in the left-most integral expresses the *abstract* idea that we are integrating over the full sphere (4π steradians). This notation makes no assumption about the coordinate system that we will use to actually perform the integration. The integral just to the right of it translates the abstract integration over all directions into a specific double integral using our coordinates θ and ϕ, based on (2.45). If we were to use a different coordinate system to describe directions, then the integrals on the right-hand side might take a different form, but the final result for this problem, if everything is done correctly, should still be 4π steradians!

Problem 2.11: Consider a cloud that, when viewed from a point on the surface, occupies the portion of the sky defined by $\pi/4 < \theta < \pi/2$ and $0 < \phi < \pi/8$.

(a) What is the solid angle subtended by the cloud?

(b) What percentage of the sky is covered by this cloud?

Problem 2.12: The moon is at a mean distance $D_m = 3.84 \times 10^5$ km from the earth; the Sun is at a mean distance $D_s = 1.496 \times 10^8$ km. The radius of the moon is $R_m = 1.74 \times 10^3$ km; the radius of the sun is $R_s = 6.96 \times 10^5$ km.

(a) Compute the angular diameter (in degrees) subtended by the sun and the moon. (b) Compute the solid angle subtended by the sun and the moon. (c) Which appears larger from Earth, and by what percentage do the two solid angles differ? (d) If the above values were constant, would it be possible to explain the occurrence of total solar eclipses?

Formal Definition of Intensity

Having defined solid angle, we are now in a position to attach a precise definition to the term radiant intensity. In words, *intensity* $I(\hat{\mathbf{\Omega}})$ *is the flux* (measured on a surface normal to the beam) *per unit solid angle traveling in a particular direction* $\hat{\mathbf{\Omega}}$. Visualize the above definition as follows:

- Looking in the direction $-\hat{\mathbf{\Omega}}$, identify a very small element of the scene with solid angle $\delta\omega$.
- Measure, normal to the beam, the flux δF of radiation arriving just from that small region, while excluding all other contributions.

- The intensity in that direction is then given by

$$\boxed{I(\hat{\mathbf{\Omega}}) = \frac{\delta F}{\delta \omega}\ .} \tag{2.47}$$

This recipe is strictly valid only when $\delta\omega$ is vanishingly small, but it's a reasonable approximation for finite solid angles as well, as long as the maximum arc subtended by the region in question is much less than one radian — say 5° or less — *and* as long as the intensity is uniform throughout the region.

Problem 2.13: The broadband flux of solar radiation that reaches the top of the atmosphere is approximately 1370 W m^{-2}, when measured on a plane normal to the beam. Combine this with the solid angle you derived in a previous problem to compute the average radiant intensity of the sun's surface.

Conservation of Intensity

You have probably known since you were small that the ability of a light source to illuminate an object weakens rapidly with increasing distance. A flashlight illuminates a book under the bedcovers much more brightly than it does an animal lurking in those bushes at the edge of the campsite. Likewise, the earth is much more brightly illuminated by the sun than is Pluto.

You might hastily infer from these observations that the *intensity* of radiation associated with a given point at the source is also a function of the distance of the observer. If you were talking about the incident *flux*, you would be right. This is not the case for intensity however. On the contrary, *in a vacuum or other transparent medium without reflecting surfaces, radiant intensity is conserved along any optical path.*

To verify this principle in a simple case, tape a sheet of white paper to the wall at eye level. Look at it from a couple feet away; then back up and look at it from across the room. Although its apparent size (solid angle) changes with distance, its apparent brightness does not (assuming you didn't adjust the room lights!).

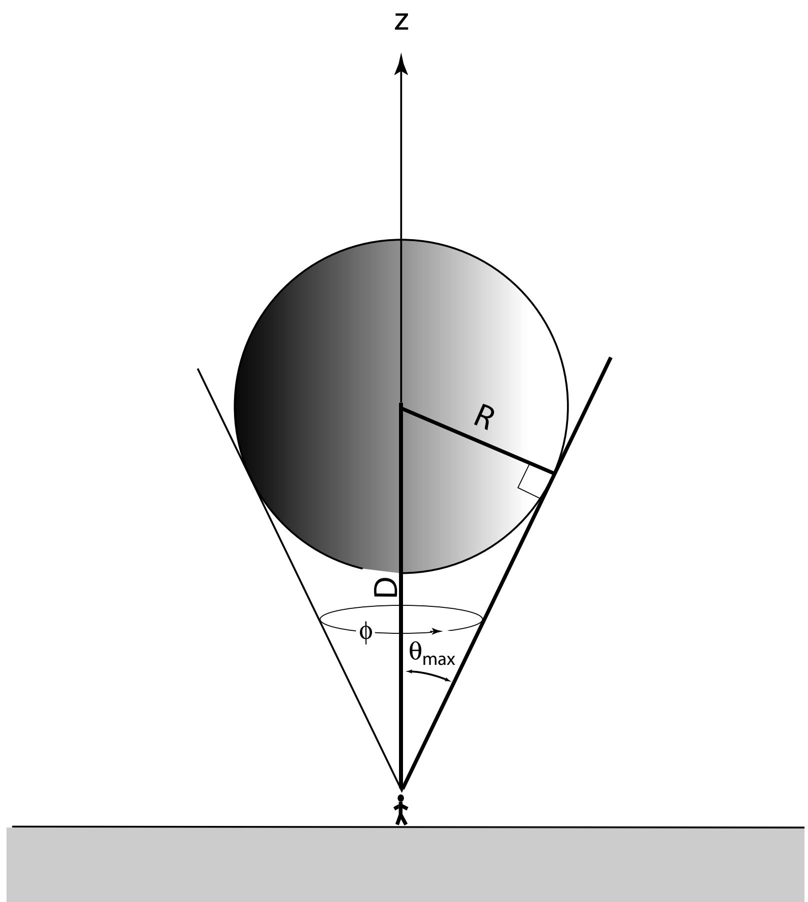

Fig. 2.5: Geometric framework for computing the solid angle subtended by a sphere of radius R whose center is a distance D from the observer.

Problem 2.14: Verify the above principle for the Sun's disk by deriving a formula for its average intensity as seen from a distance D from the Sun's center, given that its total radiant power output is P and its radius is R_s. For simplicity, assume that the Sun's intensity I is the same at all points on the visible disk (this is not strictly true). You will need to derive an exact expression for the solid angle subtended by the Sun's disk for arbitrary $D > R_s$. If done correctly, your final solution for I should not depend on D. (Hint: To find the solid angle subtended by the Sun, assume that it is directly overhead, so that the center coincides with the z-axis and the edge is defined by $\theta = \theta_{max}$ (see Fig. 2.5). Then integrate (2.45) with the appropriate limits.)

The same principle applies to optical systems such as lenses, mirrors, prisms, etc., as long as we ignore losses due to absorption and/or partial reflections (e.g., from the surface of lenses). A magnifying glass makes objects appear larger, but it has no effect on the object's brightness. The same is true of binoculars and telescopes.[4]

Intensity and Polarization†

For many applications, one need only keep track of the total *scalar* intensity I of a stream of radiation as defined above. However, we previously alluded to the polarization properties of EM radiation, and it is sometimes necessary to keep track of these as well. One reason might be a need for greater accuracy in radiative transfer calculations, as disregarding polarization almost always entails an approximation, especially when scattering by particles or surfaces is important. Another occasion arises when you are measuring radiation of one particular polarization (e.g, linear vertical or horizontal). This is often the case for microwave remote sensing instruments.

