Lecture 14.

Review for Exam 1.

Electromagnetic radiation exhibits the dual nature:

wave properties and particulate properties

Wave nature of radiation:

Electromagnetic waves are characterized by wavelength λ (or frequency $\tilde{\nu}$,or wavenumber ν) and speed

Relation between λ, ν and $\tilde{\nu}$:

$$\nu = \tilde{\nu} / c = 1/\lambda$$

Particulate nature of radiation:

can be described in terms of particles of energy, called photons.

$$E_{photon} = h\,\tilde{\nu} = h\,c/\lambda = h\,c\nu$$

h is Plank's constant ($h = 6.6256 \times 10^{-34}$ J s).

Flux (or irradiance) is defined as radiant energy in a given direction per unit time per unit wavelength (or frequency) range per unit area perpendicular to the given direction:

$$dF_\lambda = \frac{dE_\lambda}{dt\,dA\,d\lambda}$$

UNITS: (J sec^{-1} m^{-2} μm^{-1}) = (W m^{-2} μm^{-1})

The radiative **flux** is the integration of normal component of monochromatic **intensity** over some solid angle.

$$F_\lambda = \int_\Omega I_\lambda \cos(\theta) d\Omega$$

$$F_\lambda^\uparrow = \int_{2\pi} I_\lambda^\uparrow(\vec{\Omega})\vec{n}\,d\Omega$$

Net radiative flux

Monochromatic **net flux** **is the integration of normal component of monochromatic intensity over the all solid angles (over 4π):**

$$F_{net,\lambda} = F_{\lambda}^{\uparrow} - F_{\lambda}^{\downarrow} = \int_{0}^{2\pi} \int_{-1}^{1} I_{\lambda}(\mu,\varphi)\mu d\mu d\varphi$$

What is the net flux of the isotropic radiative field?

<u>Extinction (scattering +absorption) and emission.</u>

Extinction is a process that decreases the radiant **intensity**, while **emission** increases it.

Extinction (or attenuation) is due to **absorption** and **scattering.**

Absorption is a process that removes the radiant energy from an electromagnetic field and transfers it to other forms of energy.

Scattering is a process that <u>**does not**</u> remove energy from the radiation field, but may redirect it.

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The fundamental law of extinction is the **Beer-Bouguer-Lambert law**: the extinction process is linear in the intensity of radiation and amount of matter, provided that the physical state (i.e., T, P, composition) is held constant.

Extinction: $dI_\lambda = -\beta_{e,\lambda} I_\lambda ds$ **Emission:** $dI_\lambda = \beta_{e,\lambda} J_\lambda ds$

where $\beta_{e,\lambda}$ is the volume extinction coefficient (LENGTH^{-1});
J_λ is the source function.

$$\beta_{e,\lambda} = \beta_{a,\lambda} + \beta_{s,\lambda}$$

$$dI_\lambda = -\beta_{e,\lambda} I_\lambda ds + \beta_{e,\lambda} J_\lambda ds$$

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The differential form of radiative transfer equation

$$dI_\lambda = -\beta_{e,\lambda} I_\lambda ds + \beta_{e,\lambda} J_\lambda ds$$

$$\frac{dI_\lambda}{\beta_{e,\lambda} ds} = -I_\lambda + J_\lambda$$

Using $d\tau_\lambda = -\beta_{e,\lambda}(s)ds$ We have $-\frac{dI_\lambda}{d\tau_\lambda} = -I_\lambda + J_\lambda$

$$\frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - J_\lambda$$

Elementary solution:

$$I_\lambda(s_1) = I_\lambda(0)\exp(-\tau_\lambda(s_1;0)) + \int_0^{s_1} \exp(-\tau_\lambda(s_1;s)) J_\lambda \beta_{e,\lambda} ds$$

See p.12

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Solution of the radiative transfer equation in the plane-parallel atmosphere (called the integral form)

$$I_\lambda^{\uparrow}(\tau;\mu;\varphi) = I_\lambda^{\uparrow}(\tau_*;\mu;\varphi)\exp(-\frac{\tau_* - \tau}{\mu}) + \frac{1}{\mu}\int_{\tau}^{\tau_*}\exp(-\frac{\tau' - \tau}{\mu})\ J_\lambda^{\uparrow}(\tau';\mu;\varphi)d\tau'$$

$$I_\lambda^{\downarrow}(\tau;-\mu;\varphi) = I_\lambda^{\downarrow}(0;-\mu;\varphi)\exp(-\frac{\tau}{\mu}) + \frac{1}{\mu}\int_{0}^{\tau}\exp(-\frac{\tau - \tau'}{\mu})\ J_\lambda^{\downarrow}(\tau';-\mu;\varphi)d\tau'$$

Lecture 4

Blackbody emission

Planck function, $B_\lambda(T)$, gives the **intensity (or radiance)** emitted by a blackbody having a given temperature.

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5(\exp(hc/k_B T\lambda) - 1)}$$

Stefan-Boltzmann law:

$$\mathbf{F = \sigma_b T^4 = \pi B(T)}$$

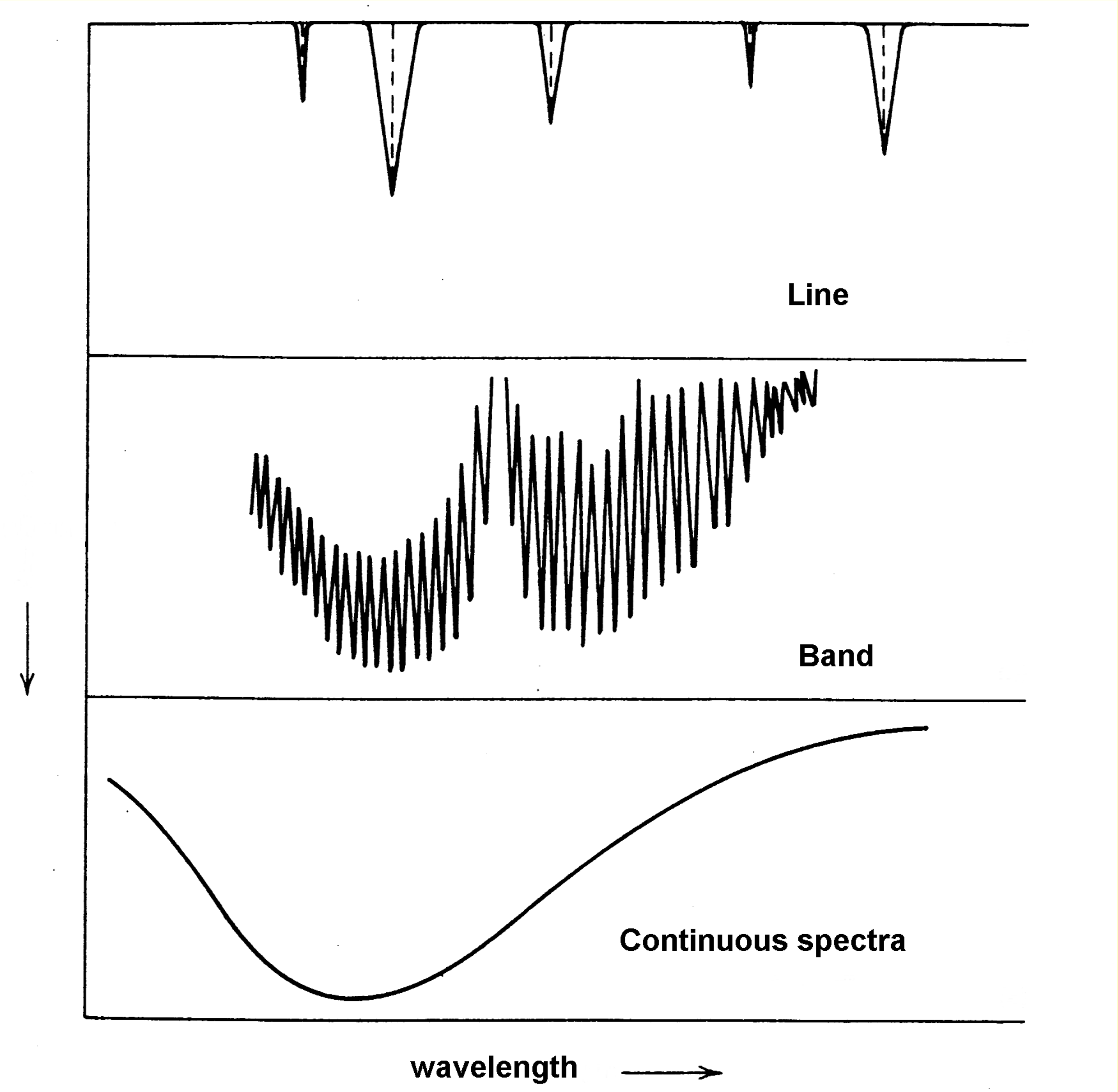
Wien displacement law:

$$\mathbf{\lambda_m = 2898 / T}$$

Kirchhoff law:

$$\mathbf{\varepsilon_\lambda = A_\lambda}$$

Molecular Absorption/Emission Spectra



Lorentz profile of a spectral line is used to characterize the **pressure broadening** and is defined as:

$$f_L(\nu-\nu_0)=\frac{\alpha/\pi}{(\nu-\nu_0)^2+\alpha^2}$$

α is the half-width of a line at the half maximum (in cm^{-1}), (often called the **line width**)

$$\alpha(P,T)=\alpha_0\frac{P}{P_0}\left(\frac{T_0}{T}\right)^n$$

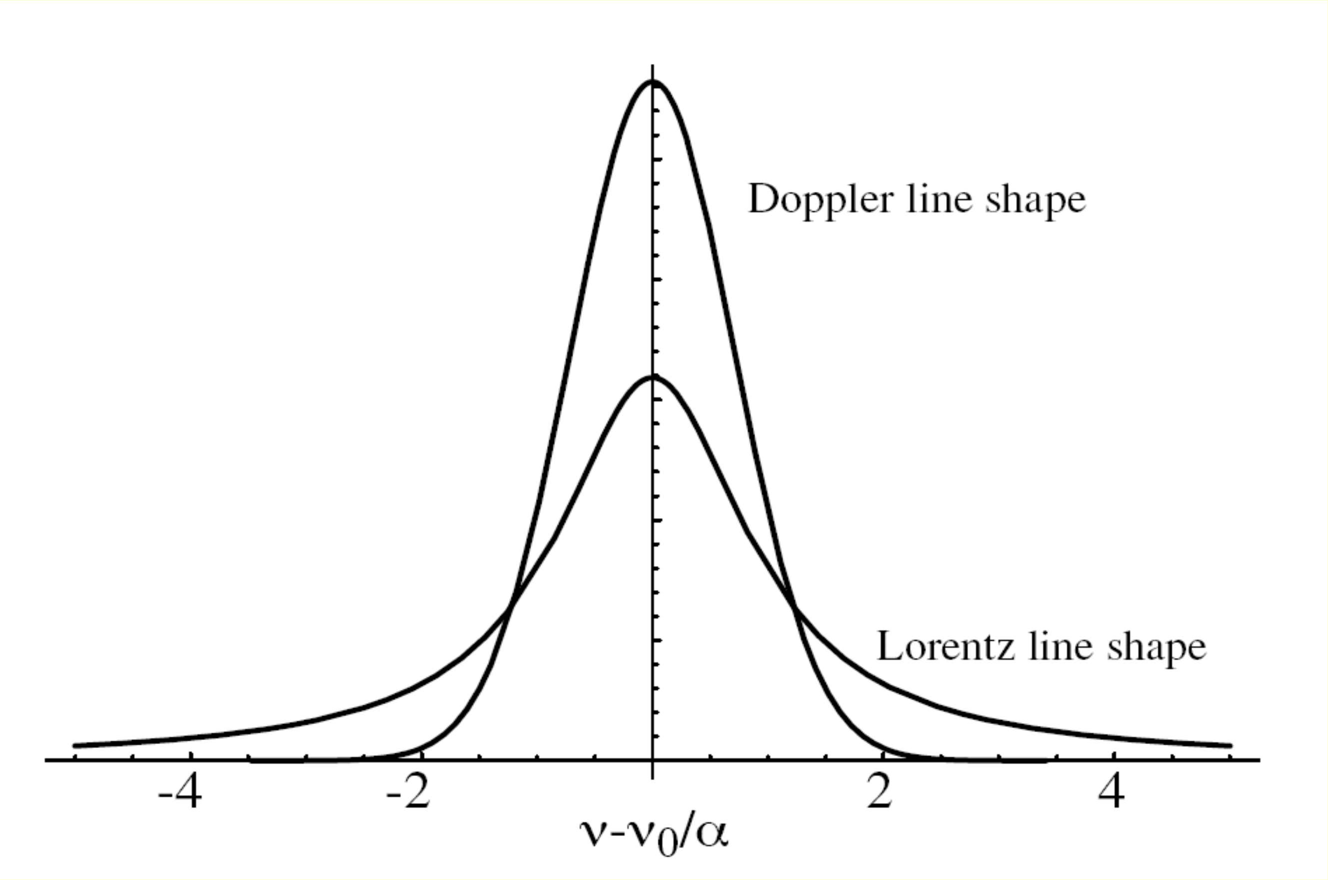
Doppler profile is defined in the absence of collision effects (i.e., no pressure broadening) as:

