Lecture 3

Basic radiometric quantities. The Beer-Bouguer-Lambert law.

Concepts of extinction (scattering plus absorption) and emission. Schwarzschild's equation.

1. Basic introduction to electromagnetic field: Definitions, dual nature of electromagnetic radiation, electromagnetic spectrum.
2. Basic radiometric quantities: energy, intensity, and flux.
3. The Beer-Bouguer-Lambert law. Concepts of extinction (scattering + absorption) and emission. Optical depth.
4 Differential and integral forms of the radiative transfer equation.

Required reading:

L02: 1.1, 1.4

1. Basic introduction to electromagnetic field.

Electromagnetic radiation is a form of transmitted energy**.** ***Electromagnetic radiation*** is so-named because it has electric and magnetic fields that simultaneously oscillate in planes mutually perpendicular to each other and to the direction of propagation through space. Electromagnetic radiation exhibits the **dual nature**: it has wave properties and particulate properties.

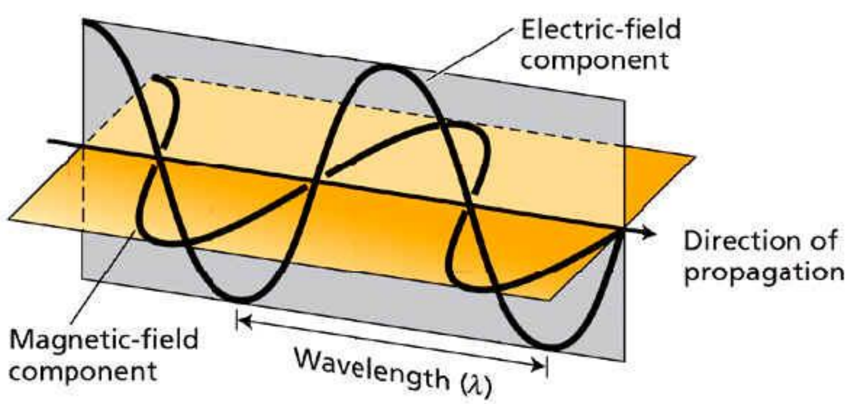


Figure 3.1 Schematic representation of electromagnetic radiation as a traveling wave.

***Wave nature of radiation*:** radiation can be thought of as **a traveling wave** characterized by the **wavelength (or frequency, or wavenumber)** and **speed.**

NOTE: speed of light in a vacuum: c = 2.9979×10^8 m/s $\cong$ 3.00×10^8 m/s.

Wavelength, λ, is the distance between two consecutive peaks or troughs in a wave.

Frequency, $\tilde{\nu}$, is defined as the number of waves (*cycles*) per second that pass a given point in space.

Wavenumber, ν, is defined as a count of the number of wave crests (or troughs) in a given unit of length.

Relation between λ, ν **and** $\tilde{\nu}$: $\nu = \tilde{\nu}/c = 1/\lambda$ [3.1]

NOTE: The frequency is a more fundamental quantity than the wavelength

Wavelength units: LENGTH,

Angstrom (A): 1 A = 1×10^{-10} m; Nanometer (nm): 1 nm=1×10^{-9} m;

Micrometer (μm): 1 μm = 1×10^{-6} m;

Frequency units: unit cycles per second 1/s (or s^{-1}) is called Hertz (abbreviated Hz)

Wavenumber units: $LENGTH^{-1}$ (often in cm^{-1})

- As a transverse wave, EM radiation can be polarized. **Polarization** is the distribution of the electric field in the plane normal to propagation direction.

***Particulate nature of radiation*:**

Radiation can be also described in terms of particles of energy, called **photons**.

The energy of a **photon** is:

$$E_{photon} = h\,\tilde{\nu} = h\,c/\lambda = h\,c\nu \qquad [3.2]$$

where ***h*** is Plank's constant ($h = 6.6256 \times 10^{-34}$ J s).

NOTE: Plank's constant ***h*** is very small!

- Eq. [3.2] relates energy of each photon of the radiation to the electromagnetic wave characteristics ($\tilde{\nu}$, ν or λ).

- **<u>Spectrum of electromagnetic radiation</u>**

is the distribution of electromagnetic radiation according to energy (or, equivalently, according to the wavelength, wavenumber, or frequency).