When polarization must be considered, we require a representation of intensity that is capable of providing complete information about the state of polarization. One such representation gives the

[4] But wait, you say. Telescopes definitely do make stars appear brighter: many that are invisible to the naked eye become clearly visible through a telescope. How can this behavior be reconciled with the previous assertions? The explanation is that stars subtend a angle far too small for the eye to resolve. As a result, the eye responds not to the intensity of the star but rather to the total flux integrated over a finite solid angle. That solid angle is determined by the eye's resolving power. Thus, moderately near-sighted individuals will see a few bright stars but will miss many more that are easily detectable by sharp-eyed people. A telescope increases the solid angle subtended by a star, and thus the total flux from that direction, making it more easily visible to everyone.

intensity as Intensity!as Stokes vector a four-element vector

$$\boxed{\vec{\mathbf{I}} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} .} \tag{2.48}$$

The elements of this vector are called the *Stokes parameters*. The first element, I, is the same as the scalar intensity we have already discussed. The remaining elements Q, U, V contain information concerning the *degree* of polarization (recall that incoherent radiation can be polarized to any degree, whereas coherent radiation is always fully polarized), about the *preferred orientation* of the polarization, and about the *nature* of the polarization – circular, linear, or something in between. In particular, the *degree of polarization* is defined as $\sqrt{Q^2 + U^2 + V^2}/I$. The ratios $\sqrt{Q^2 + U^2}/I$ and V/I, respectively, are *the degree of linear polarization* and *the degree of circular polarization*.

Thus, for completely unpolarized radiation, $Q = U = V = 0$, and for fully polarized radiation

$$I^2 = Q^2 + U^2 + V^2. \tag{2.49}$$

It is beyond the scope of this introductory text to give a detailed electromagnetic definition of each of the Stokes parameters.[5] However, some illustrative examples are given in Table 2.1.

How is the vector representation of intensity actually used? In the scalar case (i.e., when we're ignoring polarization), most interactions of radiation with matter can be described via the multiplication of the intensity by a scalar coefficient, which we will arbitrarily denote A. Thus,

$$I_{\text{new}} = A \cdot I_{\text{old}}. \tag{2.50}$$

For example, if the process in question is a reflection from a surface, the coefficient A might represent a scalar reflectivity value ranging from 0 to 1.

[5] A good overview is given in Section 2.3 of S94.

Table 2.1: Examples of Stokes parameter values.

Description	**[I,Q,U,V]**
Vertically polarized	$[1,1,0,0]$
Horizontally polarized	$[1,-1,0,0]$
Linearly polarized at +45°	$[1,0,1,0]$
Linearly polarized at −45°	$[1,0,-1,0]$
Right circularly polarized	$[1,0,0,1]$
Left circularly polarized	$[1,0,0,-1]$
Unpolarized	$[1,0,0,0]$

In the fully polarized case, the scalar coefficient A is replaced by a 4×4 *Mueller matrix* $\mathbf{A}$, so that the new intensity is described via the matrix operation

$$\vec{\mathbf{I}}_{\mathrm{new}} = \mathbf{A}\vec{\mathbf{I}}_{\mathrm{old}}. \tag{2.51}$$

Thus, the Mueller matrix describes not only how the overall intensity changes but also how the polarization changes. For example, an optical device represented by the following Mueller matrix $\mathbf{A}$ will transform a beam of radiation having arbitrary polarization into one that is 100% right-circularly polarized:

$$\mathbf{A} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}. \tag{2.52}$$

Problem 2.15: Demonstrate that (2.52) does what is claimed.

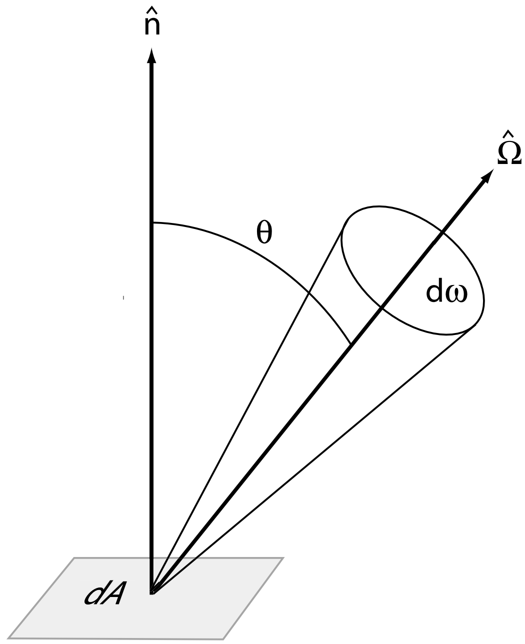

Fig. 2.6: The flux density of radiation carried by a beam in the direction $\hat{\mathbf{\Omega}}$ through a surface element dA is proportional to $\cos\theta = \hat{\mathbf{n}} \cdot \hat{\mathbf{\Omega}}$.

2.7.3 Relationship between Flux and Intensity

We previously defined the flux F as the total power incident on a unit area of surface. We then defined intensity in terms of a flux contribution arriving from a very small element of solid angle $d\omega$ centered on a given direction of propagation $\hat{\mathbf{\Omega}}$. It follows that the flux incident on, passing through, or emerging from an arbitrary surface is given by an integral over the relevant range of solid angle of the intensity.

Let us start by considering the flux emerging *upward* from a horizontal surface: it must be an integral of the intensity $I(\hat{\mathbf{\Omega}})$ over all possible directions $\hat{\mathbf{\Omega}}$ directed skyward; i.e., into the 2π steradians of solid angle corresponding to the upper hemisphere. There is one minor complication, however. Recall that intensity is defined in terms of flux per unit solid angle *normal to the beam*. For our horizontal surface, however, only one direction is normal; radiation from all other directions passes through the surface at an oblique angle (Fig.

2.6). Thus, we must weight the contributions to the flux by the cosine of the incidence angle relative to the normal vector $\hat{\mathbf{n}}$. For the upward-directed flux $F^{\uparrow}$, we therefore have the following relationship:

$$F^{\uparrow} = \int_{2\pi} I^{\uparrow}(\hat{\mathbf{\Omega}})\hat{\mathbf{n}} \cdot \hat{\mathbf{\Omega}}\, d\omega. \tag{2.53}$$

The above expression is generic: it doesn't depend on one's choice of coordinate system. In practice, it is convenient to again introduce spherical polar coordinates, with the z-axis normal to the surface:

$$\boxed{F^{\uparrow} = \int_0^{2\pi}\int_0^{\pi/2} I^{\uparrow}(\theta,\phi)\cos\theta\sin\theta\; d\theta d\phi\;,} \tag{2.54}$$

where we have used (2.45) to express $d\omega$ in terms of θ and ϕ.

For the downward flux, we integrate over the lower hemisphere, so we have

$$\boxed{F^{\downarrow} = -\int_0^{2\pi}\int_{\pi/2}^{\pi} I^{\downarrow}(\theta,\phi)\cos\theta\sin\theta\; d\theta d\phi\;.} \tag{2.55}$$

Since I is always positive, the above definitions always yield positive values for $F^{\uparrow}$ and $F^{\downarrow}$.

Key fact: For the special case that the intensity is *isotropic* — that is, I is a constant for all directions in the hemisphere, then the above integrals can be evaluated to yield

$$\boxed{F = \pi I\;.} \tag{2.56}$$

Key fact: The *net flux* is defined as the difference between upward- and downward-directed fluxes:

$$\boxed{F_{\text{net}} \equiv F^{\uparrow} - F^{\downarrow}\;,} \tag{2.57}$$

which can be expanded as

$$F_{\text{net}} = \int_0^{2\pi}\int_0^{\pi} I(\theta,\phi)\cos\theta\; d\theta d\phi = \int_{4\pi} I(\hat{\mathbf{\Omega}})\hat{\mathbf{n}} \cdot \hat{\mathbf{\Omega}}\, d\omega. \tag{2.58}$$

Note, by the way, that the notation used throughout this subsection implies that we are relating a *broadband intensity* to a *broadband flux*. Identical relationships hold between the *monochromatic intensity* I_λ and the monochromatic fluxes $F_\lambda^\uparrow$ and $F_\lambda^\downarrow$.

Problem 2.16: If the intensity of radiation incident on a surface is uniform from all directions and denoted by the constant I, verify that the total flux is πI, as stated by (2.56). Note that this approximately describes the illumination of a horizontal surface under a heavily overcast sky. It also describes the relationship between the flux and intensity of radiation *leaving* a surface, if that surface is emitting radiation of uniform intensity in all directions.

