Lecture 24.

Radiation and climate. Simple climate models.

1. Planetary radiative equilibrium and radiative energy distribution (see Lecture 2).

2. Radiative and radiative-convective equilibrium.

3. Examples of simple energy balance models.

Appendix. Derivation of the Eddington gray radiative equilibrium.

Required reading:

L02: 8.3; 8.5; 8.6.1

Additional reading:

Trenberth, K. E., J. T. Fasullo, and J. Kiehl, 2009: Earth's global energy budget. *Bull. Amer. Meteor. Soc.*, **90**, No. 3, 311-324, doi: 10.1175/2008BAMS2634.1.

1. Planetary radiative equilibrium and radiative energy distribution

(see materials of Lecture 2)

Radiation is a key factor controlling the Earth's climate.

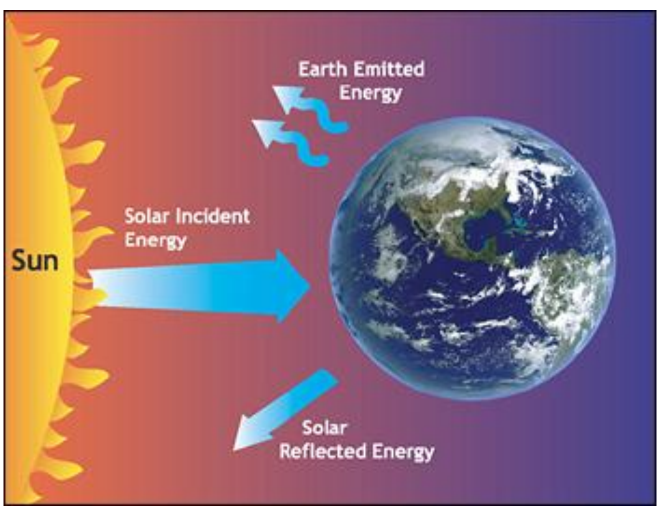
✓ Radiation equilibrium at the top of the atmosphere (TOA) represents the fundamental mode of the climate system.

Planetary radiative equilibrium (over the entire planet and long time interval):

TOA incoming radiation $\equiv$ TOA outgoing IR radiation

TOA incoming radiation = incoming solar radiation – reflected solar radiation

TOA outgoing IR radiation = outgoing IR radiation (emitted by the atmosphere-surface system)