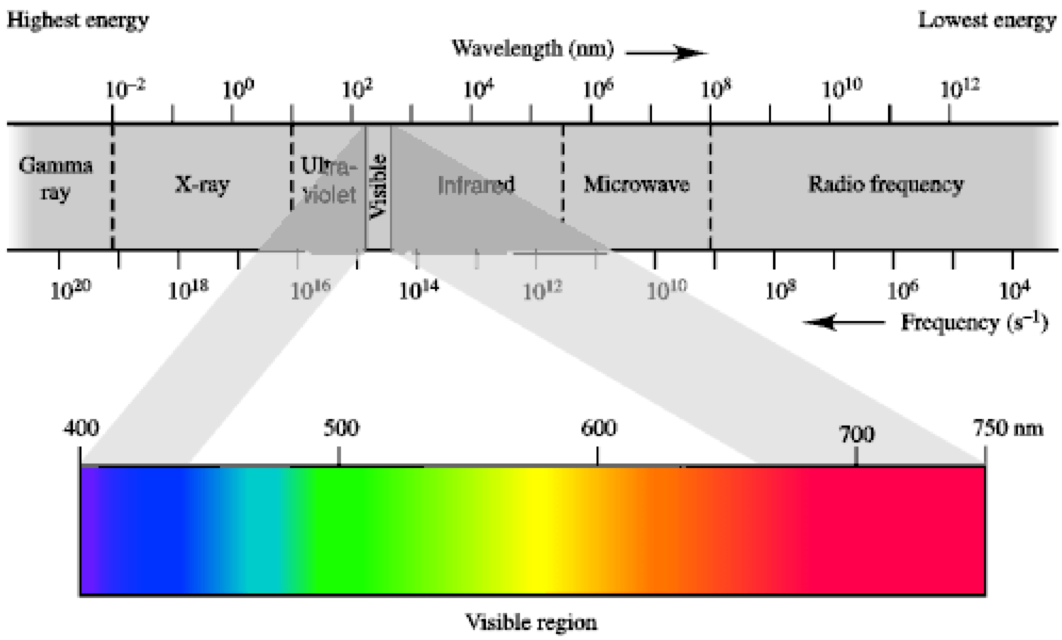


Figure 3.2 The electromagnetic spectrum.

Table 3.1 Relationships between radiation components studied in this course.

Name of spectral region	**Wavelength region, µm**	**Spectral equivalence**
Solar	0.1 - 4	Ultraviolet + Visible + Near infrared = Shortwave
Terrestrial	4 - 100	Far infrared = Longwave
Infrared	0.75 - 100	Near infrared + Far infrared
Ultraviolet	0.1 - 0.38	Near ultraviolet + Far ultraviolet = UV-A + UV-B + UV-C + Far ultraviolet
Shortwave	0.1 - 4	Solar = Near infrared + Visible + Ultraviolet
Longwave	4 - 100	Terrestrial = Far infrared
Visible	0.38 - 0.75	Shortwave - Near infrared - Ultraviolet
Near infrared	0.75 - 4	Solar - Visible - Ultraviolet = Infrared - Far infrared
Far infrared	4 - 100	Terrestrial = Longwave = Infrared - Near infrared
Thermal	4 - 100	Terrestrial = Longwave = Far infrared

2. Basic radiometric quantities.

Flux and intensity are the two measures of the strength of an electromagnetic field that are central to most problems in radiative transfer science.

Intensity (or radiance) is defined as radiant energy in a given direction per unit time per unit wavelength (or frequency) range per unit solid angle per unit area perpendicular to the given direction (see figure 3.3):

$$dI_{\lambda} = \frac{dE_{\lambda}}{\cos(\theta) d\Omega dt dA d\lambda} \quad [3.3]$$

I_{λ} is called the **monochromatic** intensity.

- ✓ Monochromatic does not mean at a single wavelengths λ, but in a very narrow (infinitesimal) range of wavelength dλ centered at λ.

NOTE: *same name*: intensity = specific intensity = radiance

UNITS: from Eq.[3.3]: (J sec^{-1} sr^{-1} m^{-2} μm^{-1}) = (W sr^{-1} m^{-2} μm^{-1})