Problem 2.17: Compute the flux from an overhead spherical sun, as seen from a planet in an orbit of radius D, given that the sun has radius R_s and a uniform intensity I_s. Make no assumptions about the size of D relative to R_s. Use two different methods for your calculation:

(a) Method 1: Integrate the intensity over the solid angle subtended by the sun, with the usual cosine-weighting relative to the local vertical.

(b) Method 2: Compute the flux density emerging from the surface of the sun, translate that into a total power emitted by the sun, and then distribute that power over the surface of a sphere of radius D.

Do your two solutions agree?

2.8 Applications to Meteorology, Climatology, and Remote Sensing

Of fundamental importance to the global climate is the input of energy from the sun and its spatial and temporal distribution. This input is a function of two variables: 1) the flux of solar radiation incident on the top of the atmosphere, and 2) the fraction of that flux that is absorbed by either the surface or the atmosphere at each

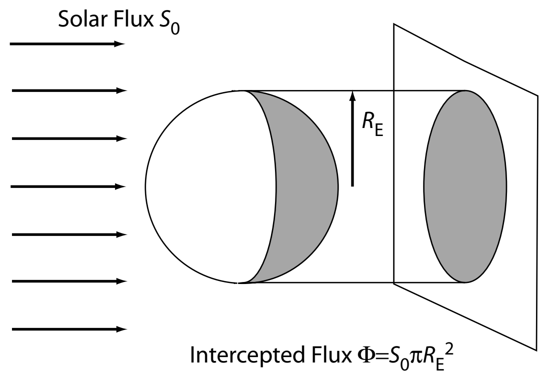

Fig. 2.7: The total flux of solar radiation intercepted by the earth is equal to the product of the incident flux density S_0 and the area of the earth's shadow.

point in the earth-atmosphere system. The second of these depends in a complex way on distributions of clouds and absorbing gases in the atmosphere, as well as on the absorbing properties of the surface. These are all issues that will be taken up in the remainder of this book. The first variable, however, can already be understood in terms of the material presented in this chapter.

2.8.1 Global Insolation

The first question that may be asked is, how much *total* solar radiation Φ is incident on the earth's atmosphere, on average? This question is easily solved by computing how much of the Sun's output is intercepted by the earth's disk. That is, given that the mean solar flux at Earth's mean distance from the Sun is $S_0 = 1370$ W m^{-2}, what cross-sectional area is presented to that flux by the earth (i.e., how big of a shadow does the earth cast)? The answer of course[6] is $A = \pi R_E^2$, where the effective radius of the earth is $R_E = 6356$ km (Fig. 2.7). Thus

$$\Phi = S_0 \pi R_E^2 = 1.74 \times 10^{17} \text{ W} \tag{2.59}$$

This result is consistent with a mean distance of the earth from the sun ($\overline{D}_s$) of 1.496×10^8 km. The reality, however, is that the

[6]Strictly speaking, this is an approximation that is valid only because the earth's radius is far smaller than the radius of the earth's orbit.

earth's orbit is slightly elliptical, with the D_s varying from 1.47×10^8 km near January 3 (perihelion) to 1.52×10^8 km on about July 5 (aphelion). Thus, the top-of-the-atmosphere (TOA) solar flux S varies seasonally from as little as 1330 W m^{-2} in July to as much as 1420 W m^{-2} in January.

Problem 2.18: Derive an expression for S as a function of S_0, $\overline{D}_s$, and D_s. Show that $\Delta S/S_0 \approx -2\Delta D_s/\overline{D}_s$. That is, a positive 1% change in D_s leads to a negative 2% change in S.

2.8.2 Regional and Seasonal Distribution of Insolation

At the distance of the earth from the sun, there is a more or less constant flux of solar radiation of $S_0 = 1370$ W m^{-2}. As noted above, there is actually some deviation from this value over the course of the year, owing to the slightly varying distance of the earth from the sun. Also, the power output P from the sun itself varies slightly over time, due to factors such as sunspot activity as well as other longer term variations that are neither well-measured nor well-understood.

Even ignoring minor variations in S_0 itself, it is clear that solar radiation is not uniformly incident on the earth. The night side of the earth receives no solar radiation at all. And even on the daylight side, the flux of solar radiation measured on a unit horizontal area at the top of the atmosphere depends on the angle of incidence of the sun. If the sun is directly overhead (solar zenith angle $\theta_s = 0$), then the flux is equal to S_0, but if $\theta_s > 0$, then a unit area normal to the sun's rays projects onto a larger area on the earth's surface (Fig. 2.8). Thus, the solar flux measured on a unit horizontal area is given by

$$F = S_0 \cos\theta_s \, . \tag{2.60}$$

Now consider the *total* insolation [energy per unit area] at the top of the atmosphere at a single location over the course of a 24-hour period. This insolation is given by

$$W = \int_{t_{\text{sunrise}}}^{t_{\text{sunset}}} S_0 \cos\theta_s(t) \; dt. \tag{2.61}$$

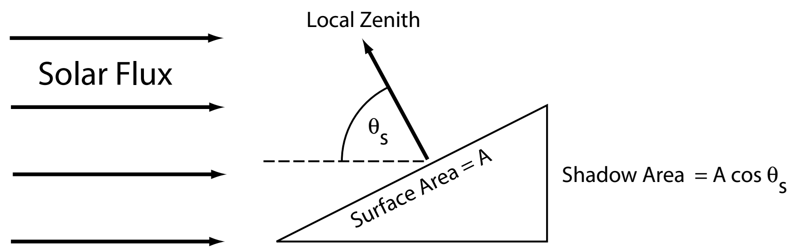

Fig. 2.8: The relationship between local solar zenith angle θ_s and insolation on a local horizontal plane.

As you can probably guess from this expression (and from everyday experience), W depends on two readily identifiable factors: (1) the length of the day $t_{\text{sunset}} - t_{\text{sunrise}}$, and (2) the average value of $\cos\theta_s(t)$ during the time the sun is up.

At the equator, the length of a day is 12 hours year-round, but the maximum elevation the sun reaches in the course of the day varies with the time of year. Twice a year, at the time of the vernal and autumnal equinox (approximately March 21 and September 21, respectively), the sun passes directly overhead at noon. At other times of year, the minimum zenith angle achieved in the course of the day is equal to the angle of tilt of the earth's axis toward or away from the sun, up to a maximum of 23° at the time of the summer and winter solstices (June 21 and December 21, respectively).

At latitudes poleward of 23°, the sun is never directly overhead, and the minimum zenith angle is always greater than zero. During the summer season, the sun can reach a point fairly high in the sky, whereas in the winter season, the maximum elevation angle is much lower. Moreover, the days are longer in the summer hemisphere than in the winter hemisphere. Indeed, poleward of the arctic or antarctic circles, there is a substantial period of time during the winter when the sun never comes up at all, while during the corresponding period of high summer, the sun never sets. At the poles themselves, the situation is very simple: the sun is up continuously for one half of the year, and the solar zenith angle θ_s is nearly constant over a 24-hour period.[7]

[7] Atmospheric refraction allows the sun to be visible from a location on the earth's surface when it is actually about 0.5°, or approximately the diameter of the

