

Lecture 10.

Terrestrial infrared radiative processes. Part 3:

K-distribution approximation.

1. K-distribution approximation (KD).
2. Correlated k-distribution approximation (CKD).

Required reading:

L02:4.3

Additional reading:

Fu, Q., and K.N. Liou, On the correlated k-distribution method for radiative transfer in nonhomogeneous atmospheres. *Journal of the Atmospheric Sciences* 49, 2139-2156, 1992.

Mlawer, E.J., S.J. Taubman, P.D. Brown, M.J. Iacono and S.A. Clough: RRTM, a validated correlated-k model for the longwave. *J. Geophys. Res.*, **102**, 16,663-16,682, 1997.

1. K-distribution method (KD).

The KD method is developed to compute the **spectral transmittance** (hence the spectral intensity and spectral fluxes) based on grouping of gaseous absorption coefficients.

NOTE: The k-distribution approach was proposed by Ambartzumian in the 30th as an alternative to the computationally expensive line-by-line methods.

- The KD method benefits from the fact that the same value of $k_{a,v}$ is encountered many times over a given spectral interval => thus to eliminate the redundancy, one can group the values of $k_{a,v}$ and perform the transmittance calculation only once for a given value of $k_{a,v}$.

Strategy:

Consider a **homogeneous** atmospheric layer. Spectral transmission is

$$T_{\nu}(u) = \frac{1}{\Delta\nu} \int_{\Delta\nu} \exp(-k_{a,\nu}u) d\nu$$

In the homogeneous atmospheric layer, spectral transmittance is independent of the ordering of $k_{a,\nu}$ in a given spectral range, i.e., the order in which the wavenumbers are summed does not matter => so let's sum them up from low to high $k_{a,\nu}$.

Thus, we want to replace the integration over the wavenumber by integration over k . It can be done by introducing a **normalized probability distribution function** for $k_{a,\nu}$ (will be denoted by k below to simplify the notation):

$$T_{\nu}(u) = \frac{1}{\Delta\nu} \int_{\Delta\nu} \exp(-k_{a,\nu}u) d\nu = \int_0^{\infty} \exp(-ku) f(k) dk \quad [10.1]$$

where we set $\int_0^{\infty} f(k) dk = 1$

$f(k)$ is the fraction of the spectral band with absorption coefficient between k and $k+dk$

$f(k)$ is a smooth function.

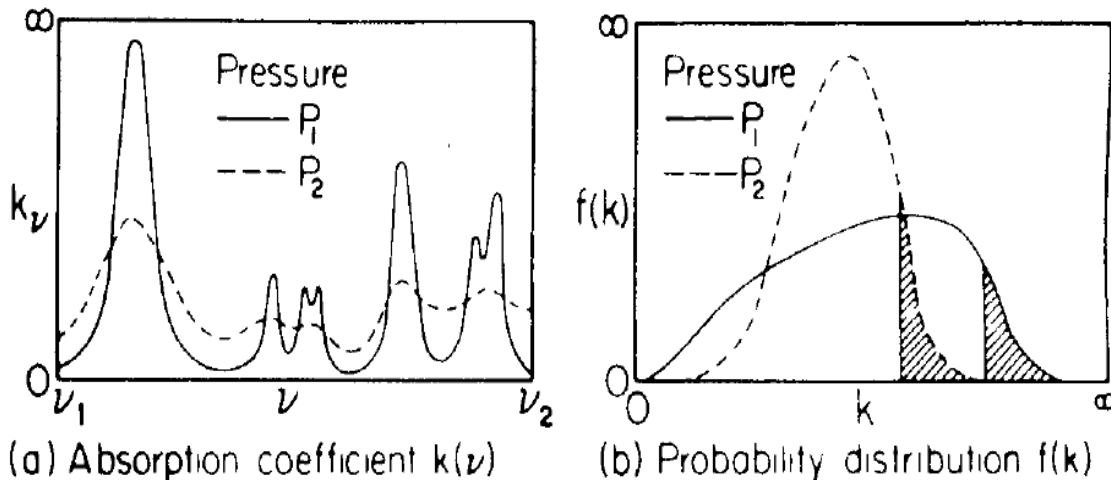


Figure 10.1 (a) A schematic of absorption line spectra at two different pressure. (b) The two probability density function $f(k)$ associated with (a). The shaded area shows the strongest absorption.