Global annual mean Earth's energy balance

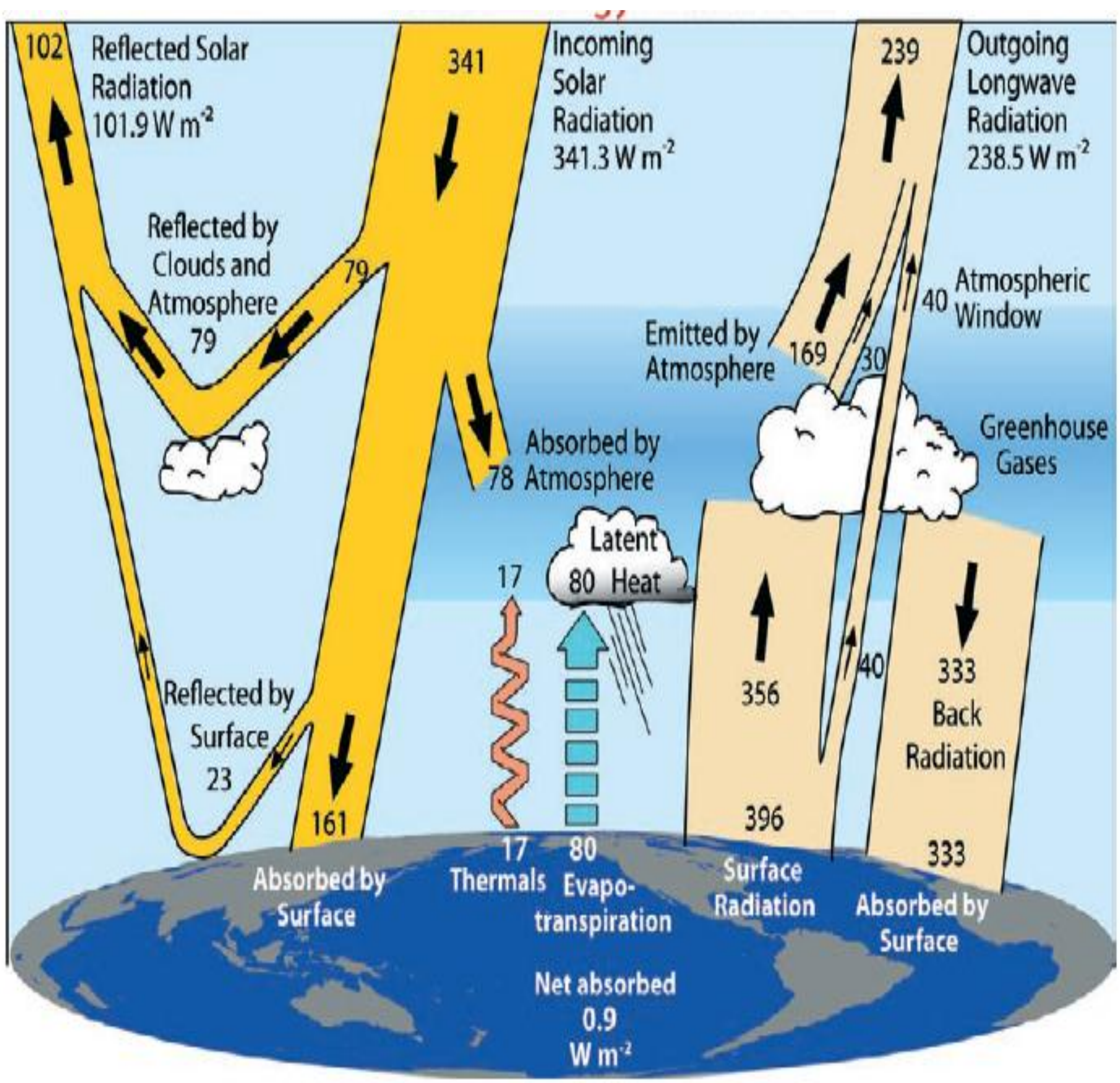


Figure 24.1 Earth's energy balance diagram from Trenberth et al. (2009). The global annual mean Earth's energy budget for the Mar 2000 to May 2004 period (W/m^2). The broad arrows indicate the schematic flow of energy in proportion to their importance.

2. Radiative and radiative-convective equilibrium models.

A hierarchy of climate models:

- Climate models can be classified by their dimensions:

Zero Dimensional Models (0-D):

consider the Earth as a whole (no change by latitude, longitude, or height)

One Dimensional Models (1-D):

allow for variation in one direction only (e.g., resolve the Earth into latitudinal zones or by height above the surface of the Earth)

Two Dimensional Models (2-D):

allow for variation in two directions at once (e.g., by latitude and by height)

Three Dimensional Models (3-D)

allow for variation in three directions at once (i.e., divide the earth-atmosphere system into domains, each domain having its own independent set of values for each of the climate parameters used in the model.

- Climate models can be classified by the basic physical processes included into the consideration:

Energy Balance Models:

0-D or 1-D models (e.g., allow to change the albedo by latitude) calculate a balance between the incoming and outgoing radiation of the planet;

Radiative Convective Models:

1-D models to model the temperature profile the atmosphere by considering radiative and convective energy transport up through the atmosphere.

General Circulation Climate Models:

2-D (longitude-averaged) or 3-D climate models solve a series of equations and have the potential to model the atmosphere very closely.

Radiative and radiative-convective equilibrium models.

In a real atmosphere, solar heating rates do not equal to IR cooling rates. This imbalance is the key driver of atmospheric dynamics.

Let's consider a hypothetical motionless atmosphere, radiative transfer processes only. Then the climate state (temperature profile) is determined by the radiative equilibrium. The radiative equilibrium climate model is a model that predicts the atmosphere temperature profile of an atmosphere in radiative equilibrium $\frac{dF_{net}}{dz} = 0$

- ✓ Under the "gray atmosphere" assumption, we can solve for the temperature profile analytically.

Eddington gray radiative equilibrium results: (see Appendix for the full derivation).

Assumptions:

1) Radiative equilibrium: $\frac{dF_{net}}{dz} = 0$
2) Gray atmosphere in longwave
3) No scattering and black surface in longwave
4) No solar absorption in the atmosphere
5) Eddington approximation: $I(\mu) = I_0 + I_1\mu$

Longwave flux profile:

$$F^{\uparrow}(\tau) = F_{sun}(1 + \frac{3}{4}\tau) \qquad \text{and} \qquad F^{\downarrow}(\tau) = F_{sun}(\frac{3}{4}\tau) \qquad [24.1]$$

where $F_{sun} = (1 - \bar{r})F_0 / 4$

Atmosphere blackbody emission and temperature profiles:

$$B(\tau) = \frac{F_{sun}}{2\pi}(1 + \frac{3}{2}\tau) \qquad \text{and} \qquad T^4(\tau) = T_e^4(\frac{1}{2} + \frac{3}{4}\tau) \qquad [24.2]$$

Surface temperature is discontinuous with the atmosphere (hotter):

$$B_s = B(\tau^*) + \frac{F_{sun}}{2\pi} \qquad \text{and} \qquad T_s^4 = T_e^4(1 + \frac{3}{4}\tau^*) \qquad [24.3]$$

Implications:

- ✓ Greenhouse effect – larger **τ*** increases surface temperature
- ✓ Runaway greenhouse effect - **τ*** increases => T_s increases
- ✓ Positive feedback – higher temperature => greenhouse gases

Eddington gray radiative equilibrium temperatures

If one wants to have the temperature profile in terms of height, one needs to relate optical depth to height.