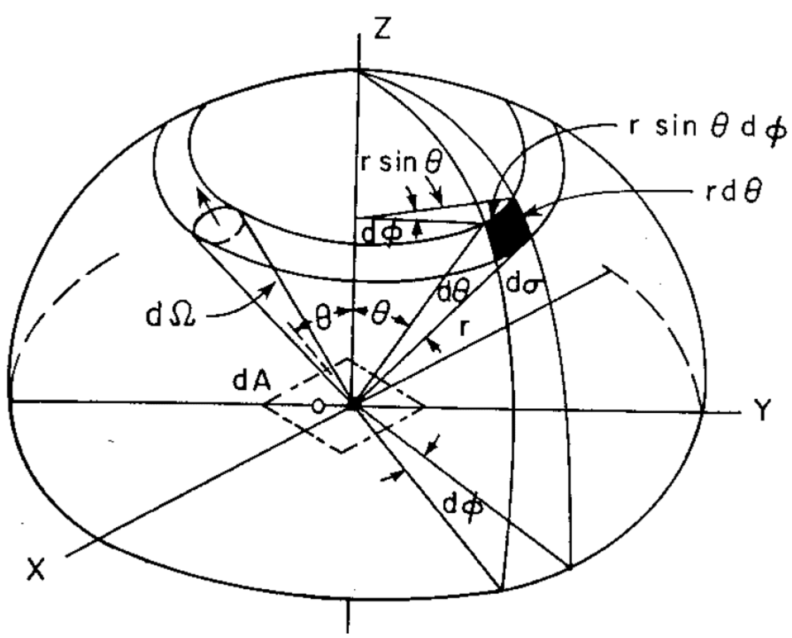
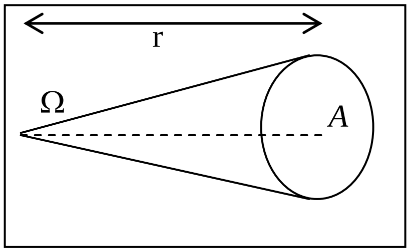


Figure 3.3 Illustration of differential solid angle in spherical coordinates.

Solid angle is the angle subtended at the center of a sphere by an area on its surface numerically equal to the square of the radius

$$\Omega = \frac{A}{r^2}$$

UNITS: of a solid angle = steradian (sr)



A differential solid angle can be expressed as

$$d\Omega = \frac{dA}{r^2} = \sin(\theta)\,d\theta\,d\phi\,,$$

using that a differential area is

$dA = (r\,d\theta)\,(r\sin(\theta)\,d\phi)$

EXAMPLE: Solid angle of a unit sphere = 4π

EXAMPLE: What is the solid angle of the Sun from the Earth if the distance from the Sun from the Earth is $D=1.5x10^8$ km and Sun's radius is $R_s = 6.96x10^5$ km.

$$\Omega = \frac{\pi R_s^2}{D^2} = 6.76x10^{-5}\,sr$$

Properties of intensity:

- ✓ In general, intensity is a function of the coordinates ($\vec{r}$), direction ($\vec{\Omega}$), wavelength (or frequency), and time. Thus, it depends on seven independent variables: three in space, two in angle, one in wavelength (or frequency) and one in time.
- ✓ Intensity, as a function of position and direction, gives a complete description of the electromagnetic field.
- ✓ If intensity does not depend on the direction, the electromagnetic field is said to be **isotropic.** If intensity does not depend on position the field is said to be **homogeneous**.

<u>Flux (or irradiance)</u> is defined as radiant energy in a given direction per unit time per unit wavelength (or frequency) range per unit area perpendicular to the given direction:

$$dF_\lambda = \frac{dE_\lambda}{dtdAd\lambda} \qquad [3.4]$$

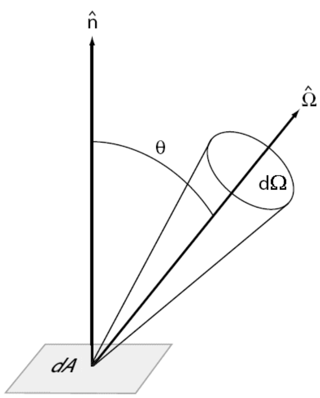
UNITS: from Eq.[3.4]: (J sec^{-1} m^{-2} μm^{-1}) = (W m^{-2} μm^{-1})

From Eqs. [3.3]-[3.4]:

$$F_\lambda = \int_\Omega I_\lambda \cos(\theta) d\Omega \qquad [3.5]$$

Monochromatic **flux** is the integration of normal component of monochromatic **intensity** over a certain solid angle.