The **cumulative probability function** can be defined as

$$g(k) = \int_0^k f(k) dk \quad [10.2]$$

and $g(0)=0$; $g(\infty)=1$ and $dg(k)=f(k)dk$.

$g(k)$ is the fraction of the spectrum with absorption coefficient below k .

NOTE: By definition, $g(k)$ is a monotonically increasing and smooth function in k -space, Therefore, $k(g)$, as an inverse function of $g(k)$, is a smooth function in g -space.

Hence, the **spectral transmittance** can be written as

$$T_v(u) = \frac{1}{\Delta\nu} \int_{\Delta\nu} \exp(-k_{a,v}u) d\nu = \int_0^\infty \exp(-ku) f(k) dk = \int_0^1 \exp(-k(g)u) dg \quad [10.3]$$

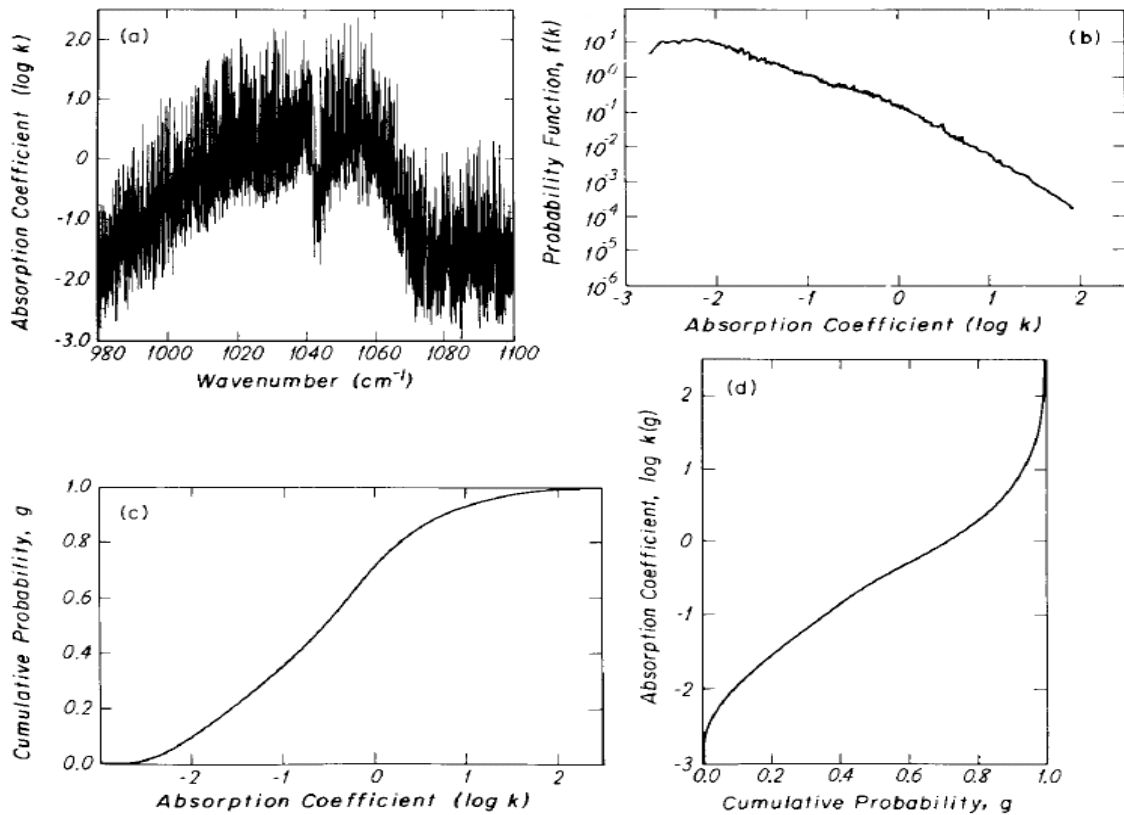


Figure 10.2 (a) Absorption coefficient $k_{a,v}$ (in $\text{cm}^{-1} \text{atm}^{-1}$) as a function of wavenumber in $9.6 \mu\text{m}$ ozone band (resolution of 0.05 cm^{-1} , $p=30 \text{ mb}$, $T=200\text{K}$). (b) The probability density function $f(k)$ of the absorption coefficient. (c) The cumulative probability distribution function as a function of k . (d) Same as (c) but k vs. g .

