Lecture 16.

Light scattering and absorption by atmospheric particulates.

Scattering and absorption by spherical particles.

Virtual machines: https://mycloud.gatech.edu/

- ***Scattering domains:***

Rayleigh scattering: 2πr/λ <<1 and *m* is arbitrary (applies to scattering by molecules and small aerosol particles);

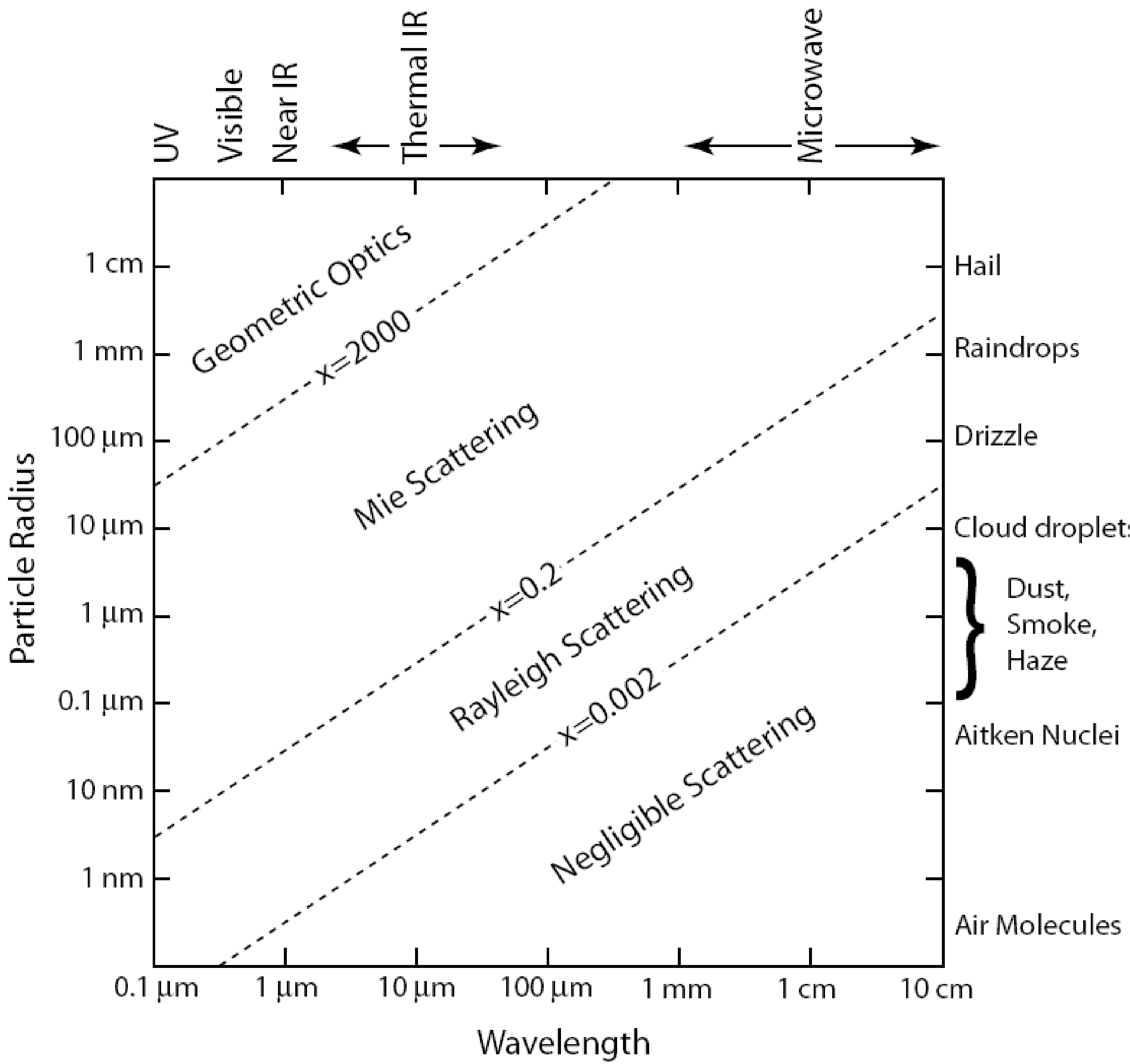
Rayleigh-Gans scattering: $\frac{2\pi r}{\lambda}|m-1| << 1$ and $|m-1| << 1$

(not useful for atmospheric application);

Mie-Debye scattering: 2πr/λ and *m* are both arbitrary but for spheres only (applies to scattering by aerosol and cloud particles)

Geometric optics: 2πr/λ >>1 and *m* is real (applies to scattering by large cloud droplets).