Monochromatic **<u>upwelling (upward) hemispherical flux</u>** on a horizontal plane is the integration of the normal component of monochromatic **intensity** over the solid angle of the hemisphere



$$F_\lambda^\uparrow = \int_{2\pi} I_\lambda^\uparrow(\vec{\Omega}) \vec{n} d\Omega \qquad [3.6]$$

where $\cos(\theta) = \vec{n}\vec{\Omega}$

Eq. [3.6] in spherical coordinates gives

$$F_\lambda^\uparrow = \int_0^{2\pi}\int_0^{\pi/2} I_\lambda^\uparrow(\theta,\varphi)\cos(\theta)\sin(\theta)d\theta d\varphi = \int_0^{2\pi}\int_0^1 I_\lambda^\uparrow(\mu,\varphi)\mu d\mu d\varphi \qquad [3.7]$$

where ***μ = cos(θ).***

<u>Downwelling (downward) hemispherical flux</u> (i.e., integration over the lower hemisphere)

$$F_\lambda^\downarrow = -\int_0^{2\pi}\int_{\pi/2}^{\pi} I_\lambda^\downarrow(\theta,\varphi)\cos(\theta)\sin(\theta)d\theta d\varphi = -\int_0^{2\pi}\int_0^{-1} I_\lambda^\downarrow(\mu,\varphi)\mu d\mu d\varphi =$$

$$= \int_0^{2\pi}\int_0^1 I_\lambda^\downarrow(-\mu,\varphi)\mu d\mu d\varphi \qquad [3.8]$$

Monochromatic **<u>net flux</u>** is the integration of normal component of monochromatic **intensity** over the entire solid angle (over 4π). **Net flux** for a horizontal plane is the difference in **upwelling and downwelling hemispherical fluxes**:

$$F_{net,\lambda} = F_\lambda^{\uparrow} - F_\lambda^{\downarrow} = \int_0^{2\pi} \int_{-1}^{1} I_\lambda(\mu,\varphi)\mu d\mu d\varphi \qquad [3.9]$$

<u>Actinic flux</u> is the total spectral energy at point (used in photochemistry):

$$F_{act,\lambda} = \int_{4\pi} I_\lambda(\Omega) d\Omega \qquad [3.10]$$

<u>Spectral integration:</u>

Radiative quantities can be spectrally integrated (e.g., in energy balance calculations: SW and LW).

For example, the *downwelling shortwave (SW) flux* is

$$F^{\downarrow} = \int_{0.1\mu m}^{4.0\mu m} F_\lambda^{\downarrow} d\lambda \qquad [3.11a]$$

and the *upwelling SW flux* is

$$F^{\uparrow} = \int_{0.1\mu m}^{4.0\mu m} F_\lambda^{\uparrow} d\lambda \qquad [3.11b]$$

Similarly, the *downwelling and upwelling longwave (LW) fluxes are*

$$F^{\downarrow} = \int_{4\mu m}^{100\mu m} F_\lambda^{\downarrow} d\lambda \qquad [3.11c]$$

$$F^{\uparrow} = \int_{4\mu m}^{100\mu m} F_\lambda^{\uparrow} d\lambda \qquad [3.11d]$$

Photosynthetically Active Radiation (**PAR**) designates the spectral range of solar light from 0.4 to 0.7 μm that photosynthetic organisms are able to use in the process of photosynthesis:

$$F_{PAR}^{\downarrow} = \int_{0.4\mu m}^{0.7\mu m} F_{\lambda}^{\downarrow} d\lambda \qquad [3.12]$$

EXAMPLE: Convert between radiance in *per wavelength* to radiance *per wavenumber* units at λ = 10 μm. Given $\boldsymbol{I_{\lambda}}$ = 9.9 W m^{-2} $sr^{-1}\mu m^{-1}$. What is $\boldsymbol{I_{\nu}}$?

$\boldsymbol{I_{\nu}}$ = (9.9 W m^{-2} $sr^{-1}\mu m^{-1}$) (10 μm) (10^{-3} cm) = 0.099 W m^{-2} $sr^{-1}(cm^{-1})^{-1}$

3. The Beer-Bouguer-Lambert law. Concepts of extinction (scattering + absorption) and emission.

- **Extinction** and **emission** are two main types of the interactions between an electromagnetic radiation field and a medium (e.g., an atmosphere).

General definition:

Extinction is a process that decreases the radiant **intensity**, while **emission** increases it.

NOTE: “same name”: **extinction = attenuation**

Radiation is **emitted** by **all** bodies that have a temperature above absolute zero (O K) (often referred to as **thermal emission**).

- **Extinction** is due to **absorption** and **scattering**.

Absorption is a process that removes the radiant energy from an electromagnetic field and transfers it to other forms of energy.