Because both $g(k)$ and $k(g)$ are smooth functions, the above integral can be calculated by a finite sum as

$$\begin{aligned} T_{\bar{\nu}}(u) &= \int_0^1 \exp(-k(g)u) dg \approx \sum_{i=1}^N \exp(-k(g_i)u) \Delta g_i = \\ &= \Delta g_1 e^{-k_1 u} + \Delta g_2 e^{-k_2 u} + \dots + \Delta g_N e^{-k_N u} \end{aligned} \quad [10.4]$$

where Δg_i is the quadrature weight.

Thus, the **KD method** allows calculating the spectral transmittance as a finite weighted sum of exponent in g -space, replacing the tedious wavenumber integration that is required in the line-by-line computations.

Numerical realization of the KD method:

(see illustration below)

Consider a spectral interval $\Delta \nu$ that contains numerous absorption lines.

Let's divide it into N intervals of $\Delta \nu_j$, $j = 1, 2, 3, \dots, N$

The probability distribution function can be written as

$$f(k) = \frac{1}{\Delta \nu} \frac{d\nu}{dk} = \frac{1}{\Delta \nu} \sum_j \left| \frac{\Delta \nu_j}{\Delta k} \right|$$

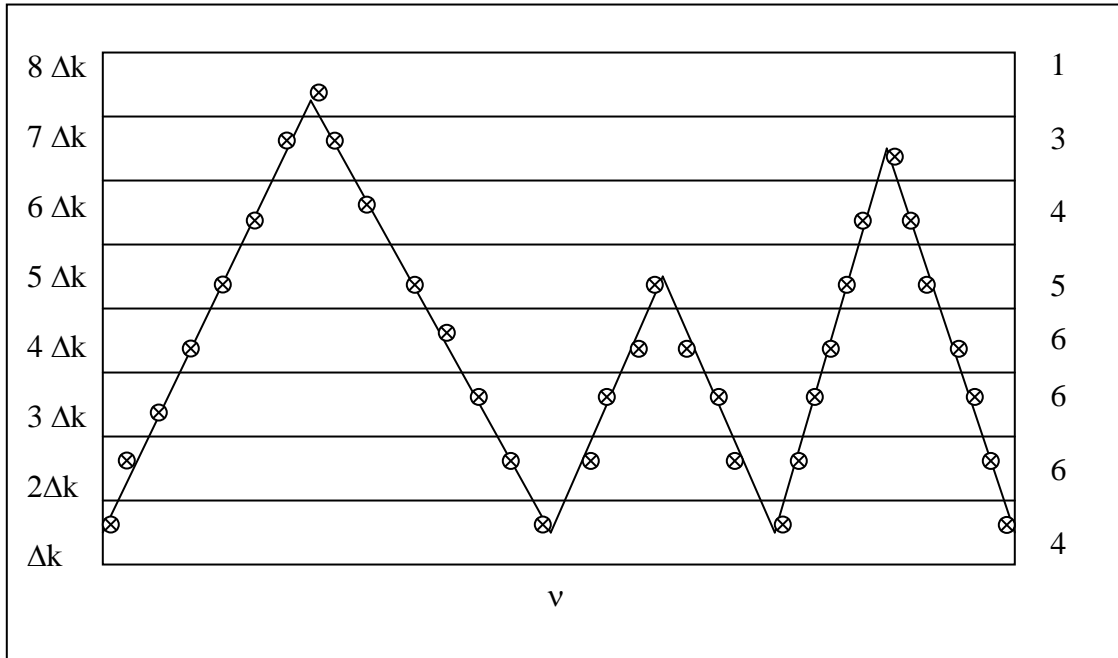
where $\Delta \nu_j$ is the subinterval of $\Delta \nu$ where k is a monotonic function of ν .