The size parameter x = 2πr/λ is a key factor determining how a particle interacts with EM radiation



UV
Visible
Near IR
Thermal IR
Microwave
Particle Radius
1 cm
1 mm
100 µm
10 µm
1 µm
0.1 µm
10 nm
1 nm
Geometric Optics
x=2000
Mie Scattering
x=0.2
Rayleigh Scattering
x=0.002
Negligible Scattering
Hail
Raindrops
Drizzle
Cloud droplets
Dust,
Smoke,
Haze
Aitken Nuclei
Air Molecules
0.1 µm
1 µm
10 µm
100 µm
1 mm
1 cm
10 cm
Wavelength

Mie theory outline:

Assumptions:

i) Particle is a **sphere** of radius ***r***

ii) Particle is "**homogeneous**" (i.e., one value of the **refractive index** $m=m_r - im_i$ at a given wavelength or **effective refractive index** if the particle is composed of more than one species)

In the far-field zone (i.e., at the large distances **R** from a sphere), the solution of the vector wave equation can be obtained as

$$\begin{bmatrix} \boldsymbol{E}_l^s \\ \boldsymbol{E}_r^s \end{bmatrix} = \frac{\exp(-\boldsymbol{ikR} + \boldsymbol{ikz})}{\boldsymbol{ikR}} \begin{bmatrix} \boldsymbol{S}_2 & 0 \\ 0 & \boldsymbol{S}_1 \end{bmatrix} \begin{bmatrix} \boldsymbol{E}_l^i \\ \boldsymbol{E}_r^i \end{bmatrix}$$

Mie scattering amplitudes:

$$S_1(\Theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left[a_n \pi_n(\cos\Theta) + b_n \tau_n(\cos\Theta) \right]$$

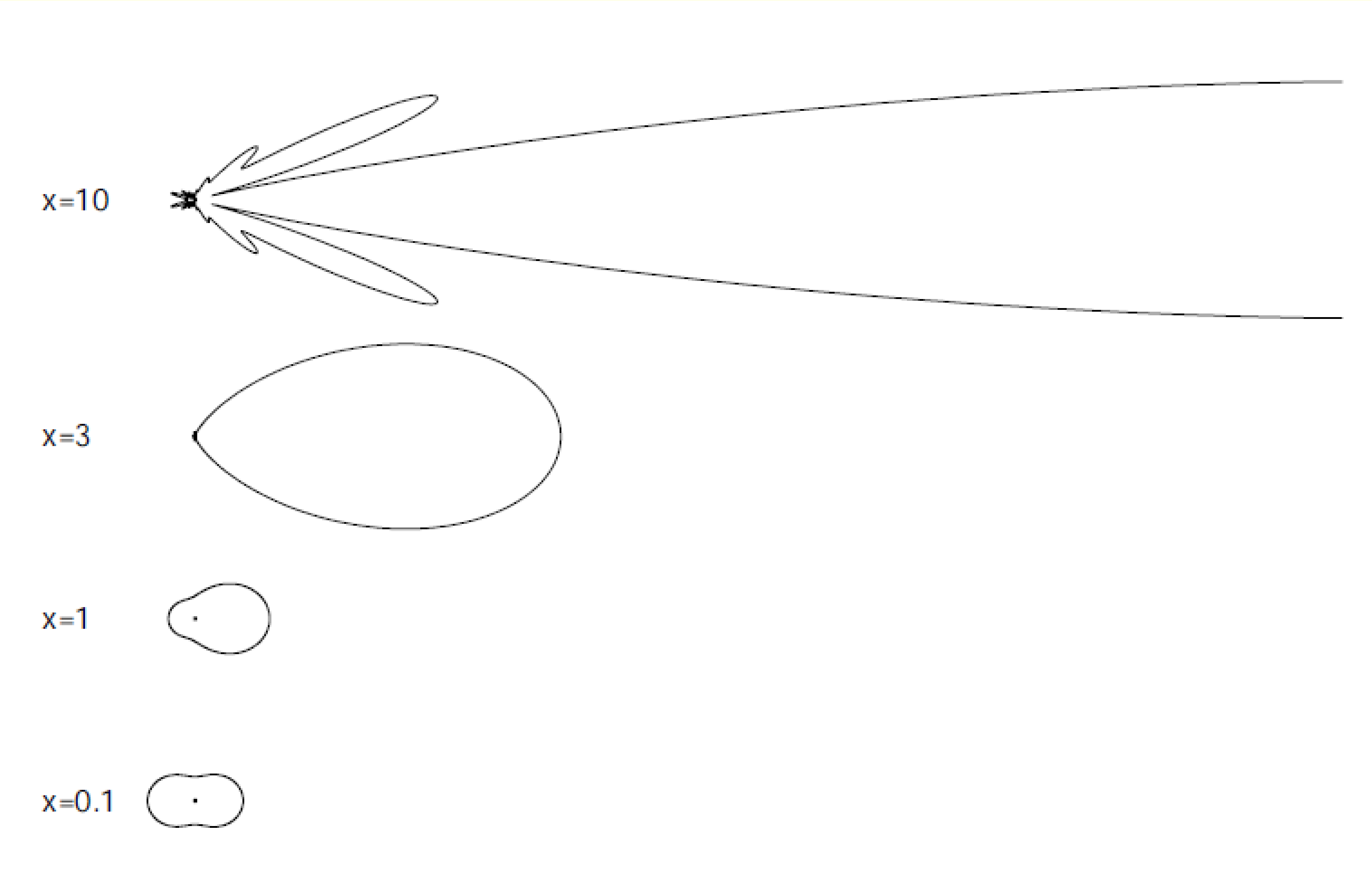
$$S_2(\Theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left[b_n \pi_n(\cos\Theta) + a_n \tau_n(\cos\Theta) \right]$$