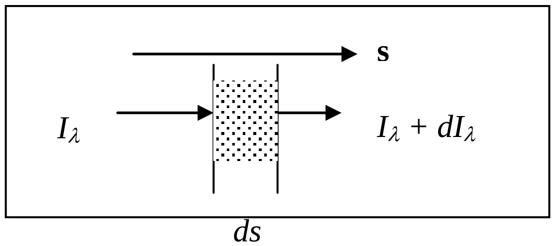
Scattering is a process that **does not** remove energy from the radiation field, but may redirect it.

NOTE: Scattering can be thought of as **absorption** of radiant energy followed by **re-emission** back to the electromagnetic field with negligible conversion of energy. Thus, scattering can remove radiant energy of a light beam traveling in one direction, but can be a "source" of radiant energy for the light beams traveling in other directions.

The fundamental law of extinction is the **Beer-Bouguer-Lambert law,** which states that the extinction process is linear in the intensity and amount of radiatively active matter, provided that the physical state (i.e., T, P, composition) is held constant.

NOTE: Some non-linear processes do occur as will be discussed later in the course.

For a small volume ΔV of infinitesimal length ***ds*** and area ΔA containing radiatively active matter, change in intensity along the path ***ds*** is proportional to the amount of matter in the path.



For extinction: $$dI_\lambda = -\beta_{e,\lambda} I_\lambda ds$$ [3.13]

For emission: $$dI_\lambda = \beta_{e,\lambda} J_\lambda ds$$ [3.14]

where $\beta_{e,\lambda}$ is the **volume extinction coefficient** (LENGTH^{-1}) and J_λ is the **source function**.

- The **source function** J_λ has emission and scattering contributions or only scattering.
- Generally, the **volume extinction coefficient** is a function of position **s.** (expressed mathematically as $\beta_{e,\lambda}$ (s)).

NOTE: Volume extinction coefficient is often referred to as the **extinction coefficient.**

Extinction coefficient = absorption coefficient + scattering coefficient

$$\beta_{e,\lambda} = \beta_{a,\lambda} + \beta_{s,\lambda} \qquad [3.15]$$

NOTE: Extinction coefficient (as well as absorption and scattering coefficients) can be expressed in different forms according to the definition of the amount of matter in the path (e.g., number concentrations, mass concentration, etc.) .

- **Volume** and **mass extinction coefficients** are most often used.

Mass extinction coefficient = volume extinction coefficient/density

UNITS: the mass coefficient is in unit area per unit mass ($LENGTH^2$ $MASS^{-1}$). For instance: (cm^2 g^{-1}), (m^2 kg^{-1}), etc.

If ρ is the density (mass concentration) of a given type of particles (or molecules), then

$$\begin{aligned} \beta_{e,\lambda} &= \rho k_{e,\lambda} \\ \beta_{s,\lambda} &= \rho k_{s,\lambda} \\ \beta_{a,\lambda} &= \rho k_{a,\lambda} \end{aligned} \qquad [3.16]$$

where the $\boldsymbol{k_{e,\lambda}}$, $\boldsymbol{k_{s,\lambda}}$ and $\boldsymbol{k_{a,\lambda}}$ are the **mass extinction, scattering, and absorption coefficients**, respectively.

NOTE: L02 uses $\boldsymbol{k_\lambda}$ for both mass extinction and mass absorption coefficients!

Using the mass extinction coefficient, the **Beer-Bouguer-Lambert (extinction) law (Eqs.[3.13-3.14])** can be expressed as

$$\begin{aligned} dI_\lambda &= -\rho k_{e,\lambda} I_\lambda ds \\ dI_\lambda &= \rho k_{e,\lambda} J_\lambda ds \end{aligned}$$

The **extinction cross section** of a given particle (or molecule) is a parameter that measures the attenuation of electromagnetic radiation by this particle (or molecule). In the same fashion, **scattering and absorption cross sections** can be defined.