$$f_D(\nu-\nu_0)=\frac{1}{\alpha_D\sqrt{\pi}}\exp\left[-\left(\frac{\nu-\nu_0}{\alpha_D}\right)^2\right]$$

α_D is the **Doppler line width**

$$\alpha_D=\frac{\nu_0}{c}(2k_BT/m)^{1/2}$$

Comparison of the Doppler and Lorentz profiles for equivalent line strengths and widths.



Absorption coefficient is defined by the position, strength, and shape of a spectral line:

$$\boldsymbol{k_{a,\nu} = S\, f(\nu - \nu_0)}$$

$$S = \int k_{a,\nu}\, d\nu \qquad \int f(\nu - \nu_0)\, d\nu = 1$$

***Dependencies*:**

S depends on **T;**

$\boldsymbol{f(\nu - \nu_0, \alpha)}$ depends on the line halfwidth **$\boldsymbol{\alpha}$ (p, T),** which depends on pressure and temperature.

Path length (or **path**) is defined as the amount of an absorber along the path

If the amount of an absorber is given in terms of mass density, **path length is**

$$u = \int_{s_1}^{s_2} \rho(s) ds$$

Homogeneous absorption path:

when $\boldsymbol{k_{a,\nu}}$ does not vary along the path => optical depth is $\boldsymbol{\tau = k_{a,\nu} u}$

Inhomogeneous absorption path: $\boldsymbol{k_{a,\nu}}$ varies along the path

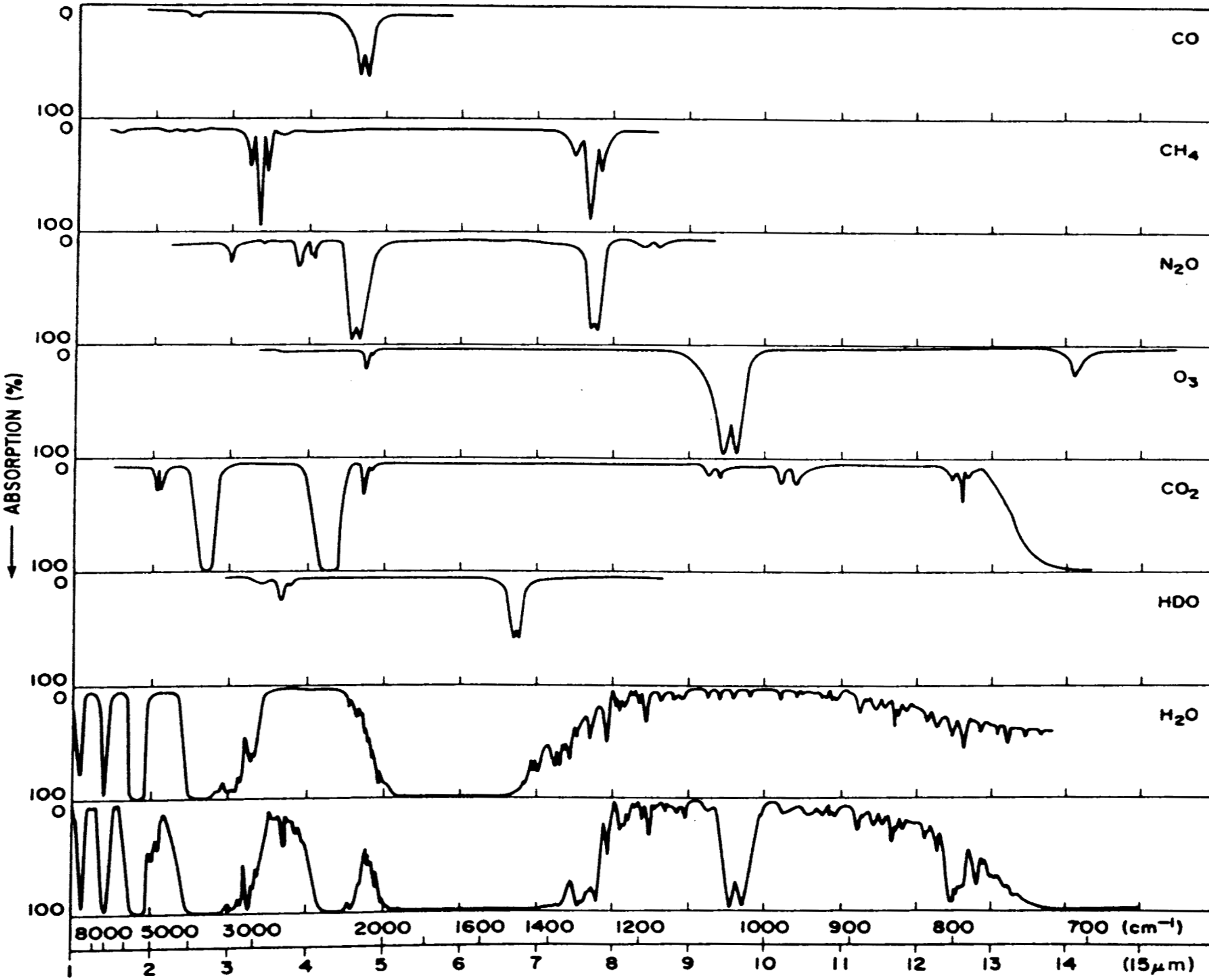
$$\tau = \int_{u_1}^{u_2} k_{a,\nu} du$$