Assume that an absorber has the exponential profile

$$\rho_a = \rho_0 \exp(-z/H_a) \qquad [24.4]$$

So the profile of optical depth is

$$\tau(z) = \bar{k}_a \int_z^\infty \rho_a(z)dz = \bar{k}_a \rho_0 H_a \exp(-z/H_a) = \tau^* \exp(-z/H_a) \qquad [24.5]$$

Temperature profile

$$T^4(z) = T_e^4(1+\frac{3}{4}\tau^* \exp(-z/H_a)) \qquad [24.6]$$

Lapse rate

$$\frac{dT}{dz}(z) = -\frac{3}{8}\frac{\tau^*}{1+\frac{3}{2}\tau^*}\frac{T(z)}{H_a}\exp(-z/H_a) \qquad [24.7]$$

Implications:

- ✓ Low **τ*** => stable atmosphere
- ✓ Smaller scale height Ha of the absorber causes steeper lapse rate
- ✓ Steepest lapse rate near the surface (z=0)

Radiative equilibrium models:

- Radiative equilibrium climate models solve for the vertical profile of temperature using accurate broadband radiative transfer models.
- Model inputs vertical profile of gases, aerosols and clouds. Iterates the temperature profile to archive equilibrium (i.e., zero heating rates or $\frac{dF_{net}}{dz} = 0$)
- Climate feedbacks can be included by having water vapor, surface albedo, clouds, etc. depend on temperature.

Solving for radiative equilibrium:

Iterate the temperature profile T(z) to get zero heating rates $\frac{\partial T}{\partial t} = 0$

1. ***Time marching method***:

   T at t+1 time step from heating rate at time t:

$$T^{t+1}(z_k) = T(z_k) + \left(\frac{\partial T(z_k)}{\partial t}\right)^t \Delta t \qquad [24.8]$$

2. ***Direct solver***:

   Use gradient information in nonlinear root solver (faster, but more complex than time marching)

Radiative equilibrium temperature profiles show (see Figure 24.2 below):

✓ CO2 –only-atmosphere has less steep profile.

✓ Earth's stratosphere warms due to UV absorption by ozone.

✓ Most greenhouse effect from water vapor.

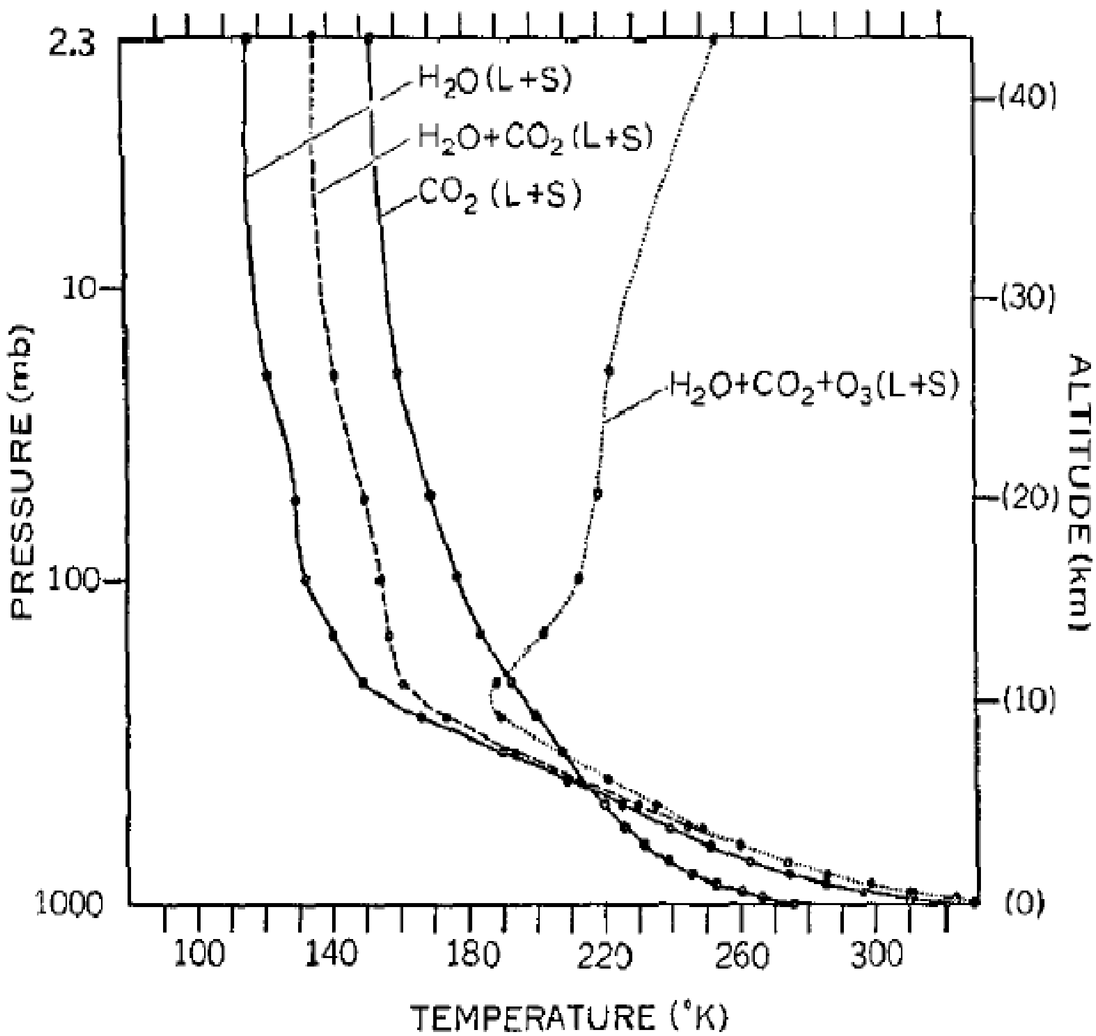


Figure 24.2 Pure radiative equilibrium temperature profiles for various atmospheric gases in a clear sky at 35 N in April. L+S means that the effects of both longwave and shortwave radiation are included (from Manabe and Strickler, 1964).