UNITS: cross section is in unit area ($LENGTH^2$)

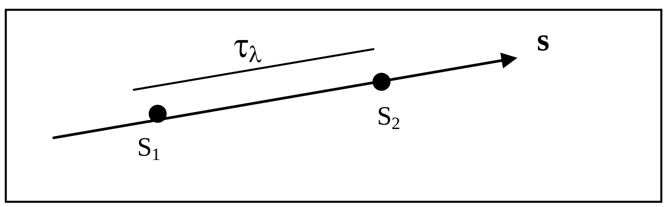
If ***N*** is the particle (or molecule) number concentration ($LENGTH^{-3}$) of a certain type of particles (or molecules), then

$$\beta_{e,\lambda} = \sigma_{e,\lambda} N$$
$$\beta_{s,\lambda} = \sigma_{s,\lambda} N \qquad [3.17]$$
$$\beta_{a,\lambda} = \sigma_{a,\lambda} N$$

where $\boldsymbol{\sigma_{e,\lambda}}$, $\boldsymbol{\sigma_{s,\lambda}}$, and $\boldsymbol{\sigma_{a,\lambda}}$ are the extinction, scattering, and absorbing cross sections,

Optical depth of a medium between points s_1 and s_2 is defined as

$$\tau_\lambda(s_2;s_1) = \int_{s_1}^{s_2} \beta_{e,\lambda}(s)ds$$



UNITS: optical depth is unitless.

NOTE: “same name”: **optical depth = optical thickness = optical path**

- **If** $\boldsymbol{\beta_{e,\lambda}}$ (s) does not depend on position (called a homogeneous optical path), then

$\boldsymbol{\beta_{e,\lambda}}$ (s) = $\boldsymbol{\beta_{e,\lambda}}$ and $\boldsymbol{\tau_\lambda(s_2;s_1) = \beta_{e,\lambda}(s_2 - s_1) = \beta_{e,\lambda} s}$

For this case, the **Extiction law** can be expressed as

$$\boldsymbol{I_\lambda = I_0 \exp(-\tau) = I_0 \exp(-\beta_{e,\lambda} s)} \qquad [3.18]$$

Optical depth can be expressed in several different ways:

$$\tau_{\lambda}(s_1;s_2) = \int_{s_1}^{s_2} \beta_{e,\lambda} ds = \int_{s_1}^{s_2} \rho k_{e,\lambda} ds = \int_{s_1}^{s_2} N\sigma_{e,\lambda} ds \qquad [3.19]$$

- If in a given volume there are several types of optically active particles each with $\beta^{i}_{e,\lambda}$, then the optical depth can be expressed as:

$$\tau_{\lambda} = \sum_{i} \int_{s_1}^{s_2} \beta^{i}_{e,\lambda}\ ds = \sum_{i} \int_{s_1}^{s_2} \rho_i \beta^{*i}_{e,\lambda} ds = \sum_{i} \int_{s_1}^{s_2} N_i \sigma^{i}_{e,\lambda} ds \qquad [3.20]$$

where ρ_i and N_i is the mass concentrations (densities) and particles concentrations of the ***i***-th type of species.

4. Differential and integral forms of the radiative transfer equation.

Let's consider a small volume ΔV of infinitesimal length ***ds*** and area ΔA containing radiatively active matter. Using the **Extinction law**, the change (loss plus gain due to both the thermal emission and scattering) of intensity along the path ***ds*** is

$$dI_{\lambda} = -\beta_{e,\lambda} I_{\lambda} ds + \beta_{e,\lambda} J_{\lambda} ds$$

Dividing this equation by $\beta_{e,\lambda} ds$, we find

$$\boxed{\frac{dI_{\lambda}}{\beta_{e,\lambda} ds} = -I_{\lambda} + J_{\lambda}} \qquad [3.21]$$

Eq. [3.21] is the **differential equation of radiative transfer (called Schwarzchild's equation).**

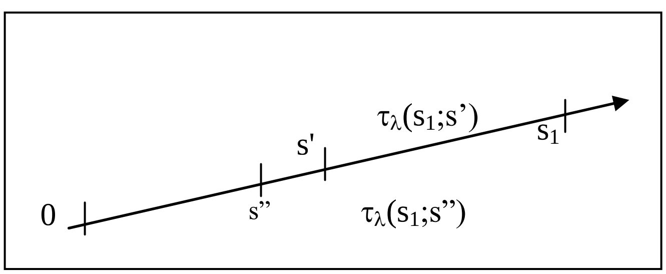
NOTE: Both I_{λ} and J_{λ} are generally functions of both position and direction.