Absorbing gas (path length u)	Absorption coefficient	Line intensity (S)
cm	cm^{-1}	cm^{-2}
$g\ cm^{-2}$	$cm^2\ g^{-1}$	$cm\ g^{-1}$
molecule cm^{-2}	cm^2/molecule	cm/molecule
cm atm	$(cm\ atm)^{-1}$	$cm^{-2}\ atm^{-1}$

Monochromatic transmittance and absorbance

$$T_\nu = \exp(-\tau_\nu)$$

$$A_\nu = 1 - T_\nu = 1 - \exp(-\tau_\nu)$$



CO
CH₄
N₂O
O₃
CO₂
HDO
H₂O
ABSORPTION (%)
0
100
8000
5000
3000
2000
1600
1400
1200
1000
900
800
700 (cm⁻¹)
1
2
3
4
5
6
7
8
9
10
11
12
13
14
(15 μm)

Spectral intensity = intensity averaged over a very narrow interval that B_ν is almost constant but the interval is large enough to consist of several absorption lines.

Narrow-band intensity= intensity averaged over a narrow band which includes a lot of lines;

Broad-band intensity= intensity averaged over a broad band (e.g., over a whole longwave region)

$$T_{\bar{\nu}}(u) = \frac{1}{\Delta\nu}\int_{\Delta\nu} T_\nu(\tau)d\nu = \frac{1}{\Delta\nu}\int_{\Delta\nu} \exp(-\tau_\nu)d\nu = \frac{1}{\Delta\nu}\int_{\Delta\nu} \exp(-k_\nu u)d\nu$$

$$A_{\bar{\nu}} = 1 - T_{\bar{\nu}}(u) = \frac{1}{\Delta\nu}\int_{\Delta\nu} (1-\exp(-\tau_\nu))d\nu = \frac{1}{\Delta\nu}\int_{\Delta\nu} (1-\exp(-k_\nu u))d\nu$$

The solutions of the radiative transfer equation for the **monochromatic upward and downward intensities** in the IR for a plane-parallel atmosphere consisting of absorbing gases (no scattering):

$$I_\nu^{\uparrow}(\tau;\mu) = I_\nu^{\uparrow}(\tau_*;\mu)\exp(-\frac{\tau_* - \tau}{\mu}) + \frac{1}{\mu}\int_\tau^{\tau_*}\exp(-\frac{\tau' - \tau}{\mu})\ B_\nu(T(\tau'))d\tau'$$

$$I_\nu^{\uparrow}(\tau;\mu) = B_\nu(\tau^*)\exp(-\frac{\tau^* - \tau}{\mu}) + \frac{1}{\mu}\int_\tau^{\tau^*}\exp(-\frac{\tau' - \tau}{\mu})\ B_\nu(\tau')d\tau'$$

$$I_\nu^{\downarrow}(\tau;-\mu) = I_\nu^{\downarrow}(0;-\mu)\exp(-\frac{\tau}{\mu}) + \frac{1}{\mu}\int_0^{\tau}\exp(-\frac{\tau - \tau'}{\mu})\ B_\nu(T(\tau'))d\tau'$$

$$I_\nu^{\downarrow}(\tau;-\mu) = \frac{1}{\mu}\int_0^{\tau}\exp(-\frac{\tau - \tau'}{\mu})\ B_\nu(\tau')d\tau'$$

For isothermal atmosphere and black body surface

$$I_\nu^{\uparrow}(0;\mu) = B_\nu(\tau^*)\exp(-\frac{\tau^*}{\mu}) + B_\nu(T_{eff})[1-\exp(-\frac{\tau^*}{\mu})]$$