Results: the radiative equilibrium surface temperature is too high and the temperature profile is unrealistic.

Problem: radiative equilibrium surface temperature lapse rate near the surface exceeds threshold for convection

Fix: assume convection limits lapse rates to < γ_c (e.g., 6.5 K/km)

- **Radiative-convective equilibrium is equilibrium of radiative and convective fluxes**

<u>Convective adjustment methods:</u>

1. Move heat like convection: if γ_c exceeded, adjust temperature so γ_c achieved and heat is conserved
2. Parameterize convective flux, e.g.

$$F_{conv} = C\left(\left|\frac{dT}{dz}\right| - \gamma_c\right) \quad \text{if} \left(\left|\frac{dT}{dz}\right| - \gamma_c\right) > 0$$

Results of the RCE model developed by Manabe and Strickler (1964):

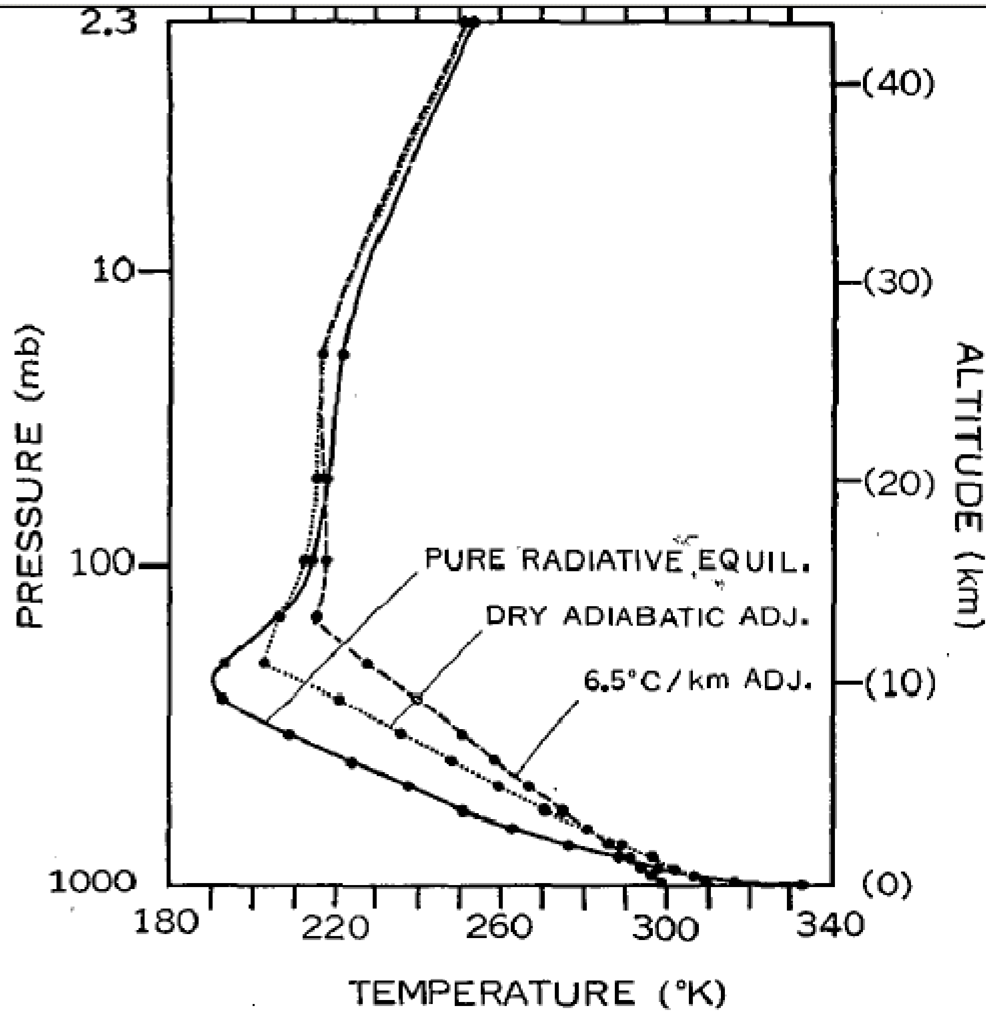


Figure 24.3 Pure radiative equilibrium and radiative-convective equilibrium temperature profiles for two values of γ_c for clear sky.