Mie coefficients:

$$a_n(x,y) = \frac{\psi_n'(y)\psi_n(x) - m\psi_n(y)\psi_n'(x)}{\psi_n'(y)\xi_n(x) - m\psi_n(y)\xi_n'(x)} \qquad b_n(x,y) = \frac{m\psi_n'(y)\psi_n(x) - \psi_n(y)\psi_n'(x)}{m\psi_n'(y)\xi_n(x) - \psi_n(y)\xi_n'(x)}$$

Mie angular functions:

$$\pi_n(\cos\Theta) = \frac{1}{\sin(\Theta)} P_n^1(\cos\Theta) \qquad \tau_n(\cos\Theta) = \frac{d}{d\Theta} P_n^1(\cos\Theta)$$



Polar plot of the Mie-derived scattering phase function $p(\Theta)$ for selected values of x

Fundamental **extinction formula (or optical theorem)**:

extinction cross section is related to scattering in forward direction

$$\sigma_e = \frac{4\pi}{k^2}\mathrm{Re}[S_{1,2}(0^0)]$$

$$S_1(0^0) = S_2(0^0) = \frac{1}{2}\sum_{n=1}^{\infty}(2n+1)(a_n + b_n)$$

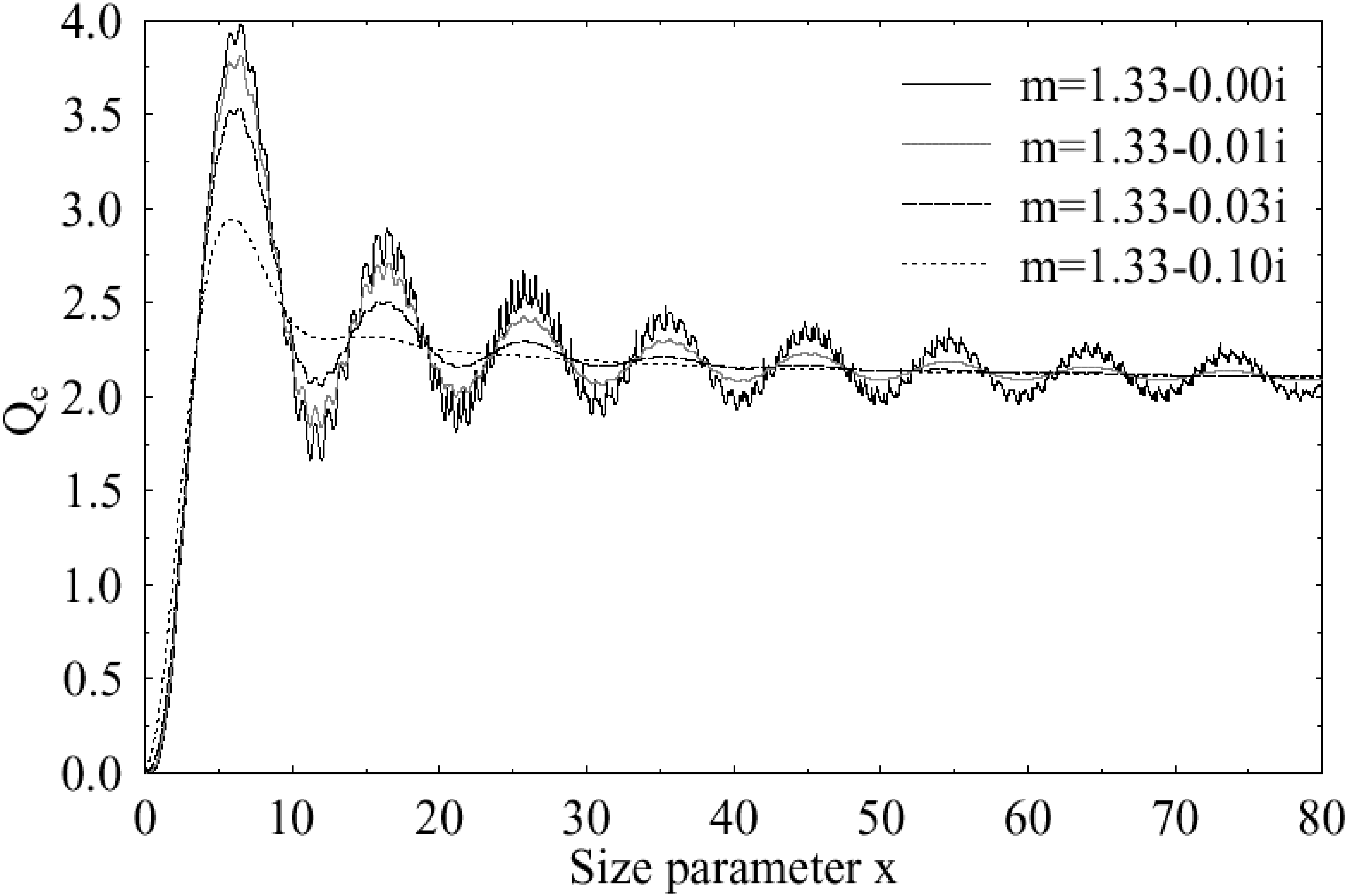
Efficiencies (or efficiency factors) for extinction, scattering and absorption are defined as