Then the cumulative probability is

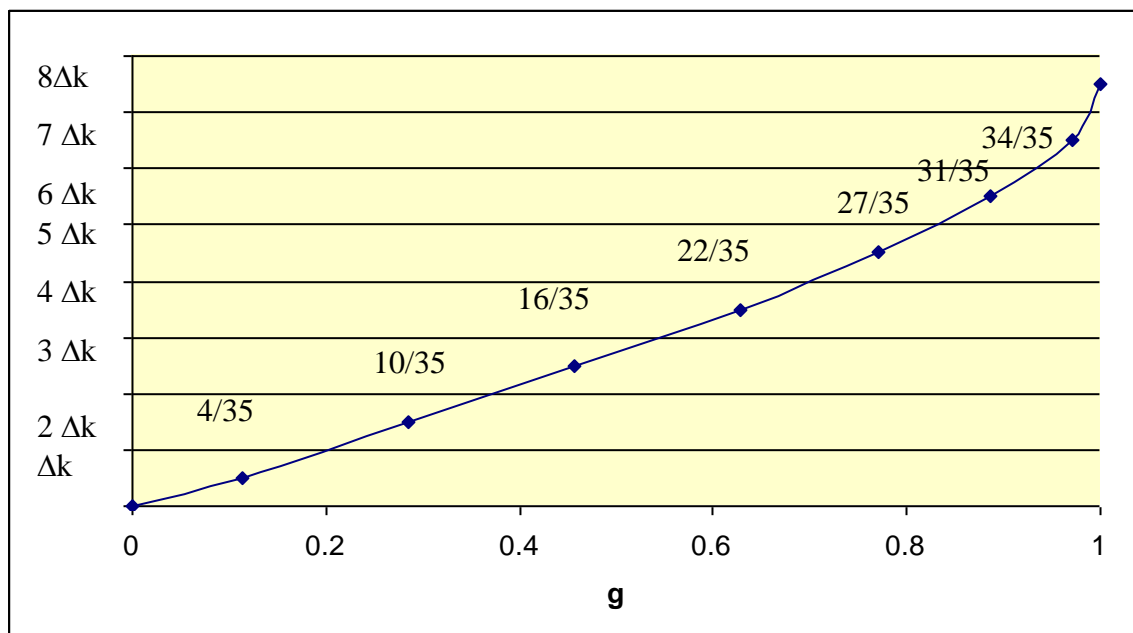
$$g(k) = \frac{1}{\Delta \nu} \sum_j \int_0^k \left| \frac{\Delta \nu_j}{\Delta k'} \right| dk' = \frac{1}{\Delta \nu} \sum_j \int_0^k \Delta \nu_j(k) = \frac{n(0, k)}{N}$$

where $n(0, k)$ is the number of computational points that contribute to k cumulatively.

Figure 10.3 How to calculate the absorption coefficient in g-space from the known absorption coefficient in the wavenumber domain. Solid line gives absorption coefficient as a function of ν . Numbers on the right side are the data points in each Δk interval (total number $N=35$).



Thus by definition, $g(j\Delta k) = n(0, j\Delta k) / N$



2. Correlated K-distribution approximation (CKD).

CKD is the extension of KD for an inhomogeneous atmosphere.

Each pressure and temperature along the path has a unique $k_{a,\nu}$ spectrum.

The **CKD method** sorts each $k_{a,\nu}(p, T)$ spectrum independently to make k-distributions $k(g, p, T)$ for each p and T.

NOTE: In practice, discrete k-distributions $k_j(p_b, T_m)$ are made for a set of pressures p_b and temperatures T_m and interpolated to any p and T.

Overlap of gases in spectral band:

What do we do about multiple gases absorbing in one spectral band?