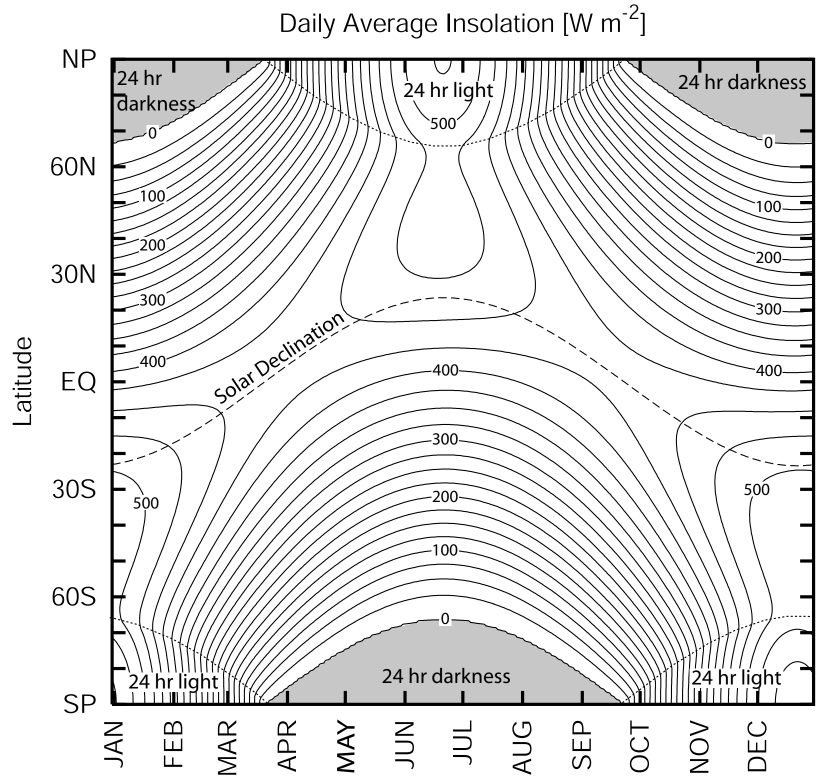

Fig. 2.9: Daily average solar flux at the top of the atmosphere, as a function of latitude and time of year. Contour values are given in units of W m^{-2}
.

The combined effects of the length of day, of the variation in $\cos\theta_s$, and of the slight variation of the earth's distance from the sun on daily insolation (at the top of the atmosphere) are depicted in Fig. 2.9. Blacked-out areas depict dates and latitudes for which the sun never emerges above the horizon. The dashed line ("decli-

sun's disk, below the horizon. Thus, the sun rises somewhat sooner and sets somewhat later than would be predicted from geometric considerations alone. Therefore, the length of continuous daylight at the North Pole (for example) is actually somewhat longer than the expected six months.

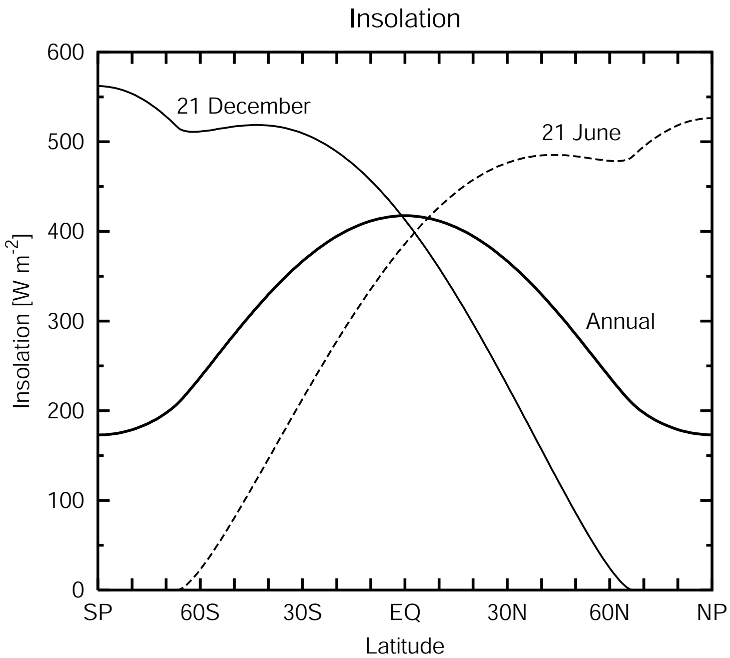

Fig. 2.10: Daily average solar flux at the top of the atmosphere as a function of latitude, for the two solstice dates and averaged over a year.

nation of the sun") indicates the dates/latitudes at which the noontime sun passes directly overhead. Not surprisingly, this curve coincides with the location of maximum daily insolation over most of the year. However, within a week or two of the summer solstice, the maximum daily insolation is found instead near the pole, where there is daylight for a full 24 hours *and* the sun is a relatively high 23° above the horizon for the entire day.

If you integrate the daily insolation at a given latitude over the entire annual cycle and then divide your result by the number of days in a year, you get the *daily average insolation*, as depicted by the heavy curve labeled "Annual" in Fig. 2.10. Also shown is the daily insolation for the two solstice dates.

In closing, I would like to remind you that the insolation discussed above describes only the amount of solar radiation incident at the top of the atmosphere. It is thus an *upper bound* on the amount of solar radiation that is available to be absorbed by the earth and atmosphere. In reality, a significant fraction of this radiation is immediately reflected back to space by clouds, aerosols, air molecules, and the underlying surface. A good part of the rest of this book is concerned with the processes that determine how much radiation is absorbed and how much is reflected.

Problem 2.19: Compute, and compare with Fig. 2.9, the daily average top-of-the-atmosphere insolation [W m^{-2}] for the following two cases: (a) the North Pole at the time of the Northern Hemisphere summer solstice; (b) the equator at the time of the equinox. Assume that the solar flux normal to the beam is a constant 1370 W m^{-2}, and note that the North Pole is inclined 23° toward the Sun at the time of the solstice.

CHAPTER 3

The Electromagnetic Spectrum

In the previous chapter, we examined how electromagnetic radiation behaves on a purely physical level, without being concerned yet with its detailed interactions with matter. One important observation was that we can treat an arbitrary radiation field as a superposition of many "pure" sinusoidal oscillations. The clearest everyday example of this is the rainbow: white sunlight interacting with raindrops is decomposed into the constituent colors red through violet, each of which corresponds to a narrow range of frequencies. Radiation associated with a given frequency and trajectory in space may be analyzed completely independently of all the others.

We also saw that there is no fundamental constraint on the frequency that EM radiation can exhibit, as long as an oscillator with the right natural frequency and/or an energy source with the minimum required energy is present (recall from Section 2.6 that a single photon has a specific energy determined by its frequency and that an oscillator cannot emit less than that minimum amount).

In a vacuum, the frequency or wavelength of a photon is of little practical consequence, as it cannot be absorbed, scattered, reflected, or refracted but rather is condemned to continue propagating in a straight line forever, regardless. In the presence of matter however, the frequency becomes an all-important property and, to a very great degree, determines the photon's ultimate fate.

There are several reasons why frequency *does* matter in the atmosphere. First of all, as already mentioned several times, the energy of a photon is given by $E = h\nu$. The rate of absorption and emission of photons by the atmosphere is strongly dependent on the precise value of that energy. Among other things, a physical or chemical event requiring a minimum input of energy ΔE_{min} cannot be initiated by a photon with a frequency of less than $\nu_{\text{min}} = \Delta E/h$. Furthermore, the quantum mechanical behavior of matter at the molecular level imposes an even stronger constraint in many cases: to be absorbed, the energy of a photon must almost exactly match a certain well-defined set of values associated with allowable energy levels in that molecule. We will examine these issues in considerable detail in Chapter 9.

Another reason arises from the wave nature of radiation, which comes to the forefront when radiation is scattered or reflected by particles or surfaces. Such interactions arise primarily when the dimensions of a particle are comparable to or larger than the wavelength. Thus, radiation in the visible band is rather weakly scattered by air molecules but strongly scattered by cloud droplets. Longer wavelengths in the microwave band (e.g., radar) are negligibly scattered by cloud droplets but rather strongly by raindrops and hailstones. Longer wavelengths still (e.g., AM radio, with wavelengths of order 10^2 m) may propagate unimpeded through any kind of weather but may be diffracted around hills and reflected by deep layers of ionized gases in the extreme upper atmosphere.

3.1 Frequency, Wavelength and Wavenumber

The most fundamental characteristic of a harmonic electromagnetic field is its frequency $\nu = \omega/2\pi$, which has units of cycles per second, or Hertz (Hz). Regardless of where you are and what other processes affect it, radiation with frequency ν will always have that frequency until such time as it is absorbed and converted into another form of energy[1].