The optical depth is

$$\tau_{\lambda}(s_1;s) = \int_{s}^{s_1} \beta_{e,\lambda}(s) ds$$

Thus

$$d\tau_{\lambda} = -\beta_{e,\lambda}(s) ds$$



Using the above expression for $d\tau_\lambda$, we can re-write Eq. [3.21] as

$$-\frac{dI_\lambda}{d\tau_\lambda} = -I_\lambda + J_\lambda$$

or as [3.22]

$$\frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - J_\lambda$$

These are other forms of the **differential equation of radiative transfer.**

Re-arranging terms in the above equation and multiplying both sides by ***exp(-τ_λ)***, we have

$$-\frac{\exp(-\tau_\lambda)dI_\lambda}{d\tau_\lambda} + \exp(-\tau_\lambda)I_\lambda = \exp(-\tau_\lambda)J_\lambda$$

and (using that ***d[I(x)exp(-x)]=exp(-x)dI(x)-exp(-x)I(x)dx***) we find

$$-d\left[I_\lambda \exp(-\tau_\lambda)\right] = \exp(-\tau_\lambda)J_\lambda d\tau_\lambda$$

Then integrating over the path from **0** to $\mathbf{s_1}$ **,** we have

$$-\int_0^{s_1} d\left[I_\lambda(s)\exp(-\tau_\lambda(s_1;s))\right] = \int_0^{s_1} \exp(-\tau_\lambda(s_1;s))J_\lambda d\tau_\lambda$$

and

$$-\left[I_\lambda(s_1) - I_\lambda(0)\exp(-\tau_\lambda(s_1;0))\right] = \int_0^{s_1} \exp(-\tau_\lambda(s_1;s))J_\lambda d\tau_\lambda$$

Thus

$$I_\lambda(s_1) = I_\lambda(0)\exp(-\tau_\lambda(s_1;0)) - \int_0^{s_1} \exp(-\tau_\lambda(s_1;s))J_\lambda d\tau_\lambda$$

and, using $d\tau_\lambda = -\beta_{e,\lambda}(s)ds$, we obtain a **solution** of the **equation of radiative transfer (often referred to as the integral form of the radiative transfer equation):**

$$I_\lambda(s_1) = I_\lambda(0)\exp(-\tau_\lambda(s_1;0)) + \int_0^{s_1} \exp(-\tau_\lambda(s_1;s))J_\lambda \beta_{e,\lambda} ds \qquad [3.23]$$

NOTE:

i) The above equation gives monochromatic intensity at a given point propagating in a given direction (often called an **elementary solution**). A completely general distribution of intensity in angle and wavelengths (or frequencies) can be obtained by repeating the elementary solution for all incident beams and for all wavelengths (or frequencies).

ii) Knowledge of the **source function** J_λ is required to solve the above equation. In the general case, the source function consists of thermal emission and scattering (or from scattering), depends on the position and direction, and is very complex. One may say that the radiative transfer equation is all about the source function.

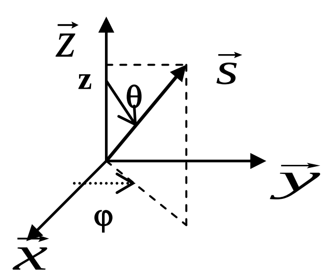
- **<u>Plane-parallel atmosphere.</u>**

✓ For many applications, the atmosphere can be approximated by **a plane-parallel model** to handle the vertical stratification of the atmospheric composition and structure.

Plane-parallel atmosphere consists of a certain number of atmospheric layers each characterized by homogeneous properties (e.g., T, P, optical properties of a given species, etc.) and bordered by the bottom and top infinite plates (called boundaries).

- Traditionally, the **vertical coordinate *z*** is used to measure linear distances in the plane-parallel atmosphere:

$$z = s\cos(\theta)$$



where θ denotes the angle between the upward normal and the direction of propagation of the light beam (or zenith angle) and φ is the azimuthal angle.

Using $ds = dz/\cos(\theta)$, the **radiative transfer equation** can be written as

$$\cos(\theta)\frac{dI_\lambda(z;\theta;\varphi)}{\beta_{e,\lambda}dz} = -I_\lambda(z;\theta;\varphi) + J_\lambda(z;\theta;\varphi) \qquad [3.24a]$$

Introducing the optical depth measured from the outer boundary downward as

$$\tau_\lambda(z_1; z) = \int_z^{z_1} \beta_{e,\lambda}(z) dz$$

and using $d\tau_\lambda = -\beta_{e,\lambda}(z)dz$ and ***μ = cos(θ),*** we have

$$\mu \frac{dI_\lambda(\tau;\mu;\varphi)}{d\tau} = I_\lambda(\tau;\mu;\varphi) - J_\lambda(\tau;\mu;\varphi) \qquad [3.24b]$$

- Eq.[3.24] can be solved to give the **upward (or upwelling) and downward (or downwelling) intensities** for a planet's atmosphere which is bounded on two sites.