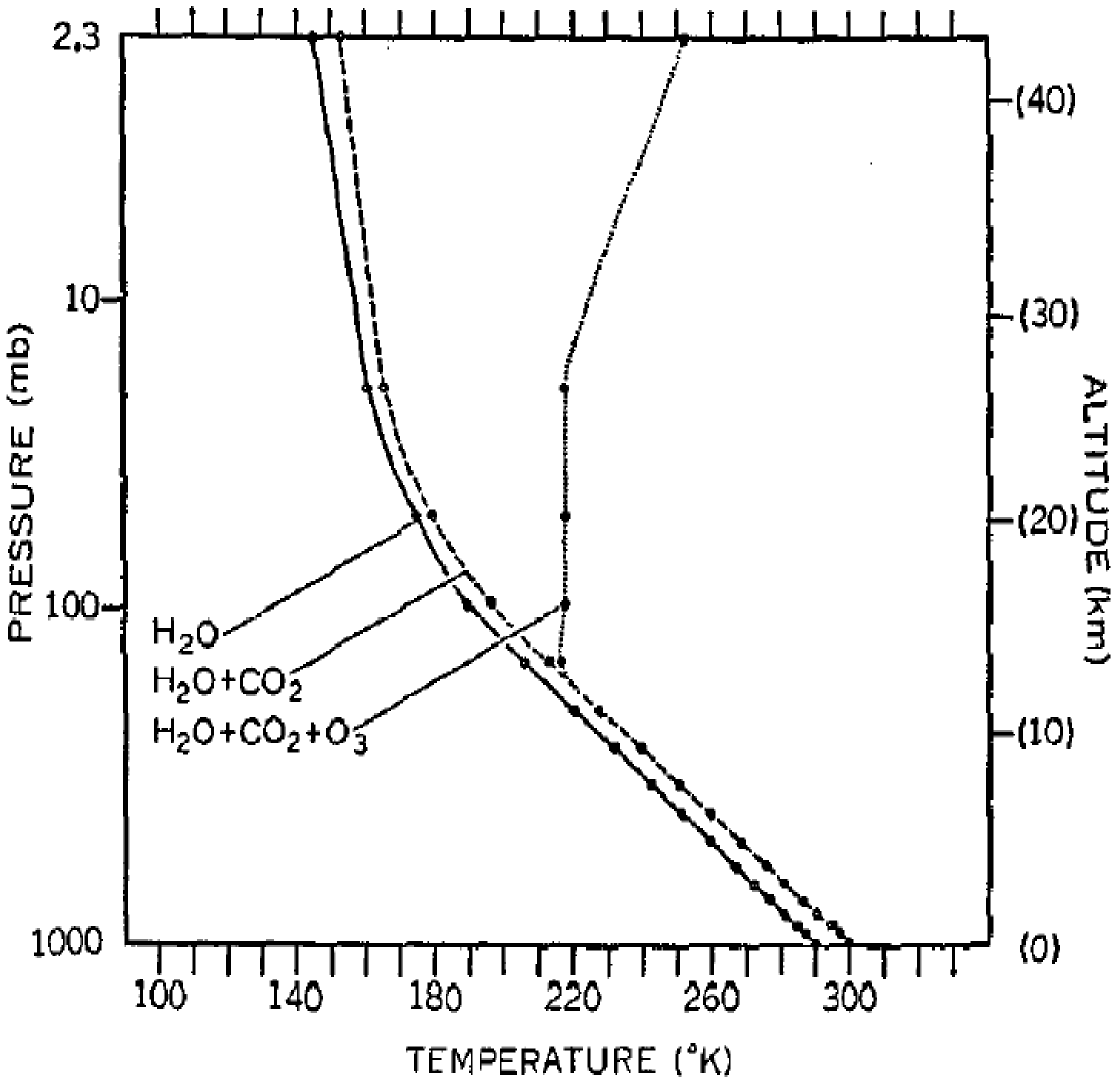


Figure 24.4 Radiative-convective equilibrium temperature profiles for various atmospheric gases in a clear sky at 35 N in April.

Comparing figures 24.2 and 24.4:

- ✓ Radiative equilibrium is fairly accurate for the stratosphere (though latitudinal and seasonal dependence is not correct)
- ✓ Convection required for to get reasonable tropospheric temperatures

3. Examples of simple energy balance models.

Let's estimate the **effective temperature** assuming that the Earth is in the radiative equilibrium. The sun emits F_s= 6.2x10^{-7} W/m^2 (a blackbody with about T= 5800K).

From the energy conservation law, we have

$$\mathbf{F_s\ 4\pi R_s^2 = F_{s0}\ 4\pi D_0^2}$$

where $\mathbf{R_s}$ is the radius of the sun (6.96x10^5 km);

$\mathbf{F_{s0}}$ is the solar flux reaching the top of the atmosphere (called the **solar constant** = about 1368 W/m^2) at the average distance of the Earth from the sun, $\mathbf{D_0}$ = 1.5x10^8 km.