[1] This assumes that you, the observer, are at a fixed distance from the source. Otherwise the frequency will be shifted by the Doppler effect.

In practice, it is usually more convenient to specify the wavelength λ rather than the frequency ν. This is because the frequencies of interest to most atmospheric scientists tend to be numerically large and unwieldy. The two parameters are related by

$$\boxed{c = \lambda\nu\ .} \tag{3.1}$$

Note that this relationship is valid for the wavelength in a vacuum. Inside a medium like air or water, the phase speed of radiation is somewhat slower than c and the actual wavelength is correspondingly shorter. The dependence of the actual wavelength on the index of refraction of the medium is important for understanding some effects such as refraction. Normally, if we refer to wavelength without further qualification, we mean wavelength in a vacuum.

For atmospheric radiation, wavelength is most commonly expressed using one of the following units, whichever is most convenient: nanometers (nm = 10^{-9} m), micrometers or *microns* (μm = 10^{-6} m), or centimeters (cm = 10^{-2} m). Other units, such as the Angstrom (10^{-10} m) are no longer widely used by meteorologists.

The description preferred by some specialists is neither wavelength nor frequency but *wavenumber* $\tilde{\nu}$, which is just the reciprocal of wavelength:

$$\boxed{\tilde{\nu} = \frac{1}{\lambda} = \frac{\nu}{c}\ .} \tag{3.2}$$

Wavenumber is usually stated in units of inverse centimeters (cm^{-1}).

3.2 Major Spectral Bands

The electromagnetic spectrum spans an enormous range of frequencies, from essentially zero to extremely high frequencies associated with energetic photons released by nuclear reactions. As a matter of convention, the spectrum has been subdivided by scientists and engineers into a few discrete spectral *bands*. The frequency and wavelength boundaries of the major spectral bands are given in Table 3.1

Table 3.1: Regions of the electromagnetic spectrum

Region	Spectral range	Fraction of solar output	Remarks
X rays	$\lambda < 0.01\ \mu$m		Photoionizes all species; absorbed in upper atmosphere
Extreme UV	$0.01 < \lambda < 0.1\ \mu$m	3×10^{-6}	Photoionizes O_2 and N_2; absorbed above 90 km
Far UV	$0.1 < \lambda < 0.2\ \mu$m	0.01%	Photodissociates O_2; absorbed above 50 km
UV-C	$0.2 < \lambda < 0.28\ \mu$m	0.5%	Photodissociates O_2 and O_3; absorbed between 30 and 60 km
UV-B	$0.28 < \lambda < 0.32\ \mu$m	1.3%	Mostly absorbed by O_3 in stratosphere; responsible for sunburn
UV-A	$0.32 < \lambda < 0.4\ \mu$m	6.2%	Reaches surface
Visible	$0.4 < \lambda < 0.7\ \mu$m	39%	Atmosphere mostly transparent
Near IR	$0.7 < \lambda < 4\ \mu$m	52%	Partially absorbed, mainly by water vapor
Thermal IR	$4 < \lambda < 50\ \mu$m	0.9%	Absorbed and emitted by water vapor, carbon dioxide, ozone, and other trace gases
Far IR	$0.05 < \lambda < 1$ mm		Absorbed by water vapor
Microwave	$\lambda >1$ mm		Clouds semi-transparent

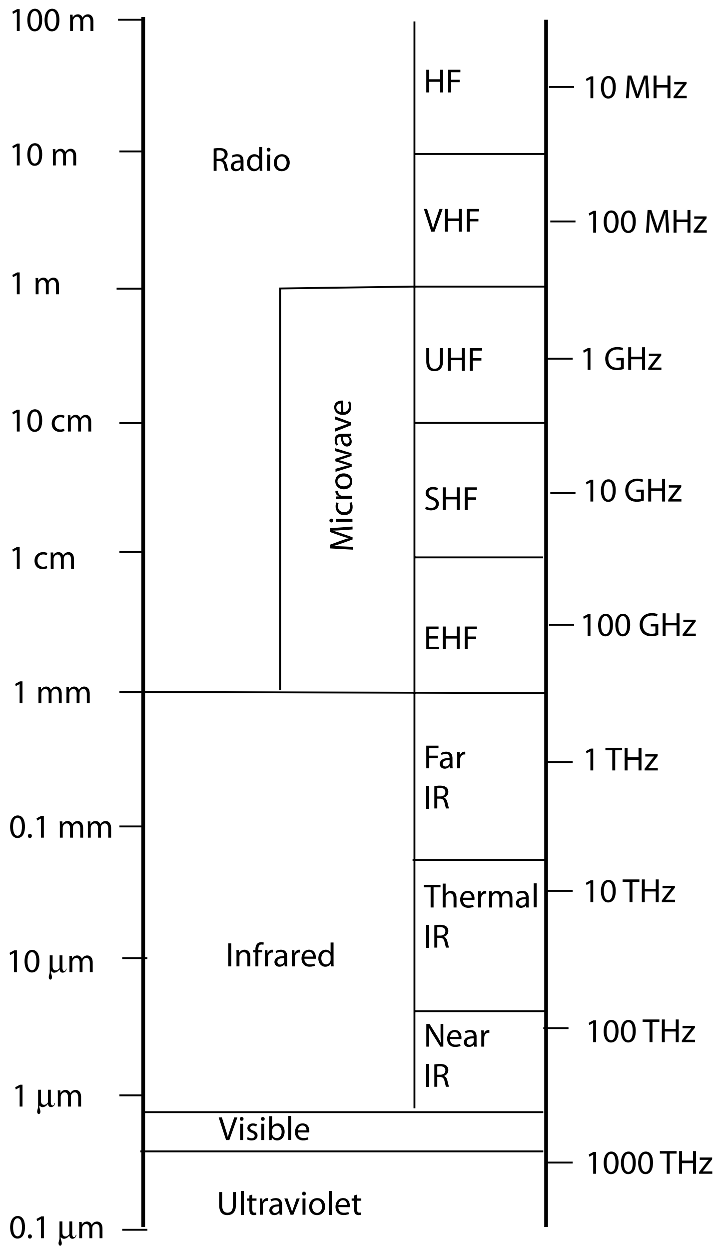

Fig. 3.1: The electromagnetic spectrum.

and Fig. 3.1. It is important to understand that there is nothing special about the precise frequencies defining the boundaries between bands; in most cases, these boundaries were decided more or less arbitrarily and have no real physical significance. There is for example no abrupt change in the behavior of radiation as one crosses from the microwave to the infrared band in the vicinity of 1 mm wavelength. The exception of course is the visible band, whose boundaries are defined by the range of wavelengths (approximately 0.4 to 0.7 μm) that the normal human eye can see. Other animal species might have defined this band differently. Many insects, for example, can see well into the ultraviolet band.

Note that there are three rather distinct ways in which a particular spectral band can make itself "interesting" to atmospheric scientists:

Diabatic heating/cooling - As pointed out in the introduction, radiative transfer is one of the most important mechanisms of heat exchange in the atmosphere, and is the *sole* mechanism for heat exchange between the earth and the rest of the universe. For reasons that will become clearer later, not all spectral bands contribute significantly in this category.

Photochemistry - Many of the chemical reactions that take place in the atmosphere, including those that produce smog, as well as some that help cleanse the air of pollutants, are driven by sunlight. In addition, the existence of the ozone layer is a direct result of photochemical processes. The photon energy $E = h\nu$ is a crucial factor in determining which spectral bands are "players" in atmospheric photochemistry.

Remote sensing - Any frequency of radiation that is absorbed, scattered or emitted by the atmosphere can potentially be exploited for satellite- or ground-based measurements of atmospheric properties, such as temperature, humidity, the concentration of trace constituents, and many other variables.

In this book, we shall restrict our attention to radiative processes relevant primarily to the troposphere and stratosphere. With this constraint in mind, we may now undertake a brief survey of the major spectral bands.