Upward intensity $I_\lambda^\uparrow$ is for $1 \geq \mu \geq 0$ (or $0 \leq \theta \leq \pi/2$);

Downward intensity $I_\lambda^\downarrow$ is for $-1 \leq \mu \leq 0$ (or $\pi/2 \leq \theta \leq \pi$)

(using that ***cos***(0)=1; ***cos***(***π***/2)=0 and ***cos***(***π***) =-1)

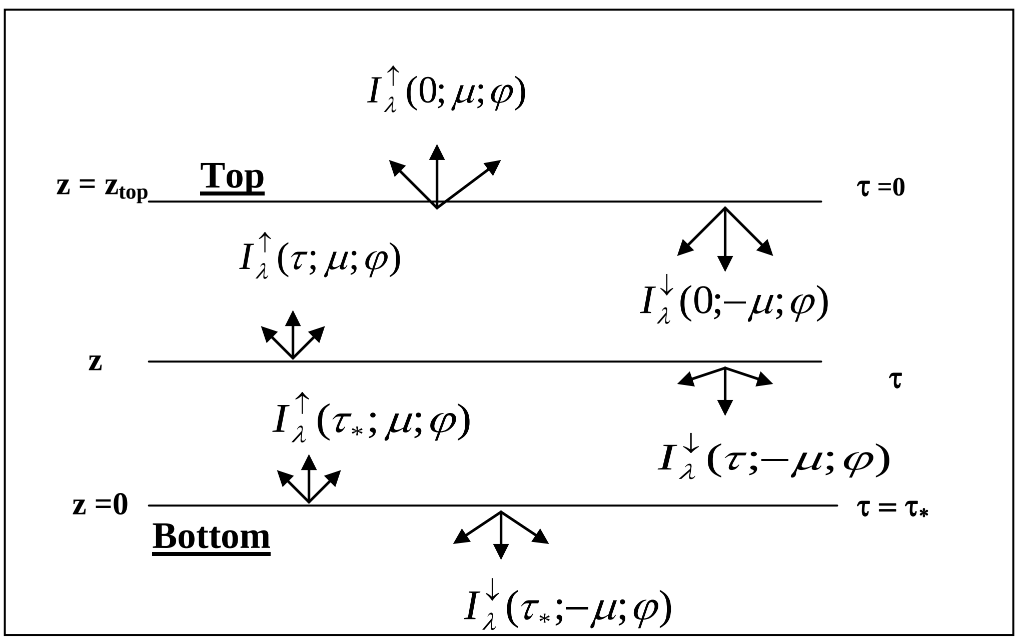


Figure 3.4 Schematic representation of the plane-parallel atmosphere.

NOTE: For downward intensity, μ is replaced by –μ.

The **radiative transfer equation** [3.24b] can be written for **upward and downward intensities:**

$$\mu \frac{dI_\lambda^{\uparrow}(\tau;\mu;\varphi)}{d\tau} = I_\lambda^{\uparrow}(\tau;\mu;\varphi) - J_\lambda^{\uparrow}(\tau;\mu;\varphi) \qquad [3.25a]$$

$$-\mu \frac{dI_\lambda^{\downarrow}(\tau;-\mu;\varphi)}{d\tau} = I_\lambda^{\downarrow}(\tau;-\mu;\varphi) - J_\lambda^{\downarrow}(\tau;-\mu;\varphi) \qquad [3.25b]$$

A solution of Eq.[3.25a] gives the upward intensity in the plane-parallel atmosphere:

$$I_\lambda^{\uparrow}(\tau;\mu;\varphi) = I_\lambda^{\uparrow}(\tau_*;\mu;\varphi)\exp(-\frac{\tau_* - \tau}{\mu}) + \frac{1}{\mu}\int_{\tau}^{\tau_*} \exp(-\frac{\tau' - \tau}{\mu})\; J_\lambda^{\uparrow}(\tau';\mu;\varphi)d\tau' \qquad [3.26a]$$

and **a solution of Eq.[3.25b] gives the downward intensity in the plane-parallel atmosphere:**

$$I_\lambda^{\downarrow}(\tau;-\mu;\varphi) = I_\lambda^{\downarrow}(0;-\mu;\varphi)\exp(-\frac{\tau}{\mu}) + \frac{1}{\mu}\int_{0}^{\tau} \exp(-\frac{\tau - \tau'}{\mu})\; J_\lambda^{\downarrow}(\tau';-\mu;\varphi)d\tau' \qquad [3.26b]$$