Thus we have $\mathbf{F_{s0} = F_s\, R_s^2 / D_0^2}$

If the instantaneous distance from the Earth to sun is $\mathbf{D}$, then the total sun energy flux $\mathbf{F_0}$ reaching the Earth is $\mathbf{F_0 = F_{s0}\,(D_0/D)^2}$

The total sun energy intercepted by the cross section of the Earth is $\mathbf{F_{s0}\ \pi R_e^2}$ where $\mathbf{R_e}$ is the radius of the Earth. This energy is spread uniformly over the entire planet (with surface area $\mathbf{4\pi R_e^2}$). Thus the amount of received energy per unit surface becomes

$$\mathbf{F_{s0}\ \pi R_e^2 / 4\pi R_e^2 = F_{s0} /4}$$

Therefore, the total energy $\mathbf{Q_{in}}$ (in W/m^2) absorbed by the earth-atmosphere system is:

$$\mathbf{Q_{in} = (1 - \bar{r})\ \ F_{s0} /4}$$

where $\bar{r}$ is the spherical (or global) albedo. Spherical albedo of the earth is about 0.3.

Assuming that the Earth is a blackbody with temperature T_e, we have:

$$\mathbf{Q_{out} = F_b = \sigma_B\ \ T_e^4}$$

where $\boldsymbol{\sigma_B}$ is the Stefan-Boltzmann constant.

From the balance of incoming and outgoing energy, the **effective temperature** of the Earth is: $\mathbf{Q_{in} = Q_{out}}$

$$\mathbf{F_{s0}\ \ (1 - \bar{r}) /4 = \sigma_B\ \ T_e^4}$$

$$\mathbf{T_e^4\ = F_{s0}\,(1 - \bar{r}) / 4\,\sigma_B}$$

$$\mathbf{T_e\ \ = 255\ K = -18^0C\ is\ very\ low!!!}$$

$\mathbf{T_e}$ is much lower than the global average surface temperature (about 288 K)

Why? Because we didn't include the greenhouse effect and ignore the temperature structure.

Table 25.2 Effective temperatures of some planets in the radiative equilibrium.

Planet	Relative distance to the sun with respect to the Earth	Global albedo	T_e(K)
Mercury	**0.39**	**0.06**	**441**
Venus	**0.72**	**0.78**	**226**
Earth	**1**	**0.3**	**255**
Mars	**1.52**	**0.17**	**217**
Jupiter	**5.2**	**0.45**	**106**

How can we measure the greenhouse effect?

Upwelling flux at the surface

$$F_s^{\uparrow} = \sigma T_s^4$$

Upwelling flux $F_{TOA}^{\uparrow}$ at the top of the atmosphere (TOA): from satellite observations

The difference between the upwelling fluxes at the surface and TOA gives a measure of greenhouse effect G:

$$G = \sigma T_s^4 - F_{TOA}^{\uparrow} \qquad [24.9]$$

NOTE: G is the amount of heat (e.g., measured in Watts) per unit area of the Earth.

What is a reasonable estimate of the greenhouse effect?

$F_{TOA}^{\uparrow}$ = 235 W/m^2

Ts = 288 K ➔ G = 390-235 = 155 W/m^2

σT_s^4 = 390 W/m^2

Simple model of greenhouse effect #1: single layer gray energy balance model:

Let's include the atmosphere assuming that it emits (absorbs) as a gray body. Assume that the atmosphere does not absorb solar radiation - all is absorbed at the surface.

TOA balance: $$\frac{F_0}{4}(1-\bar{r}) = (1-\varepsilon)\sigma T_s^4 + \varepsilon\sigma T_a^4 \quad [24.10]$$

Surface balance: $$\frac{F_0}{4}(1-\bar{r}) + \varepsilon\sigma T_a^4 = \sigma T_s^4 \quad [24.11]$$

where σ is the Stefan-Boltzmann constant and ε is the emissivity of the atmosphere.

$$\Downarrow$$

$$T_s^4 = \frac{F_0(1-\bar{r})}{2\sigma(2-\varepsilon)} \quad [24.12]$$

for ε = 0.6 => $\mathbf{T_s}$ =278 K

NOTE: increasing for ε increases $\mathbf{T_s}$. This is so-called "**runaway greenhouse effect**" :

warmer Ts => more evaporation => more water vapor => higher emissivity =>warmer Ts

Why? Because we assume that atmosphere is a gray body.

Simple model of greenhouse effect #2: single black layer with spectral window energy balance model:

A more realistic way to deal with partial longwave transparency of the atmosphere is to assume that a fraction of the spectrum is clear.

Assume that

$f = \dfrac{1}{\sigma T^4}\displaystyle\int_{\nu_1}^{\nu_2} B_\nu(T)d\nu$ is the fraction of LW spectrum which is completely transparent

and the remainder of the LW spectrum is black. Surface is black and no SW absorption by the atmosphere.