Overlap method #1: assumes that absorption spectra are independent

$$\int_{\Delta\nu} T_{\nu}^{(1)} T_{\nu}^{(2)} d\nu = T_{\bar{\nu}}^{(1)} T_{\bar{\nu}}^{(2)} = \sum_{i=1}^N \exp(-k_1(g_i)u_1) \Delta g_{1,i} \sum_j^M \exp(-k(g_j)u_2) \Delta g_{2,j}$$

$$T_{\bar{\nu}}(u_1, u_2) = \sum_{i=1}^N \Delta g_{1,i} \sum_j^M \Delta g_{2,j} \exp(-\tau_{nm})$$

where $\tau_{nm} = k_{1m}u_1 + k_{2n}u_2$,

thus we have M x N terms

Overlap method #2: introduces the mixing ratio q as an additional factor, so $k(g, p, T, q)$

Examples of correlated k-distribution radiative transfer codes:

Fu&Liou code:

Fu, Q., and K.N. Liou, On the correlated k-distribution method for radiative transfer in nonhomogeneous atmospheres. *Journal of the Atmospheric Sciences* 49, 2139-2156, 1992.

http://snowdog.larc.nasa.gov/rose/fu200503/flp200503_web.htm

<http://snowdog.larc.nasa.gov/cgi-bin/rose/flp200503/flp200503.cgi>

Fu&Liou code divides shortwave into 6 bands with total 54 k 's and longwave into 12 bands with total of 121 k 's.

Band	Region (cm ⁻¹)	N g's	Gases	Solar Flux (W/m ²)
1	50000-14500	10	O3	619.62
2	14500-7700	8	H2O	484.30
3	7700-5250	12	H2O	149.85
4	5250-4000	7	H2O	48.73
5	4000-2850	12	H2O	31.66
6	2850-2500	5	H2O	5.80
7	2200-1900	2	H2O	Overlap method
8	1900-1700	3	H2O	
9	1700-1400	4	H2O	
10	1400-1250	4	H2O, CH4, N2O	1
11	1250-1100	3	H2O, CH4, N2O	1
12	1100- 980	5	H2O, O3	1
13	980- 800	2	H2O	
14	800- 670	10	H2O, CO2	2
15	670- 540	12	H2O, CO2	2
16	540- 400	7	H2O	
17	400- 280	7	H2O	
18	280- 0	8	H2O	

RRTM code: http://rtweb.aer.com/rrtm_frame.html

Mlawer, E.J., S.J. Taubman, P.D. Brown, M.J. Iacono and S.A. Clough: RRTM, a validated correlated-k model for the longwave. *J. Geophys. Res.*, **102**, 16,663-16,682, 1997.

Table. RRTM (version 3.1) Bands and Included Species

Band	Wavenumber Range (cm-1)	1050 - 96 mb	96 - 0.01 mb	Halocarbons
1	10 - 350	H2O,N2~	H2O,N2~	----
2	350 - 500	H2O	H2O	----
3	500 - 630	H2O,CO2,N2O~	H2O,CO2,N2O~	----
4	630 - 700	H2O,CO2	CO2,O3	----
5	700 - 820	H2O,CO2,O3~	CO2,O3	CCL4
6	820 - 980	H2O,CO2~	---	CFC11^,CFC12
7	980 - 1080	H2O,O3,CO2~	O3,CO2	----
8	1080 - 1180	H2O,CO2~,O3~,N2O~	O3,CO2~,N2O~	CFC12,CFC22^
9	1180 - 1390	H2O,CH4,N2O~	CH4,N2O	----
10	1390 - 1480	H2O	H2O	----
11	1480 - 1800	H2O,O2~	H2O,O2~	----
12	1800 - 2080	H2O,CO2	---	----
13	2080 - 2250	H2O,N2O,CO2~,CO~	O3~	----
14	2250 - 2380	CO2	CO2	----
15	2380 - 2600	N2O,CO2,WV~,N2~	---	----
16	2600 - 3250	H2O,CH4	CH4	----

Some features of RRTM LW:

- ✓ k-distributions are obtained directly from a line-by-line radiative transfer code, LBLRTM.
- ✓ fluxes and cooling rates can be calculated over sixteen bands in the longwave (10-3250 cm⁻¹)
- ✓ modeled molecular absorbers are: water vapor, carbon dioxide, ozone, nitrous oxide, methane, oxygen, nitrogen and halocarbons.
- ✓ fluxes calculated by RRTM agree with those computed by LBLRTM within 1.0 W/m² at all levels, and computed cooling rates agree to within 0.1 K/day in the troposphere and 0.3 K/day in the stratosphere.