For fluxes – see 4.6.2 pp.154-157

Monochromatic **net flux** (net power per area) at a given height

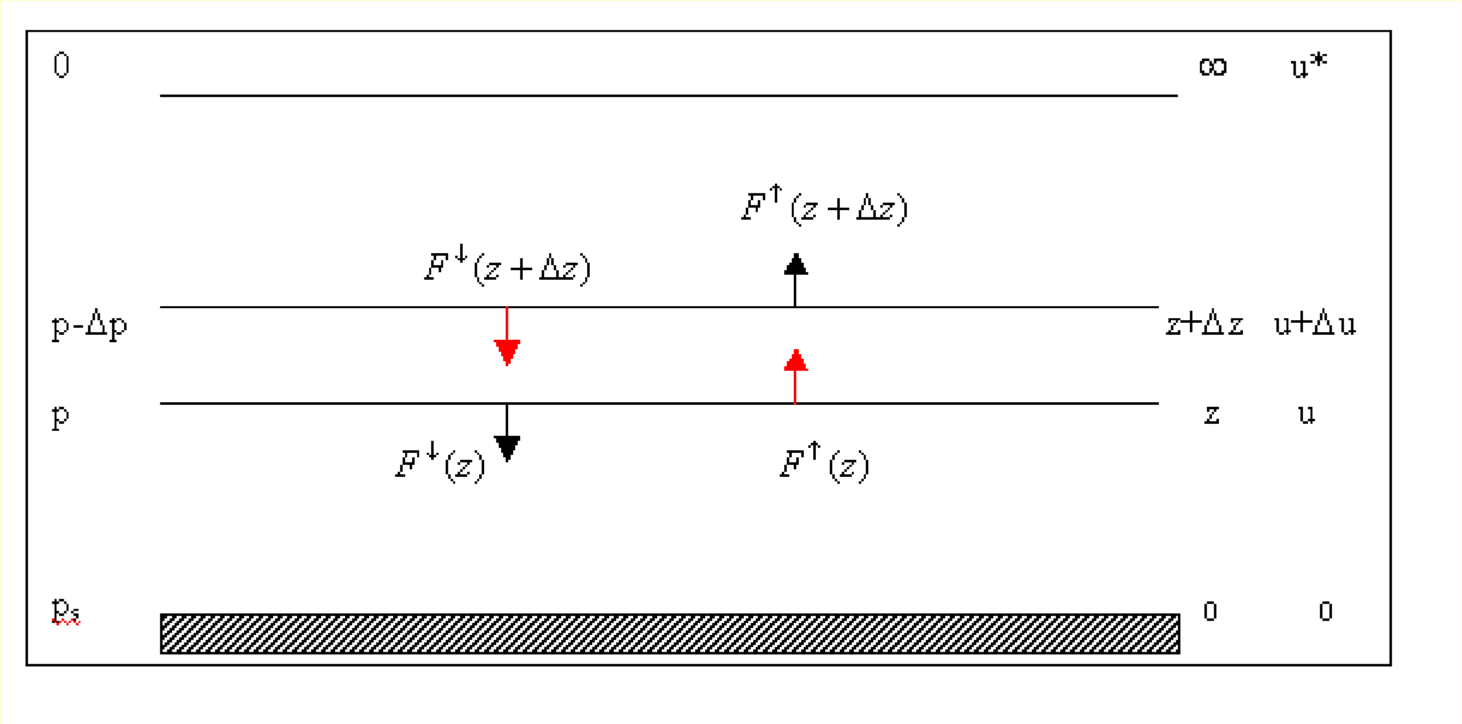
$$F_\nu(z) = F_\nu^\uparrow(z) - F_\nu^\downarrow(z)$$

and total net flux

$$F(z) = F^\uparrow(z) - F^\downarrow(z)$$

Introducing the net flux ***F*(z+Δz)** at the level **z+Δz**, the **net flux divergence** for the layer **Δz** is

$$\Delta F = F(z+\Delta z) - F(z)$$



F(z+Δz) < F(z) (hence ΔF < 0) => a layer gains radiative energy => heating

F(z+Δz) > F(z) (hence ΔF > 0) => a layer losses radiative energy => cooling

The IR **radiative heating (or cooling) rate** is defined as the rate of temperature change of the layer ***dz*** due to IR radiative energy gain (or loss):

$$\left(\frac{dT}{dt}\right)_{IR} = -\frac{1}{c_p \rho}\frac{dF_{net}}{dz} = \frac{g}{c_p}\frac{dF_{net}}{dp}$$

where $\boldsymbol{c_p}$ is the specific heat at the constant pressure

($\boldsymbol{c_p}$ = 1004.67 J/kg/K) and $\boldsymbol{\rho}$ is the air density in a given layer.

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EXAMPLE Calculate longwave cooling at night for an atmospheric layer from 0 to 1 km using the upwelling and downwelling fluxes calculated with MODTRAN for US Standard Atmosphere 1976.

Altitude (km)	IR Upwelling flux (W/m^2)	IR Downwelling flux (W/m^2)
0	390	285
1	375	250

SOLUTION:

Need to find net fluxes at each altitude

$$F(z) = F^{\uparrow}(z) - F^{\downarrow}(z)$$

At 0 km: $F_{net} = 390 - 285 = 105$ W/m^2

At 1 km: $F_{net} = 375 - 250 = 125$ W/m^2

Thus $\Delta F = 20$ W/m^2

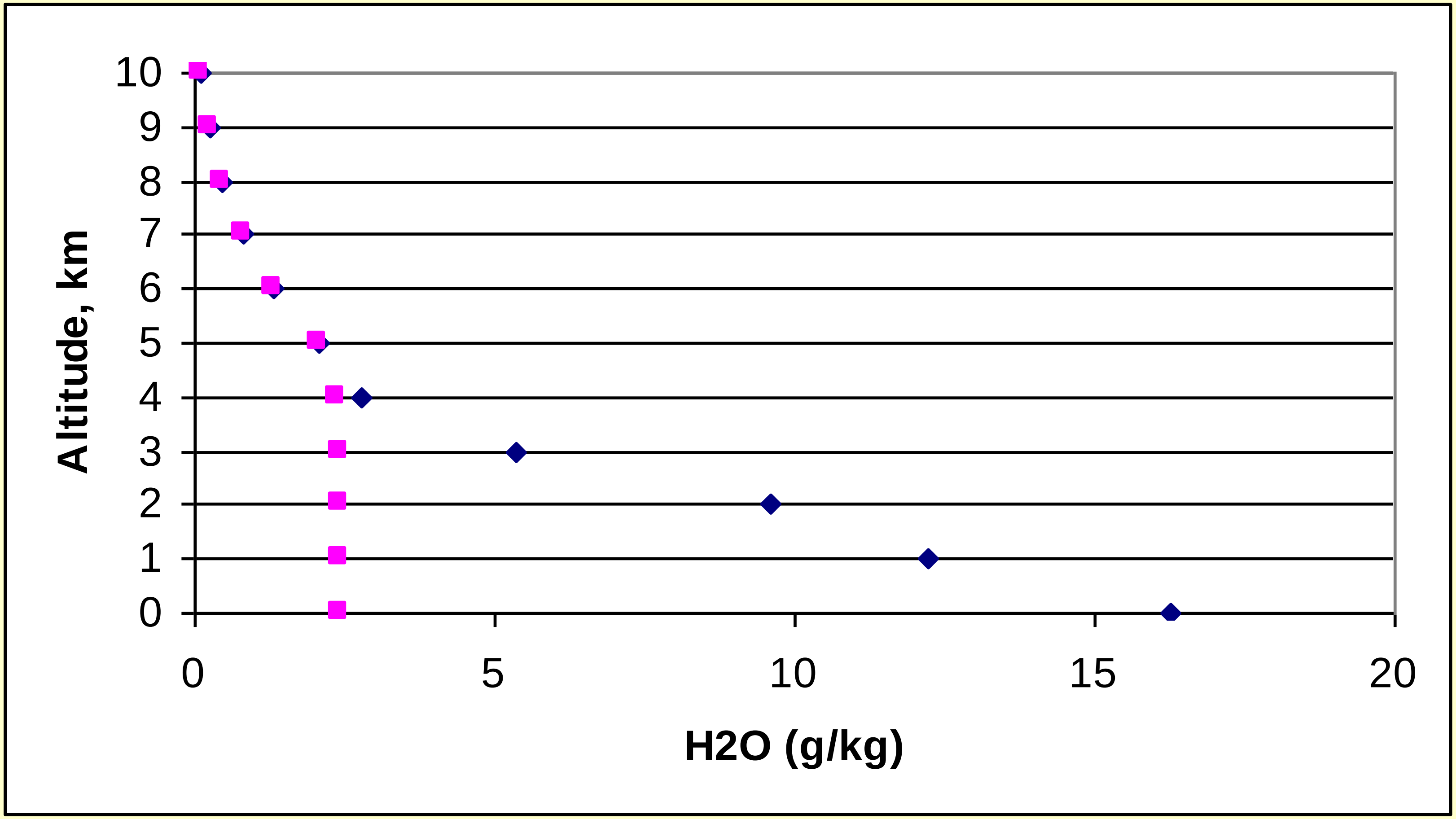
$$\left(\frac{dT}{dt}\right)_{IR} = -\frac{1}{c_p \rho}\frac{dF_{net}}{dz} = \frac{-20\ Js^{-1}m^{-2}}{(1.17\ kg/m^3)(1004\ Jkg^{-1}K^{-1})(1000\ m)}$$