$$Q_e = \frac{\sigma_e}{\pi r^2} \qquad Q_s = \frac{\sigma_s}{\pi r^2} \qquad Q_a = \frac{\sigma_a}{\pi r^2}$$

$$Q_e = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1) \operatorname{Re}[a_n + b_n]$$

$$Q_s = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1) [|a_n|^2 + |b_n|^2]$$

$$Q_a = Q_e - Q_s$$



4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Q_e
m=1.33-0.00i
m=1.33-0.01i
m=1.33-0.03i
m=1.33-0.10i
0
10
20
30
40
50
60
70
80
Size parameter x

Stokes Vector consists of four parameters **(called Stokes parameters)**:

intensity I, **the degree of polarization** Q,

the plane of polarization U,**the ellipticity** V.

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

Unpolarized light: $Q = U = V = 0$

Fully polarized light:

$$I^2 = Q^2 + U^2 + V^2$$

Linear polarized light: $V = 0$

Circular polarized light: $|V| = I$

$$I = E_l E_l^* + E_r E_r^*$$

$$Q = E_l E_l^* - E_r E_r^*$$

$$U = E_l E_r^* + E_r E_l^*$$

$$V = -i(E_l E_r^* - E_r E_l^*)$$

For unpolarized light:

$$Q = U = V = 0$$

The degree of polarization may be defined as:

$$DP = (Q^2 + U^2 + V^2)^{1/2} / I$$

The degree of linear polarization may be defined as:

$$LP = -\frac{Q}{I} = -\frac{P_{12}}{P_{11}}$$

➢ Scattering phase matrix

$\boldsymbol{I_0}$, $\boldsymbol{Q_0}$, $\boldsymbol{U_0}$ and $\boldsymbol{V_0}$ are the Stokes parameters of incident field and I, Q, U and V the Stokes parameters of scattered radiation

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \frac{\sigma_s}{4\pi R^2} P \begin{bmatrix} I_o \\ Q_o \\ U_0 \\ V_o \end{bmatrix}$$

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \frac{\sigma_s}{4\pi R^2} \begin{bmatrix} P_{11} & P_{12} & 0 & 0 \\ P_{12} & P_{11} & 0 & 0 \\ 0 & 0 & P_{33} & -P_{34} \\ 0 & 0 & P_{34} & P_{33} \end{bmatrix} \begin{bmatrix} I_o \\ Q_o \\ U_0 \\ V_o \end{bmatrix}$$

$$P_{11} = \frac{4\pi}{2k^2\sigma_s}\left[S_1 S_1^* + S_2 S_2^*\right]$$

➢ **Scattering phase function**

$$P(\theta,\varphi) = \frac{\left|S_j(\theta)\right|^2}{\pi x^2 Q_{sca}}$$

x – size parameter

Q_{sca}- scattering efficiency

$$\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} P(\theta,\varphi) sin\theta d\theta d\varphi = 1$$