3.2.1 Gamma Rays and X-Rays

Gamma rays and X-rays, which are associated with wavelengths shorter than $\sim 10^{-2}$ μm, are usually produced by nuclear decay, nuclear fission and fusion, and other reactions involving energetic subatomic particles. The most energetic of photons, gamma and X-ray radiation can easily strip electrons from, or *ionize*, atoms and decompose chemical compounds. As such, ionizing radiation poses significant hazards to life. It is therefore fortunate that the strongest natural sources are extraterrestrial — so-called *cosmic rays* — and thus affect primarily the upper levels of the atmosphere. The intensity of gamma and X-ray radiation arriving at the top of the atmosphere is typically reduced by well over half for each 100 mb of atmosphere that it traverses, so that very little of this radiation makes it to the lowest levels. But airline passengers are exposed to nonnegligible levels of cosmic radiation.

In the lower troposphere, most natural radiation observed in this spectral band is traceable to radioactive materials in the earth's crust, such as uranium and its daughter isotopes. Although such sources are widely distributed, most are (thankfully) rather weak.

The gamma and X-ray bands are the only bands that have no major significance for any of the three processes identified in the previous section. Fluxes of radiation in these bands are not large enough to have a measurable effect on the heating or cooling of the lower and middle atmosphere. For various reasons, including the absence of strong natural terrestrial sources and the relatively strong attenuation of ultrashort wavelength radiation by the atmosphere, remote sensing of the troposphere and stratosphere is not a practical proposition in these bands. Finally, although these types of radiation can potentially participate in chemical reactions, their role is minor compared with that of ultraviolet radiation (see below). In the view of lack of strong relevance of this band to meteorology, we will not consider it further in this book.

3.2.2 Ultraviolet Band

The ultraviolet (UV) band occupies the range of wavelengths from approximately 0.01 μm on the X-ray side to approximately 0.4 μm on the visible-light side. The sun is the sole significant source of natural UV radiation in the atmosphere. However, the fraction of sunlight at the top of the atmosphere that falls in this band is small, only a few percent of the total power output. Nevertheless, this contribution is very important. The UV band is further divided into the following sub-bands:

UV-A extends from 0.4 down to 0.32 μm. Radiation in this sub-band is a significant component of sunlight, comprising close to 99% of the total solar UV radiation that reaches sea level. Although UV-A radiation is invisible to the human eye, it stimulates fluorescence (the emission of visible light) in some materials — e.g., "Day-Glo" markers, highway safety cones, and yellow tennis balls. So-called "black lights" used with fluorescent posters are artificial sources of UV-A radiation. Although the wavelengths are shorter, and therefore more energetic, than those of visible light, UV-A is still relatively innocuous with respect to living organisms. This is fortunate because the atmosphere is rather transparent to UV-A.

UV-B extends from 0.32 down to 0.280 μm. Because of its even shorter wavelength, its photons are energetic enough to initiate photochemical reactions, including injury of tissues (e.g., sunburn) and even damage to cellular DNA, leading to increased risk of skin cancer in exposed individuals. Fortunately, most UV-B (approximately 99%) is absorbed by ozone in the stratosphere. However, thinning of the ozone layer by human-manufactured chemicals is believed to be responsible for a significant increase in the amount of UV-B now reaching the surface.

UV-C extends from 0.280 to $\sim$0.1 μm. The most energetic UV sub-band, virtually all UV-C radiation is absorbed in the mesosphere and uppermost stratosphere, where much of its energy is expended on the dissociation of O_2 into atomic oxygen. The remainder is absorbed by ozone.

UV radiation is interesting in all three of the respects outlined earlier. As we have already mentioned, it is a major player in atmospheric photochemistry. Also, satellite remote sensing of ozone and other stratospheric constituents is possible in this band. Finally,

Table 3.2: Relationship between color and wavelength

Wavelength interval (μm)	Color
0.39–0.46	Violet
0.46–0.49	Dark Blue
0.49–0.51	Light Blue
0.51–0.55	Green
0.55–0.58	Yellow-Green
0.58–0.59	Yellow
0.59–0.62	Orange
0.62–0.76	Red

the absorption of solar UV radiation by ozone is a major diabatic heating term in the stratosphere and mesosphere.

3.2.3 Visible Band

The visible band extends from approximately 0.4 μm to 0.7 μm. In addition to its obvious importance for human vision, its significance for the atmosphere cannot be overstated, despite the fact that it occupies a surprisingly narrow slice of the EM spectrum.

First, through an interesting coincidence, *the visible band includes the wavelength of maximum emission of radiation by the sun.* (Fig. 3.2) In fact, close to half of the total power output of the sun falls in this narrow band.

Second, *the cloud-free atmosphere is remarkably transparent to all visible wavelengths*. We may take this for granted, but for no other major spectral band is the atmosphere as uniformly transparent (Fig. 3.3). This means that the absorption of visible solar radiation occurs primarily at the surface of the earth rather than within the atmosphere itself. Thus, the atmosphere is largely heated from below and only secondarily by direct absorption of solar radiation. The thermal structure of the atmosphere would likely be quite different if the atmosphere were less transparent to this component of the solar flux.

Clouds are remarkably reflective in the visible band. Again, this might seem obvious, but it's not true for many other spectral bands. The global distribution of cloud cover has a huge influ-

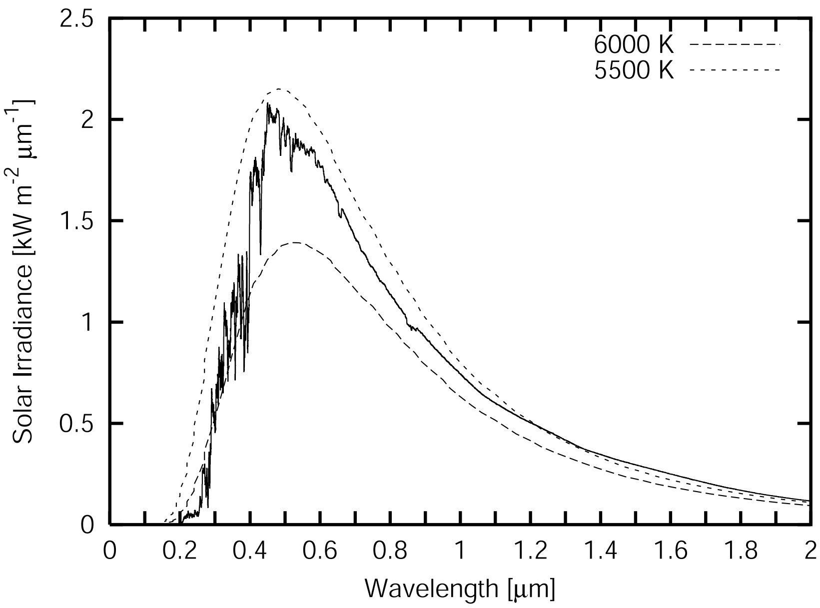

Fig. 3.2: Spectrum of solar radiation at the top of the atmosphere, at moderate spectral resolution. Dashed curves represent emission from an ideal blackbody at the indicated temperatures.

ence on the fraction of total solar radiation that gets absorbed by the earth-atmosphere system rather than being reflected back out to space.

Satellites with imaging capabilities in the visible band are able to easily detect and classify clouds. In the absence of clouds, visible imagers are able to map surface features, vegetation types, ocean color (related to biological productivity) and many other variables.