dT/dt = -1.7x10^{-5} K/s = -1.5 K/day

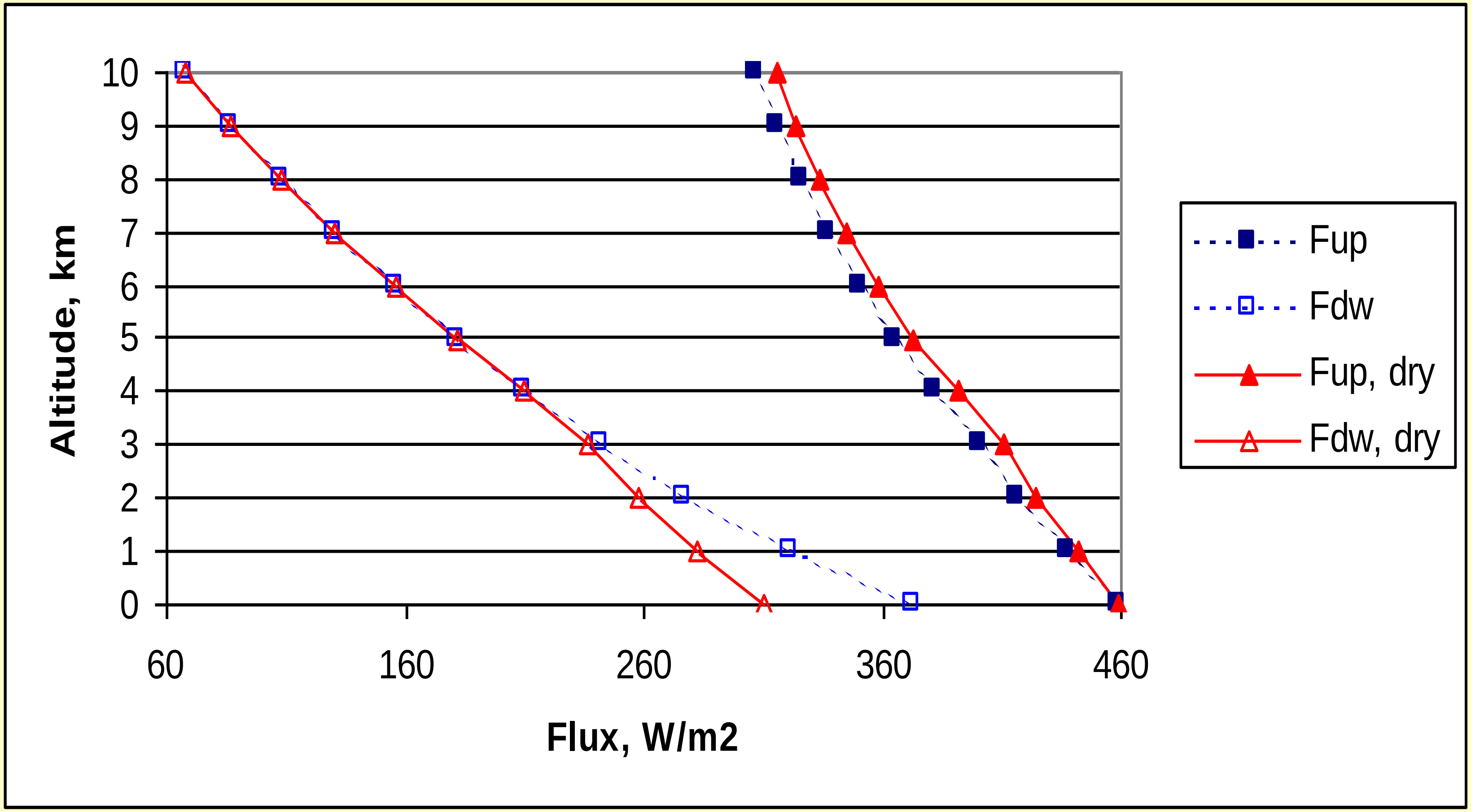
<u>Effect of the varying amount of a gas on IR radiation under the same atmospheric condition</u>