TOA balance: $$\frac{F_0}{4}(1-\bar{r}) = f\sigma T_s^4 + (1-f)\sigma T_a^4 \quad [24.13]$$

Surface balance: $$\frac{F_0}{4}(1-\bar{r}) + (1-f)\sigma T_a^4 = \sigma T_s^4 \quad [24.14]$$

Thus we can express $\mathbf{T_s}$ and $\mathbf{T_a}$ via $\mathbf{T_e}$

$$T_s = \left(\frac{2}{1+f}\right)^{1/4} T_e \qquad \text{and} \qquad T_a = \left(\frac{1}{1+f}\right)^{1/4} T_e \qquad [24.15]$$

Limits:

No window f=0 => $T_s = 2^{1/4} T_e$ and $T_a = T_e$

All window f=1 => $T_s = T_e$ and $T_a = T_e / 2^{1/4}$

Earth: f is about 0.3 => T_s = 284 K and T_a = 239 K

<u>How to make the model more realistic:</u>

Tropics: radiation excess

North poles and high latitudes: radiation deficit

$$\Downarrow$$

must include poleward transport of energy

<u>One-dimensional (latitude) energy balance model:</u>

(Budyko 1969; Sellers 1969, Cess 1976)

Atmosphere is only implicit: TOA outgoing longwave flux is parameterized as a function of T_s

Budyko's parameterization is based on monthly mean atmospheric temperature and humidity profiles, and cloud cover observed at 260 stations

$$F_{LW}(x) = a_1 + b_1 T_s(x) - [a_2 + b_2 T(x)]\eta \qquad [24.16]$$

where a_i and b_i are the empirical constants based on statistical fitting, x =sin (ϕ) and ϕ is latitude. If cloud cover is taken constant of 0.5 then

$$F_{LW}(x) = (1.55 W/m^2/K) T_s(x) - 212 W/m^2 \qquad [24.17]$$

NOTE: The approximation for linear relation between OLR and the surface temperature may be argued from the fact that the temperature profiles have more or less the same shape at all latitudes, and that OLR, which depend on temperatures at all levels, may be expressed as a function of surface temperature.

Annual mean TOA solar insolation fit well with

$$S(x) = F_0 / 4[1 - 0.482 P_2(x)] \qquad [24.18]$$

$P_2(x) = (3x^2-1)/2$ is the second Legendre polynomial.

Thus energy balance equilibrium ($\delta T/\delta t=0$) with diffuse transport:

$$-D\frac{\partial}{\partial x}(1-x^2)\frac{\partial F_{LW}}{\partial x} + F_{LW} = S(x)[1-r(x)] \qquad [24.19]$$

where D is diffusion coefficient for energy transport and r(x) is albedo.

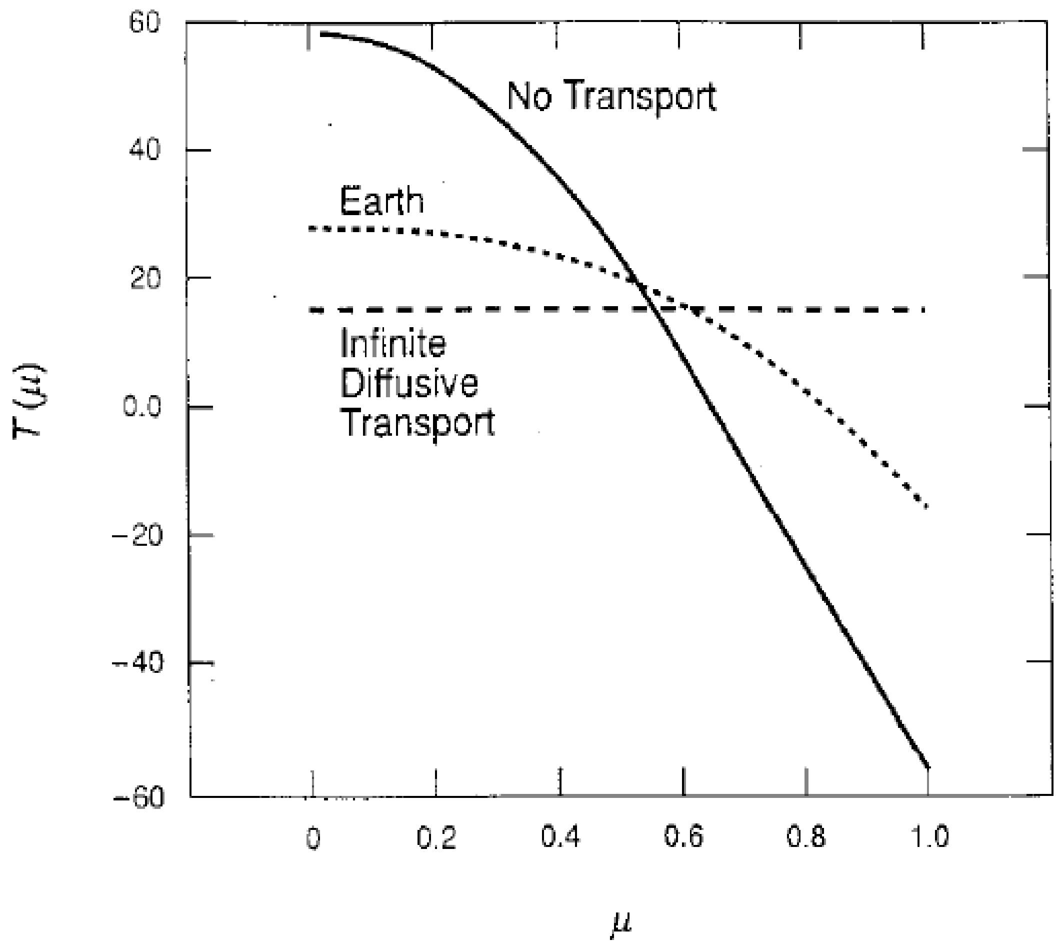


Figure 24.5 Zonally average surface temperature (K) as a function of the sine of the latitude, μ (μ is same as x in [24.16]), observed and for cases of no horizontal heat transport, and infinite horizontal heat transport (North et al., 1981).