3.2.4 Infrared Band

The infrared (IR) band extends from wavelengths of approximately 0.7 μm up to approximately 1000 μm or 1 mm. This rather broad range (over three decades of wavelength) encompasses a rich variety of absorption and emission features in the atmosphere. IR radiation is enormously important as the means by which energy is exchanged between lower and upper levels of the atmosphere and between the earth-atmosphere system and outer space. Not only does

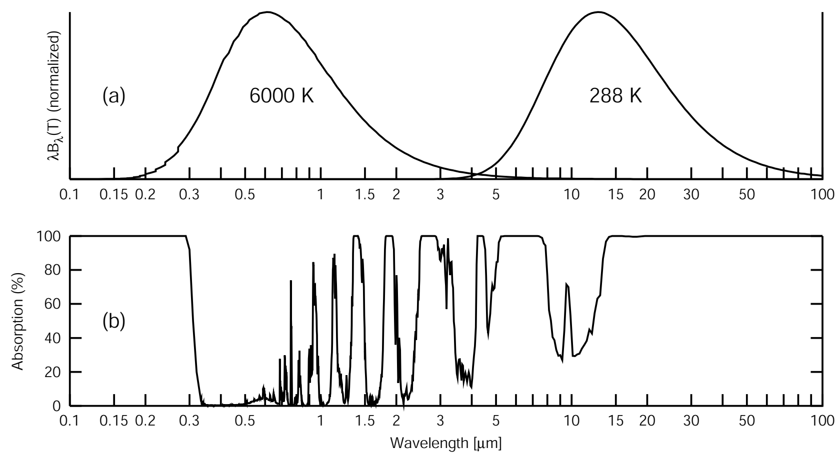

Fig. 3.3: Overview of the relationship between solar and terrestrial emission and the transmission properties the atmosphere. a) Normalized blackbody curves corresponding to the approximate temperature of the sun's photosphere (6000 K) and a typical terrestrial temperature of 288 K. b) A coarse-resolution depiction of the absorption spectrum of the cloud-free atmosphere.)

the steady-state climate of the earth depend heavily on the absorptive and emissive properties of the atmosphere in the IR band, but we now believe that the climate can change in response to human-induced increases in IR-absorbing trace constituents ("greenhouse gases") in the atmosphere, such as water vapor, carbon dioxide, methane, and chlorofluorocarbons (CFCs).

Because so many major and minor constituents of the atmosphere have distinctive (and often very strong) absorption features in the IR band, there are countless ways to exploit this band for remote sensing of temperature, water vapor, and trace constituents. On the other hand, the IR band is unimportant for atmospheric photochemistry, because photon energies are below the threshold required to dissociate most chemical compounds.

Atmospheric scientists tend to subdivide the IR band into three sub-bands: the *near IR* band, the *thermal IR* band, and the *far IR* band.

The near IR band is in one sense a continuation of the visible band, in that the primary source of this radiation in the atmosphere is the sun. It extends from 0.7 to 4 μm. Approximately half of the sun's output is found in this band, so that all but 1% of solar radiation incident on the top of the earth's atmosphere is accounted for by the UV, visible, and near IR bands together.

Unlike the case for the visible band, however, the atmosphere is not uniformly transparent to all near-IR wavelengths but rather exhibits a number of significant atmospheric absorption features. Thus, a moderate fraction of near-IR radiation from the sun is absorbed by the atmosphere in this band. For some wavelengths, e.g., near 1.3 μm, absorption is nearly total.

The range from 4 μm to 50 μm encompasses what we will refer to as the *thermal IR* band. Different sources quote various upper wavelength bounds on the thermal IR band, some as low as 15 μm. We have chosen the 50 μm bound because significant thermal energy exchanges via radiative transfer in the atmosphere occur up to approximately this limit. The thermal IR band is "where the action is" in view of both the magnitude of the energy exchanges and the enormous complexity of the atmospheric absorption spectra in this band. We will have much more to say about this band in later chapters.

For our purposes, the *far IR* band represents wavelengths between about 50 μm and 1000 μm (1 mm). Energy transfer in the atmosphere at these wavelengths is insignificant relative to that associated with the thermal IR, near IR, and visible bands. There are some potential applications of the far IR band to remote sensing, especially of cirrus clouds, but otherwise this region of the spectrum is relatively uninteresting to meteorologists.

3.2.5 Microwave and Radio Bands

Moving through the EM spectrum toward longer wavelengths (lower frequencies), one leaves the far IR band and enters the *microwave band* at a wavelength of about 1 mm, or at a frequency of about 300 GHz (GHz = gigahertz = 10^9 Hz). The lower bound (in frequency) is often taken to be around 3 GHz, or 10 cm wavelength. Thus, the microwave band encompasses two decades of frequency. At lower frequencies still, and continuing down to zero, we have the *radio band.* Note that both for historical reasons and because the numbers are low enough to be manageable, it is most common to use frequency rather than wavelength when describing microwave and radio band radiation.

From an engineering point of view, one of the distinguishing characteristics of the radio band is that the frequencies involved are low enough to be amenable to generation, amplification, and detection using traditional electronic components and circuits. By contrast, the much higher frequencies and shorter wavelengths of IR and visible radiation require mirrors, diffraction gratings, and/or lenses. The microwave band occupies a gray area, as many of the components in microwave circuits have a quasi-optical character — e.g., waveguides, resonant cavities, feedhorns, and parabolic reflectors.

The microwave band has risen greatly in prominence in recent years for its role in remote sensing of the atmosphere and surface. Radar, which was first developed during World War II, is now the principal means by which meteorologists monitor severe weather and study the dynamics of convective cloud systems. Satellites with sensors operating in the microwave band have proliferated since the mid-1970s and are now a very important component of our weather satellite programs, both for research and operationally.

The utility of the microwave band is greatly enhanced by the relative transparency of clouds, especially at frequencies well below 100 GHz. The properties of the surface and of the total atmospheric column — can be observed from space under all weather conditions except rainfall.

The radio band, which by some definitions includes the microwave band, continues down to zero frequency. Frequencies lower than around 3 GHz tend to interact very weakly with the atmosphere and therefore have only limited applicability to atmospheric remote sensing. Also, because of the long wavelengths involved, it is difficult to achieve good directionality with antennas of managable size (especially on satellites).

Two notable examples of remote sensing in the radio band do bear mentioning: 1) ground-based Doppler wind profilers operating near 915 MHz, which observe scattering from turbulence-induced fluctuations in atmospheric density and humidity, and 2) lightning detection systems, which are sensitive to low-frequency "static" emitted by lightning discharges. Apart from these cases, radio wavelengths are of very limited interest to meteorologists.

3.3 Solar and Terrestrial Radiation

In the previous section, we surveyed the entire electromagnetic spectrum with an eye toward outlining the relevance of each major band to atmospheric science. The two most important facts to emerge from this survey are the following:

- Over 99% of the energy radiated by the sun and incident on the top of the earth's atmosphere is accounted for by just three bands spanning wavelengths from 0.1 μm to 4 μm: the ultraviolet band contributes a few percent, with the remainder more or less evenly split between the visible band and the near-infrared band. We collectively refer to these bands as *solar* or *shortwave* radiation. Although solar emission in other bands may have some significance for remote sensing (e.g., sunglint from the ocean surface in the microwave band), it is insignificant for the energy budget of the atmosphere.

- Over 99% of the radiative energy emitted by the earth and atmosphere is found in the thermal infrared band from 4–50 μm. We will often refer to radiation in this band as *terrestrial* or *longwave* radiation. Emission in other bands (principally the far-IR and microwave bands) may be important for remote sensing but is essentially irrelevant for the atmospheric energy budget.

It is an interesting and convenient coincidence that a wavelength of approximately 4 μm cleanly separates the band containing most solar radiation from that containing most terrestrial emission (Fig. 3.3). For a narrow range of wavelengths in the vicinity of 4 μm, it may sometimes be necessary to consider both terrestrial and solar sources, but for most wavelengths it is just one or the other. The physical reasons for this separation will be addressed in Chapter 5; for now it is sufficient to point out that disparate temperatures of the sources (approximately 6000 K for the sun versus 250–300 K for the earth and atmosphere) are responsible.

Problem 3.1: For the given electromagnetic waves in a vacuum, compute the frequency ν in Hz, the wavenumber $\tilde{\nu}$ in cm^{-1}, and the wavelength λ in μm. Also identify the spectral band.

a) $\lambda = 0.0015$ cm, b) $\nu = 37$ GHz, c) $\tilde{\nu} = 600\ \mathrm{cm}^{-1}$, d) $\lambda = 300$ nm, e) $\nu = 3 \times 10^{14}$ Hz, f) $\tilde{\nu} = 10000\ \mathrm{cm}^{-1}$.

3.4 Applications to Meteorology, Climatology, and Remote Sensing

3.4.1 UV Radiation and Ozone

The Ozone Layer

The absorption of radiation by way of molecular photodissociation was briefly mentioned in Section 2.6. It turns out that this process has easily observable consequences for the atmosphere and, for that matter, for all of life on Earth. In particular, UV-C radiation is responsible for dissociating molecular oxygen, according to the reaction

$$O_2 + h\nu\ (\lambda < 0.2423\ \mu\mathrm{m}) \rightarrow O + O. \tag{3.3}$$

The large amount of molecular oxygen in the atmospheric column absorbs most solar radiation at wavelengths shorter than 0.24 μm by this mechanism.

The free oxygen atoms from the above reaction then can combine with O_2 to form ozone according to the reaction

$$O + O_2 + M \rightarrow O_3 + M, \tag{3.4}$$

where M is any third molecule or atom (required in order to carry away the energy released by the above reaction).

It is fortunate for us that this second reaction occurs, because ordinary oxygen by itself would continue to allow dangerous UV-C and UV-B radiation with wavelengths between 0.24 and 0.32 μm to reach the surface, posing a deadly hazard to life. But ozone happens to absorb strongly between about 0.2 and 0.31 μm via electronic transitions. It therefore "mops up" most of whatever UV-B and UV-C was not absorbed via (3.3). Of course it does little for UV-A radiation, with wavelengths longer than 0.32 μm. But radiation in this band is relatively innocuous, except perhaps when it is used to illuminate velvet blacklight posters of Elvis.

There remains a small sliver of the UV-B band between about 0.31 and 0.32 μm that manages to reach the surface without complete absorption; it is precisely this narrow sliver that is primarily responsible for sunburn. Lately, there has been considerable concern over observed declines in ozone layer density. If the decline continues, then the resulting widening of this narrow UV-B window could have serious consequences for life on Earth.

In the very process of absorbing harmful shortwave UV radiation, the ozone layer influences our environment in another very important way. The solar energy that is absorbed by ozone warms the atmosphere at those levels to a much higher temperature than would be the case without the presence of ozone. Have you ever wondered why temperature *increases* with height in the stratosphere, reaching a maximum at the stratopause before decreasing again in the mesophere? The ozone layer is responsible!

In an atmosphere without free oxygen, and therefore without ozone, the temperature structure would be much simpler: we'd have a *very* deep troposphere (temperature generally decreasing with height) transitioning directly to the thermosphere. The stratosphere and mesosphere would be missing. This is in fact what you find on Mars, whose atmosphere consists mainly of CO_2. On Earth, the temperature structure of the lower stratosphere serves as a very important "lid" on tropospheric convection and other circulations. If you already have some background in atmospheric dynamics, try to imagine how different our weather might be if the tropopause were near 50 km altitude rather than its present 5-15 km!

Photochemical Smog

We have just surveyed the role of oxygen and ozone in the middle atmosphere (stratosphere and mesosphere) in absorbing UV-C and UV-B radiation. The UV-C, we saw, is mostly absorbed by photodissociation of O_2; the UV-B was then mostly absorbed by the resulting O_3. This leaves mainly UV-A radiation to reach the troposphere.

Although less energetic than UV-B and UV-C radiation, UV-A radiation is a key player in tropospheric chemistry. Among other things, photochemical reactions involving organic molecules (e.g. unburned fuel vapors) and nitrogen oxides (produced by high temperatures in automobile engines) can lead to the formation of ozone in surface air. Although ozone in the stratosphere is highly desirable because of its UV-blocking characteristics, it is considered a serious pollutant in near-surface air where we live, because it is a strong chemical oxidant which attacks most organic substances, including the lining of your lungs. Ozone is thus one of the main ingredients of *photochemical smog.*

In its most basic form, the chemical sequence leading to ozone pollution goes like this:

1. Automobile engines and other industrial processes emit *primary pollutants*, which include unburned hydrocarbons (a.k.a. volatile organic compounds, or VOC) and nitrogen oxides — nitrogen monoxide (NO) and nitrogen dioxide (NO_2), collectively known as NO_x. The NO oxidizes, further increasing the NO_2 concentration.

2. The NO_2 is photolyzed (photodissociated) according to the reaction
$$NO_2 + h\nu\ (\lambda < 0.4\ \mu\text{m}) \rightarrow NO + O^\star, \tag{3.5}$$
where $O^\star$ is a highly reactive free oxygen atom that immediately combines with an ordinary oxygen molecule to form one of several *secondary pollutants*[2], in this case ozone:
$$O_2 + O^\star + M \rightarrow O_3 \tag{3.6}$$

3. A third reaction completes the cycle, bringing us back to our starting point:
$$O_3 + NO \rightarrow NO_2 + O_2 \tag{3.7}$$

Graphically, the cycle is shown in Fig. 3.4. Although each pass through the cycle doesn't yield a net increase of ozone, the fact that

[2]Another major secondary pollutant, also produced by photochemical action on the primary emissions, is peroxyacetyl nitrate, or PAN.

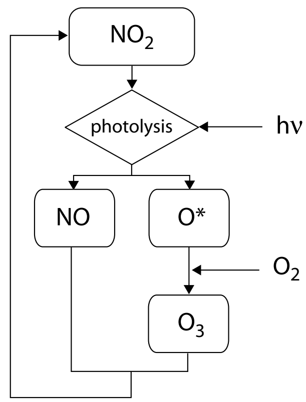

Fig. 3.4: Photolysis of NO_2 and generation of O_3.

there is (during the daytime) a continuous input of UV radiation implies that there will be constant generation of new ozone as earlier ozone molecules are destroyed, leading to a finite steady state concentration in the atmosphere. Although we won't go into the process here, one role of VOC molecules is to create additional sources of NO and NO_2, which increases the equilibrium concentration of O_3.

The location in the U.S. most stereotypically associated with photochemical smog is the Los Angeles basin, with its high quota of sunshine, high concentration of automobiles, and shallow pool of stagnant air hemmed in by mountains.

The Hydroxyl Radical

So far we have seen an example of a photochemical process that is "good" (ozone production in the stratosphere) and one that is "bad" (ozone production in the troposphere). UV-A and visible radiation are responsible for other photochemical processes in the troposphere besides smog, not all of which yield harmful byproducts. One in particular is quite beneficial: the production of short-lived hydroxyl (OH) radicals by the following pair of reactions:

$$O_3 + h\nu\ (\lambda < 0.34\ \mu\text{m}) \rightarrow O_2 + O^\star \tag{3.8}$$

$$H_2O + O^\star \rightarrow 2OH \tag{3.9}$$

where the ozone in the first of these equations occurs at low levels even in unpolluted air. What is interesting about the hydroxyl radical is that it is highly reactive and acts to break down a wide range of undesirable pollutants in the atmosphere, such as carbon monoxide and methane. In fact, OH radicals are sometimes referred to as the atmosphere's "detergent." Without the daily action of hydroxyl radicals, the air would be much dirtier, on average, than it is.

Among the very few pollutants that are impervious to breakdown by OH radicals are the synthetic compounds known as chlorofluorocarbons (CFCs). These molecules are so exceptionally stable that they can persist in the atmosphere for as long as it takes — often a year or more — for them to get circulated from the troposphere into the stratosphere. Only there are CFC molecules exposed to enough UV-B and UV-C radiation to get broken down. Unfortunately, one byproduct of the photolysis is a free chlorine atom. A single Cl atom has the capacity to catalytically destroy many ozone molecules. Therefore, the release of CFCs into the atmosphere over the past few decades has led to a marked decrease in the steady state concentration of ozone in the stratosphere. This is especially the case in the polar regions during springtime, where complex chemical processes involving polar stratospheric cloud particles and sunlight greatly accelerates the destruction of ozone. As noted above, this thinning of the ozone layer is a point of great concern, due to the hazards of UV-B radiation to life. Fortunately, progress has been made in slowing the release of CFCs into the atmosphere, and the ozone layer is now expected to gradually recover over the next few decades.