Consider the standard tropical atmosphere and “dry” tropical atmosphere:

same atmospheric characteristics, except the amount of H_2O

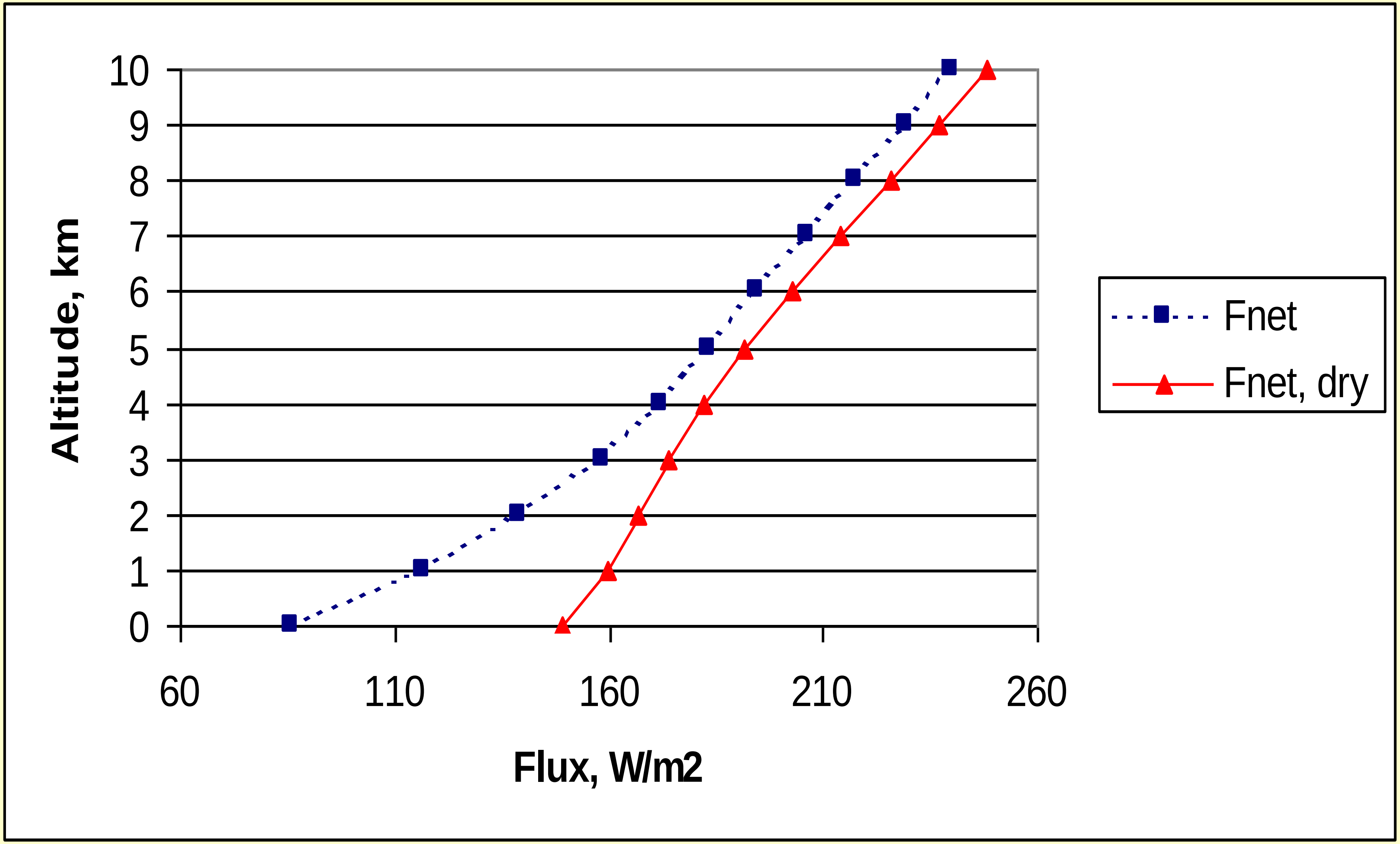


IR fluxes for tropical (dotted lines) and dry tropical atmospheres (solid lines)

