Homework Assignment #2 (total 70 points)

Due: March 14

Problem 1 (30 points)

Rayleigh scattering for spherical particles is a limiting case of Mie scattering as the size parameter x -> 0. In this limit the scattered spherical wave Mie coefficients $\boldsymbol{a_n}$ and $\boldsymbol{b_n}$ (see Lecture 15) are all negligible except for a_1 which is

$$a_1 = \frac{2i}{3}\frac{m^2-1}{m^2+2}x^3$$

Using the Mie theory results (Lecture 15), derive the following quantities for Rayleigh scattering

a) scattering efficiency Q_s and the extinction efficiency Q_e;

b) scattering amplitudes $\mathbf{S_1(\Theta)}$ and $\mathbf{S_2(\Theta)}$;

c) scattering phase function $\mathbf{P(\Theta)}$;

d) Consider the case of a nonabsorbing sphere in this limit. What is the extinction efficiency derived above in this case? Is this result physical (compare with the scattering)? What might be the cause of this dilemma? In this limit of Mie theory what process does Q_e really measure?

Problem 2 (40 points)

A volcanic eruption results in a layer of sulfuric acid aerosol droplets in the Earth's lower stratosphere. The layer is 5 km thick and has the particle size distribution characterized by a lognormal function with N = 100 cm^{-3} number concentration, r_o= 0.2 μm median radius and $\ln\sigma$ =0.5 standard deviation.

a) Calculate the effective radius of this size distribution. What is the size parameter of the effective radius for a wavelength of λ =10 μm?

b) Calculate the optical depth of the aerosol layer at λ =10 μm. The refractive index of sulfuric acid at this wavelength is m = 2.094-0.306i.

c) If the temperature of the stratospheric aerosol layer is 215 K and the surface/lower atmosphere at 10 μm radiates like a blackbody at 290 K, what is the fractional change in upwelling longwave flux at 10 μm due to the volcanic aerosols?