NOTE: with no meridional transport, the poles are way too cold!

***Appendix*. Derivation of the Eddington gray radiative equilibrium.**

Assumptions:

6) Radiative equilibrium: $\frac{dF_{net}}{dz} = 0$
7) Gray atmosphere in longwave
8) No scattering and black surface in longwave
9) No solar absorption in the atmosphere
10) Eddington approximation: $I(\mu) = I_0 + I_1 \mu$

Since the atmosphere is gray (all wavelength are equivalent), one can write the wavelength integrated thermal emission radiative transfer equation (no scattering)

$$\mu \frac{dI}{d\tau} = I - B$$

where ***I*** is the integrated radiance (W m^{-2} st^{-1}), $\boldsymbol{\tau}$ increases downward , and $\boldsymbol{\mu}$ >0 in the upward direction. Note that deriving the variation of B with the optical depth $\boldsymbol{\tau}$ is equivalent to determining the temperature profiles since the blackbody emission is a function of temperature only.

Using the Eddington approximation, the net flux (positive upward) becomes

$$F_{net} = 2\pi \int_{-1}^{1} I\mu d\mu = \frac{4\pi}{3} I_1$$

The radiative equilibrium assumption implies that F_{net} (and I_1) is constant with optical depth.

Integrating the above radiative transfer equation over dμ gives

$$2\pi \frac{d}{d\tau} \int_{-1}^{1} I\mu d\mu = 2\pi \int_{-1}^{1} Id\mu - 2\pi \int_{-1}^{1} Bd\mu$$

$$\frac{dF_{net}}{d\tau} = 4\pi I_0 - 4\pi B$$

Under the radiative equilibrium assumption, we have

$$I_0 = B$$

Integrating the radiative transfer equation over μdμ gives

$$2\pi \frac{d}{d\tau} \int_{-1}^{1} I\mu^2 d\mu = 2\pi \int_{-1}^{1} I\mu d\mu - 2\pi \int_{-1}^{1} B\mu d\mu$$

Since B is isotropic the last term drops out leaving

$$\frac{4\pi}{3} \frac{dI_0}{d\tau} = F_{net} = \frac{4\pi}{3} I_1$$

$$\frac{dB}{d\tau} = I_1$$

Thus, the solution for B is simply a linear function of optical depth:

$$B(\tau) = B(0) + I_1 \tau$$

Constants B(0) and I_1 need to be determined from the boundary conditions.

Top of the atmosphere:

First boundary condition: no thermal downwelling flux

$$F^{\downarrow}(0) = 2\pi \int_{-1}^{0} I\mu d\mu = \pi B(0) - \frac{2\pi}{3} I_1 = 0$$

so we have

$$I_1 = \frac{3}{2} B(0) \quad \text{or} \quad F_{net} = 2\pi B(0)$$

Second boundary condition: upwelling longwave flux is equal to the absorbed solar flux F_{sun}:

$$F^{\uparrow}(0) = 2\pi \int_{0}^{1} I\mu d\mu = \pi B(0) + \frac{2\pi}{3} I_1 = F_{sun}$$

(Recall that the absorbed solar flux F_{sun} is $F_{sun} = (1-\bar{r})F_0 / 4)$)

Putting in $I_1 = \frac{3}{2} B(0)$ gives

$$F_{sun} = 2\pi B(0) = F_{net}$$

So now we have the B(0) and I_1 and thus the atmosphere Planck function profile is determined

$$B(\tau) = \frac{F_{sun}}{2\pi} (1 + \frac{3}{2} \tau)$$

The final step is to apply the boundary condition at the surface to obtain the surface temperature T_s. This boundary condition is that the emitted flux by the surface equals to the sum of the downwelling shortwave and longwave flux at the black surface:

$$F_{sun} + F^{\downarrow}(\tau^*) = \pi B_s$$

where $F^{\downarrow}(\tau^*) = \pi B(\tau^*) - \frac{2\pi}{3} I_1$

Using $F_{sun} = \frac{4\pi}{3} I_1$ gives the emission from the surface

$$B_s = B(\tau^*) + \frac{F_{sun}}{2\pi}$$

which is discontinuous with the atmospheric emission.

The previous results can be expressed in terms of temperature by

$$T^4(\tau) = T_e^4\left(\frac{1}{2} + \frac{3}{4}\tau\right)$$

where $\sigma T_e^4 = F_{sun}$

$$T_{top}^4 = \frac{1}{2} T_e^4$$

$$T_s^4 = T_e^4\left(1 + \frac{3}{4}\tau^*\right)$$

For F_0 = 1366 W/m^2 and $\bar{r} = 0.3$:

T_e = 255K and a “top” temperature T_t = 214 K

Assuming a global averaged surface air temperature of $T(\tau^*)$ = 288 K gives a gray body optical depth of τ^* = 1.5, and a surface skin temperature of T_s = 308 K