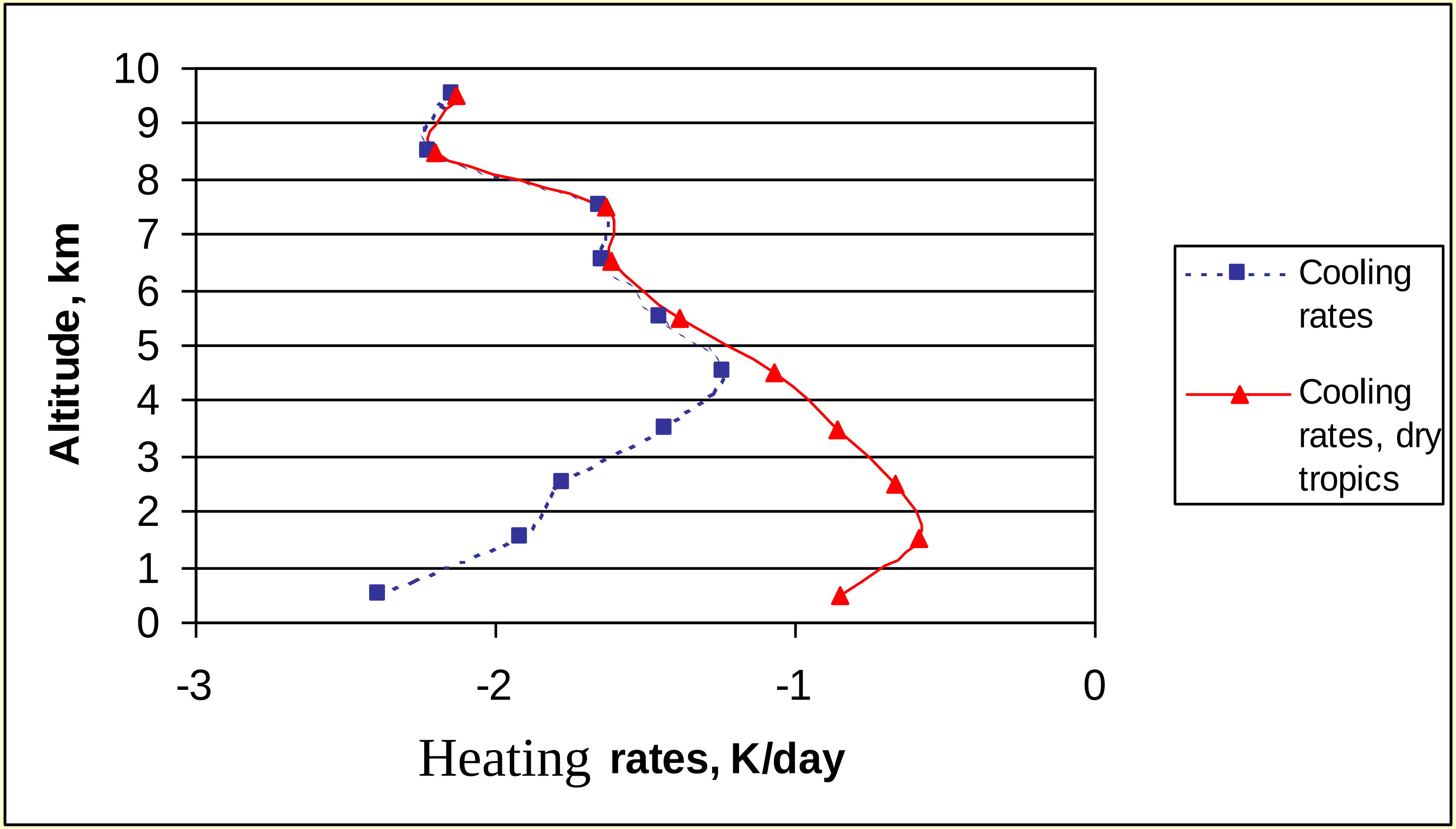


- H_2O increases in a layer => $F^{\downarrow}$ increases because more IR radiation emitted in a layer => $F^{\downarrow}(surface)$ increases
- H_2O increases in a layer => $F^{\uparrow}$ decreases because more IR radiation absorbed but reemitted at the lower temperature => $F^{\uparrow}(TOA)$ decreases
- Increase of an IR absorbing gas contributes to the greenhouse effect.

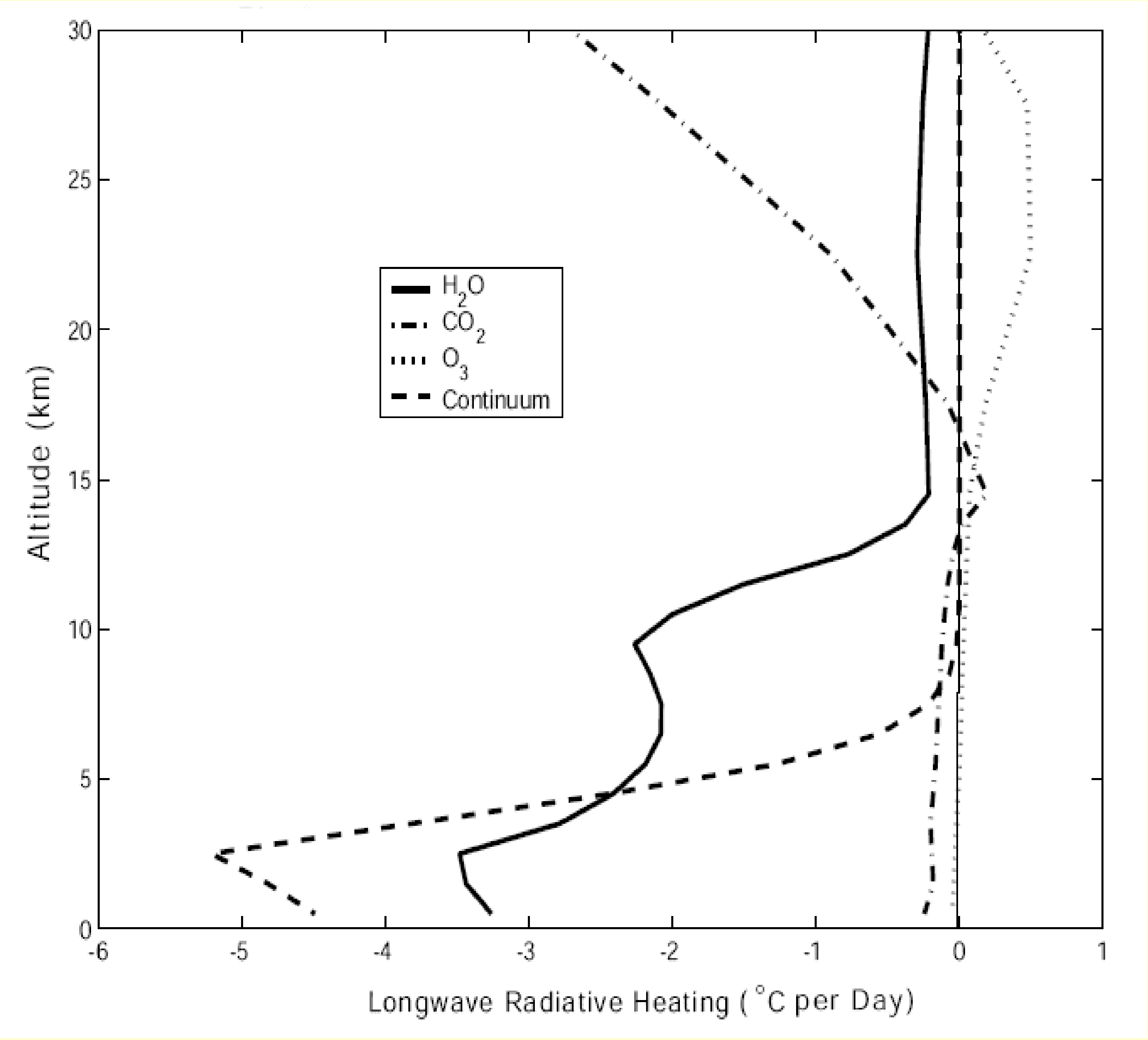
IR <u>net</u> fluxes for tropical (dotted lines) and dry tropical atmospheres (solid lines)



IR cooling rates for tropical (dotted lines) and dry tropical atmospheres (solid lines)



IR cooling rates of individual gases:



IR cooling rates in different cloud-free atmospheres: