Lecture 20.

Methods for solving the radiative transfer equation with multiple scattering. Part 2: Inclusion of surface reflection and emissivity. Exact methods: Discrete-ordinate, Adding-doubling, and Monte Carlo.

1. Inclusion of the surface reflection into the radiative transfer equation.
2. Inclusion of the surface emissivity into the radiative transfer equation.
3. Discrete-ordinate method for homogeneous and inhomogeneous atmosphere.
4. Principles of invariance and Adding-doubling method.
5. Monte Carlo method.

Required reading:

L02: 6.3.5, 6.2, 6.3.1- 6.3.4, 6.4, 6.7

Ricchiazzi, P., S. R. Yang, et al. SBDART: A research and teaching software tool for Plane-parallel radiative transfer in the earth's atmosphere. Bulletin of the American Meteorological Society 79, 2101-2114, 1998.

Advanced reading:

Thomas G.E. and K. Stamnes, Radiative transfer in the atmosphere and ocean, 2000, Chapter 8.1-8.10 .

Liu Q. and F. Weng, Advanced Doubling–Adding Method for Radiative Transfer in Planetary Atmospheres. Journal of Atmospheric Sciences 63, 3459–3465, 2006.

Stamnes, K., S. Tsay, W. Wiscombe and K. Jayaweera, Numerically stable algorithm for discrete-ordinate-method radiative transfer in multiple scattering and emitting layered media." Appl. Opt., 27, 2502-2509, 1998.

The International Intercomparison of 3D Radiation Codes (I3RC): http://i3rc.gsfc.nasa.gov/

Cahalan, R. F., et al., 2005: The International Intercomparison of 3D Radiation Codes (I3RC): Bringing together the most advanced radiative transfer tools for cloudy atmospheres. Bull. Amer. Meteor. Soc. **86** (9), 1275-1293.

Davis, A. B., and A. Marshak, 2010: Solar radiation transport in the cloudy atmosphere: a 3D perspective on observations and climate impacts. Reports on progress in Physics, 73, 026801, doi:10.1088, http://i3rc.gsfc.nasa.gov/Publications/Davis_Marshak_2010.pdf

1. Inclusion of the surface reflection into the radiative transfer equation.

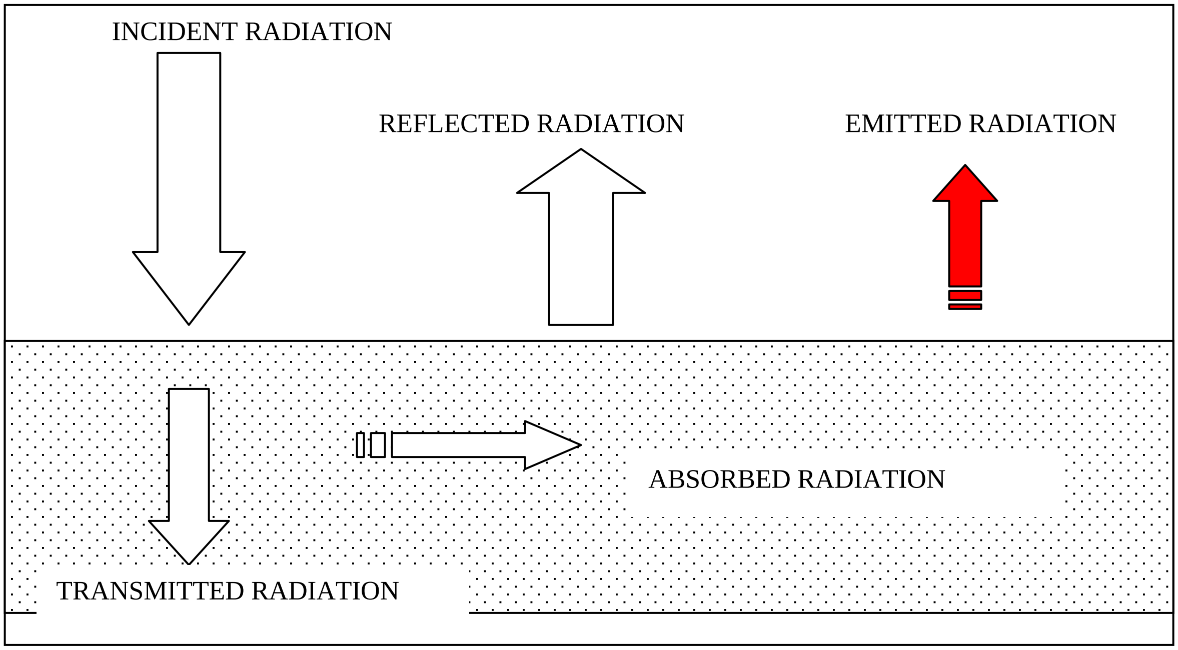
The ocean and land surfaces can modify the atmospheric radiation field by

a) reflecting a portion of the incident radiation back into the atmosphere;

b) transmitting some incident radiation;

c) absorbing a portion of incident radiation (Kirchhoff's law);

d) emitting the thermal radiation (Kirchhoff's law);



Types of reflection

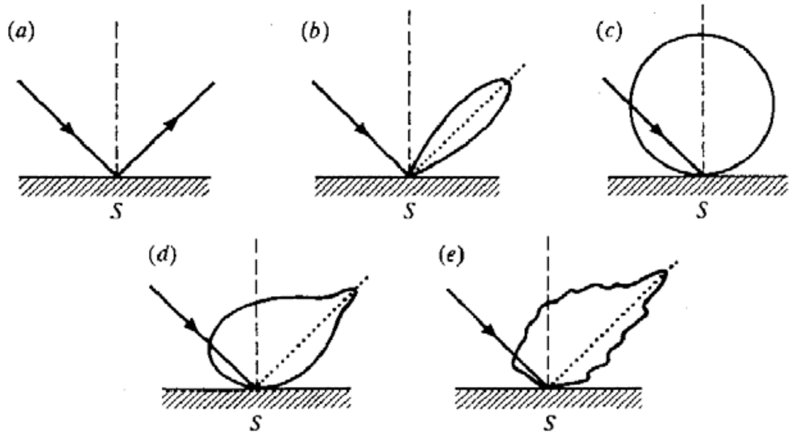


Figure 20.1 Schematic illustration of different types of surface scattering. The lobes are polar diagrams of the scattered radiation: (a) specular, (b) quasi-specular, (c) Lambertian, (d) quasi-Lambertian, (e) complex.

Two extreme types of the surface reflection:

specular reflectance and **diffuse reflectance**.

Specular reflectance is the reflectance from a perfectly smooth surface (e.g., a mirror):

Angle of incidence =Angle of reflectance

- ✓ Reflection is generally **specular** when the "roughness" of the surface is smaller than the wavelength used. In the solar spectrum (0.4 to 2 μm), reflection is therefore specular on smooth surfaces such as polished metal, still water or mirrors.

NOTE: While incoming solar light is unpolarized, reflected waves are generally polarized and Fresnel's laws can be used to determine polarization.

- ➢ Practically all real surfaces are not smooth and the surface reflection depends on the incident angle and the angle of reflection. Reflectance from such surfaces is called the **diffuse reflectance**.

Bi-directional reflectance distribution function (BRDF), $\rho(\mu,\varphi,-\mu',\varphi')$ is introduced to characterize the angular dependence in the surface reflection and defined as the ratio of the reflected intensity to the energy flux in the incident beam:

$$\rho(\mu,\varphi,-\mu',\varphi') = \frac{\pi dI^{\uparrow}(\tau^*,\mu,\varphi)}{I^{\downarrow}(\tau^*,-\mu',\varphi')\mu' d\Omega'} \qquad [20.1]$$

NOTE: Each type of surfaces has a specific spectral **BRDF**.

Reciprocity law: $\rho(\mu,\varphi,-\mu',\varphi') = \rho(-\mu',\varphi',\mu,\varphi)$

Upwelling radiance is an integral over BRDF and downwelling radiance

$$I_r^{\uparrow}(\tau^*,\mu,\varphi) = \frac{1}{\pi}\int_0^{2\pi}\int_0^{1}\rho(\mu,\varphi,-\mu',\varphi')I^{\downarrow}(\tau^*,-\mu',\varphi')\mu' d\mu' d\varphi' \qquad [20.2]$$

Surface albedo is defined as the ratio of the surface upwelling to downwelling flux:

$$r_{sur} = \frac{F^{\uparrow}}{F^{\downarrow}} = \frac{1}{\pi} \frac{\int_0^{2\pi}\int_0^1\int_0^{2\pi}\int_0^1 \rho(\mu,\varphi,-\mu',\varphi')I^{\downarrow}(-\mu',\varphi')\mu' d\mu' d\varphi' \mu d\mu d\varphi}{\int_0^{2\pi}\int_0^1 I^{\downarrow}(-\mu',\varphi')\mu' d\mu' d\varphi'} \qquad [20.3]$$

NOTE: Eq.[20.3] is similar to Eq.[20.4] , except that the latter is written for the collimated (direct) incident field.

- A surface is called the **Lambert surface** if it obeys **the Lambert's Law.**

Lambert's Law of diffuse reflection: the diffusely reflected light is isotropic and unpolarized independently of the state of polarization and the angle of the incidence light. **For the Lambert surface,** BRDF is independent on the directions of incident and observed light beam.

$$\rho(\mu,\varphi,-\mu',\varphi') = \rho_L \qquad [20.3]$$

For the Lambert surface, from Eq.[20.2], we have

$$I_r^{\uparrow}(\tau^*,\mu,\varphi) = \frac{\rho_L}{\pi} F^{\downarrow} \qquad [20.4]$$

and from Eq.[20.3], we have $r_{sur} = \rho_L$ [20.5]

- ***<u>Reflectance from ocean surfaces</u>***

Ocean reflection depends on the ocean surface (and near surface) conditions: waves, whitecaps and suspended particulates. Many models have been developed to account for these factors by introducing corrections to the Fresnel reflection. One of the most widely used models is the Cox and Munk (1954) model.

- ***<u>Reflectance from land surfaces (soils, vegetation):</u>***

BRDFs are controlled by the physical structure of the surface (*e.g.*, the density and three-dimensional arrangement of plant leaves and stems, and the surface roughness of the soil substrate) and the optical properties of its component elements (*e.g.*, the spectral reflectance and transmittance of leaves, stems and soil facets). Numerous models have been developed to describe and account for these relationships. These models are generally formulated as follows

$$\rho_{\lambda} = f_{\lambda,iso} + f_{\lambda,geo}k_{geo} + f_{\lambda,vol}k_{vol}$$

where k_{geo} and k_{vol} are the model `kernels', and $f_{\lambda\ iso}$, $f_{\lambda\ geo}$ and $f_{\lambda\ vol}$ are spectrally-dependent weighting factors. The `kernels' are trigonometric functions that describe the shape of the BRDF in terms of the solar illumination and sensor view angles. They are derived from approximations to, and simplifications of, the principles of geometrical-optics (k_{geo}) and radiative-transfer theory (k_{vol}). Each kernel is multiplied by a factor, f_{λ}, that weights the relative contribution of surface-scattering ($f_{\lambda\ geo}$) and volume-scattering ($f_{\lambda\ vol}$) to the measured BRDF. The term $f_{\lambda\ iso}$ is included to account for isotropic scattering from the surface (in practice, this includes contributions from both single-scattering and multiple-scattering).

Thus, the model is formulated so that bidirectional reflectance is a linear combination of three terms weighted by three parameters: an **isotropic** function accounting for bidirectional reflectance with nadir viewing and the overhead sun; a **geometric** function accounting for the effects of shadows and the geometrical structure of protrusions from a background surface; and a **volume scattering** function based on radiative transfer accounting for reflectance by a collection of randomly-dispersed facets.

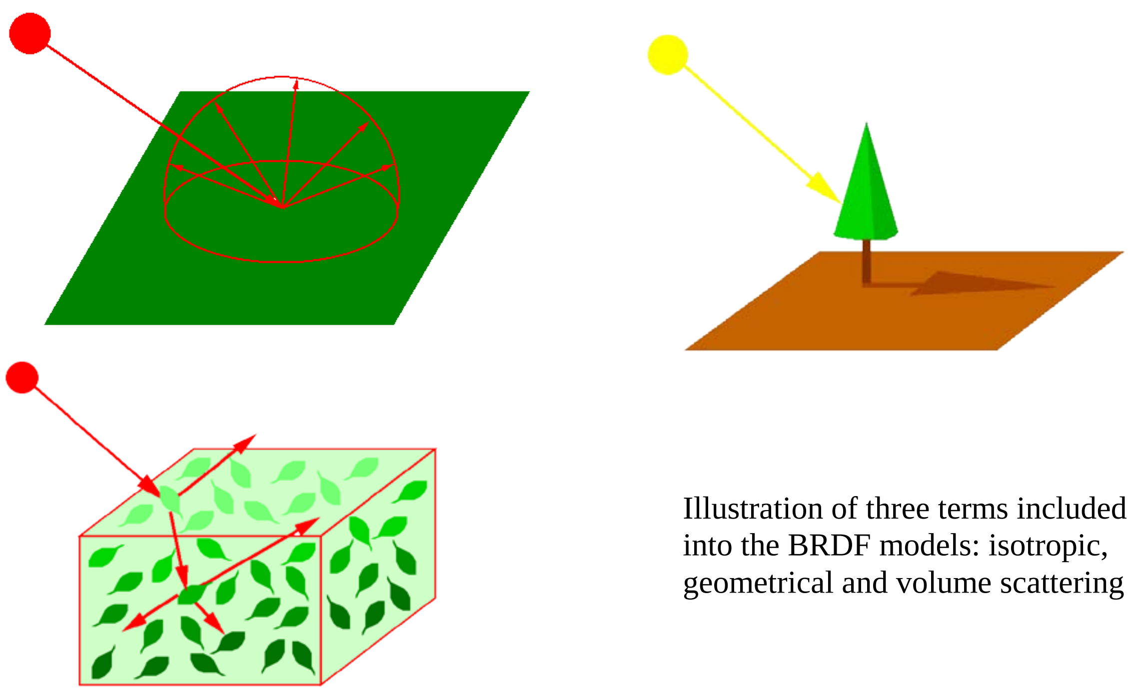
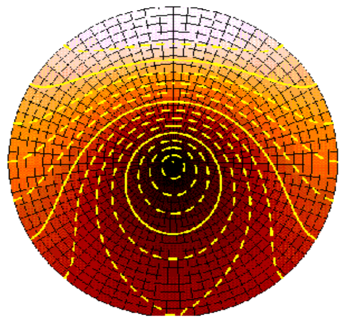


Illustration of three terms included into the BRDF models: isotropic, geometrical and volume scattering

- ✓ A notable feature of the BRDF for natural surfaces is the **hot spot** - a peak in reflectance for direct backscatter ($\Theta=180^0$).

Hot spot are caused by:

1) the lack of observed shadows and
2) specular reflection from oriented leaves



- ➢ **In general, the surface reflectance is a function of wavelength**.

Examples of the surface albedo at about 550 nm: fresh snow/ice =0.8-0.9; desert=0.3, soils=0.1-0.25; ocean=0.05.

Examples of the spectral surface reflectance.

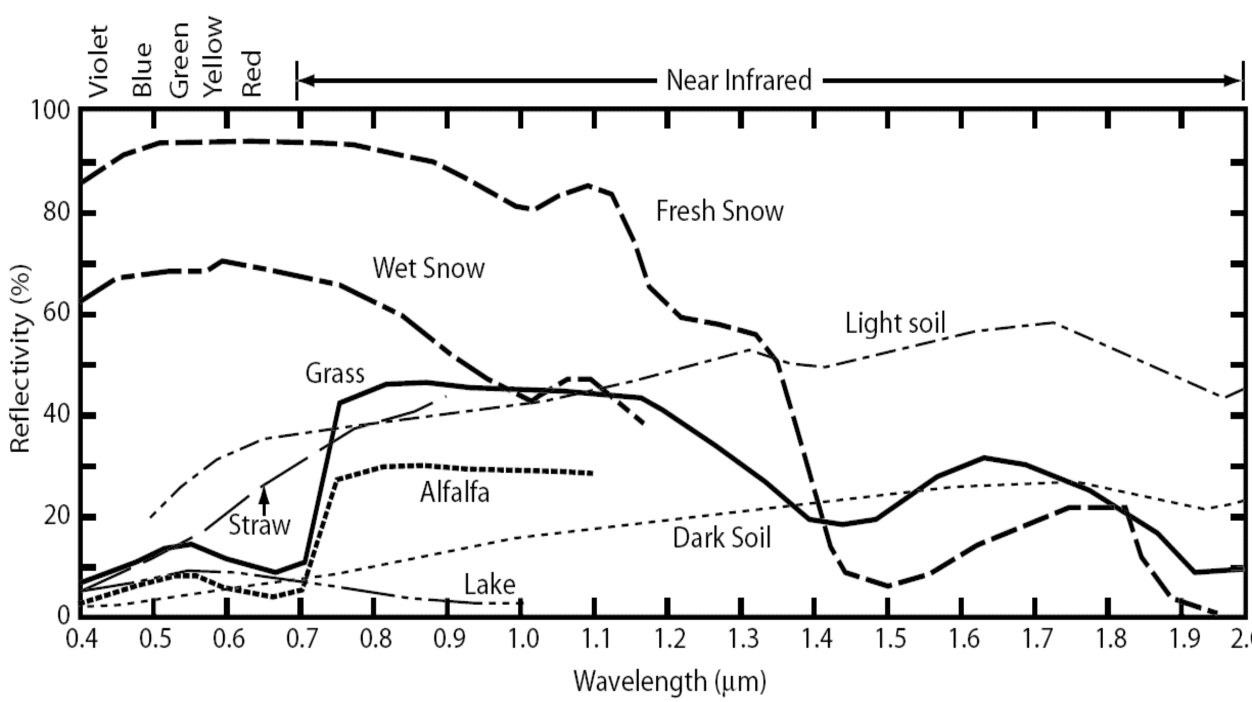


Figure 20.2 Typical shortwave spectral reflectances of various natural surfaces.

Inclusion of the surface reflection into the radiative transfer equation:

Let's include the contribution from the Lambert surface.

Lambert surface: $I^{\uparrow}(\tau^*, \mu, \varphi) = I_{sur} = const$ [20.6]

Generalizing the definitions for the reflection and transmission functions (i.e., Eqs.[20.1]—[20.2]), we may express the reflected diffuse intensity $I_r^{\uparrow}(0, \mu, \varphi)$ and transmitted diffuse intensity $I_t^{\downarrow}(\tau^*, -\mu, \varphi)$ as

$$I_r^{\uparrow}(0, \mu, \varphi) = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 R(\mu, \varphi, \mu', \varphi') I_{inc}(-\mu', \varphi')\mu' d\mu' d\varphi' \qquad [20.7]$$

$$I_t^{\downarrow}(\tau^*, -\mu, \varphi) = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 T(\mu, \varphi, \mu', \varphi') I_{inc}(-\mu', \varphi')\mu' d\mu' d\varphi' \qquad [20.8]$$

The reflected intensity at the top of the layer including the surface reflection may be written as

$$I^*(0, \mu, \varphi) = I^{\uparrow}(0, \mu, \varphi) + \frac{1}{\pi}\int_0^{2\pi}\int_0^1 T(\mu, \varphi, \mu', \varphi') I_{sur}\mu' d\mu' d\varphi' + I_{sur}\exp(-\tau^*/\mu) \qquad [20.9]$$

NOTE: The second term on the right-hand side gives the contribution from the surface reflected intensity which is diffusely transmitted to the top of the layer, whereas the third term gives the contribution from the surface reflected intensity which is the directly transmitted.

We can re-write Eq.[20.9] as

$$I^*(0, \mu, \varphi) = \mu_0 F_0 R(\mu, \varphi, \mu_0, \varphi_0) + I_{sur}\gamma(\mu) \qquad [20.10]$$

where $\gamma(\mu) = \exp(-\tau^*/\mu) + t(\mu)$

Now, let's consider the diffuse transmitted intensity. Isotropic intensity $\boldsymbol{I_{sur}}$, propagating upward in the layer after being scattered by the Lambertion surface, can be partially reflected back to the surface and, hence, contribute to the downward intensity in the additional amount

$$I^{\downarrow}_{add}(-\mu) = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 R(\mu,\varphi,\mu',\varphi') I_{sur}\mu' d\mu' d\varphi' = I_{sur} r(\mu)$$

Thus, the transmitted intensity including the surface contribution is

$$I^*(\tau^*,-\mu,\varphi) = I^{\downarrow}(\tau^*,-\mu,\varphi) + I_{sur} r(\mu) = \mu_0 F_0 T(\mu,\varphi,\mu_0,\varphi_0) + I_{sur} r(\mu)$$

[20.11]

Both Eqs.[20.10] and [20.11] have $\boldsymbol{I_{sur}}$. Thus, we need to find $\boldsymbol{I_{sur}}$.

$$\pi I_{sur} = (Surface\ albedo)\ x(Downward\ \ flux)$$

The downward flux has three components:

(1) Transmitted direct flux = $\mu_0 F_0 \exp(-\tau^*/\mu_0)$

(2) Transmitted diffuse flux=

$$\int_0^{2\pi}\int_0^1 I^{\downarrow}(\tau^*,-\mu,\varphi)\mu d\mu d\varphi = \int_0^{2\pi}\int_0^1 \frac{\mu_0 F_0}{\pi} T(\mu,\varphi,\mu_0,\varphi_0)\mu d\mu d\varphi = \mu_0 F_0 t(\mu_0)$$

(3) Fraction of $\boldsymbol{I_{sur}}$ reflected by the atmosphere back to the surface =

$$\int_0^{2\pi}\int_0^1 I^{\downarrow}_{add}(-\mu)\mu d\mu d\varphi = \pi I_{sur}\bar{r}$$

Therefore

$$\pi I_{sur} = r_{sur}\left(\mu_o F_0 \exp(-\tau^*/\mu_0) + \mu_0 F_0 t(\mu_0) + \pi I_{sur}\bar{r}\right)$$

and rearranging term, we have

$$I_{sur} = \frac{r_{sur}}{1 - r_{sur}\bar{r}} \frac{\mu_0 F_0}{\pi} \gamma(\mu_0)$$

Therefore, the diffuse reflected and transmitted intensities, accounting for the surface contribution are

$$\boxed{I^*(0,\mu,\varphi) = I^{\uparrow}(0,\mu,\varphi) + \frac{r_{sur}}{1 - r_{sur}\bar{r}} \frac{\mu_0 F_0}{\pi} \gamma(\mu_0)\gamma(\mu)} \quad [20.12a]$$

$$\boxed{I^*(\tau^*,-\mu,\varphi) = I^{\downarrow}(\tau^*,-\mu,\varphi) + \frac{r_{sur}}{1 - r_{sur}\bar{r}} \frac{\mu_0 F_0}{\pi} \gamma(\mu_0) r(\mu)} \quad [20.12b]$$

Integrating Eq.[20.12a, b] over the solid angle, we find diffuse fluxes

$$F^*(0) = F^{\uparrow}(0) + \frac{r_{sur}}{1 - r_{sur}\bar{r}} \mu_0 F_0 \gamma(\mu_0)\bar{\gamma} \qquad [20.13a]$$

$$F^*(\tau^*) = F^{\downarrow}(\tau^*) + \frac{r_{sur}}{1 - r_{sur}\bar{r}} \mu_0 F_0 \gamma(\mu_0)\bar{r} \qquad [20.13a]$$

where $\bar{\gamma} = \bar{t} + 2\int_0^1 \exp(-\tau^* / \mu_0)\mu_0 d\mu_0$

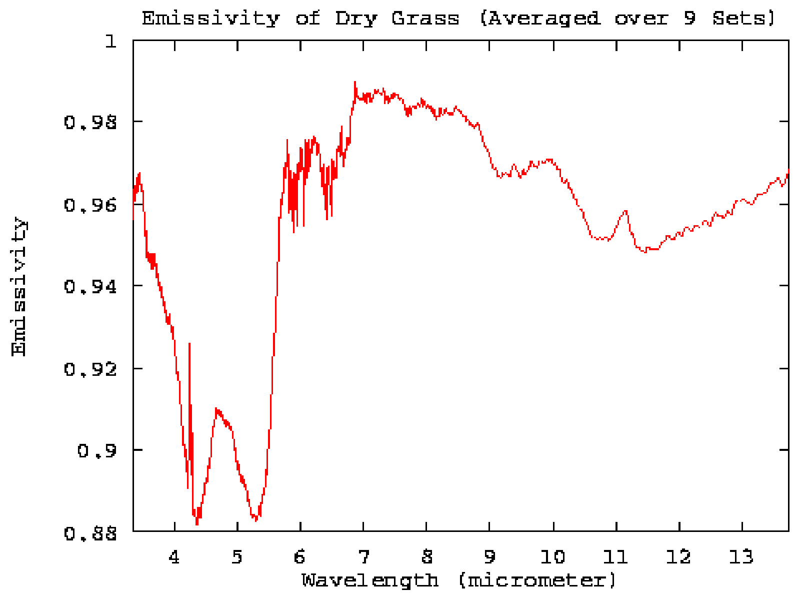
NOTE: $\bar{t}$ and $\bar{r}$ were defined in Lecture 20 (see Eq.[20.8] and [20.9]).

NOTE: For non-Lambert surface, the inclusion of the surface reflection is a complex boundary problem.

2. Inclusion of the surface emissivity into the radiative transfer equation.

In general, emissivity depends on the direction of emission, surface temperature, wavelength and some physical properties of the surface (e.g., the refractive index).

- ✓ In the thermal IR (λ> 4 μm), nearly all surfaces are efficient emitters with the emissivity > 0.8 and their emissivity depends little on the direction (about 1-3% angular variation)



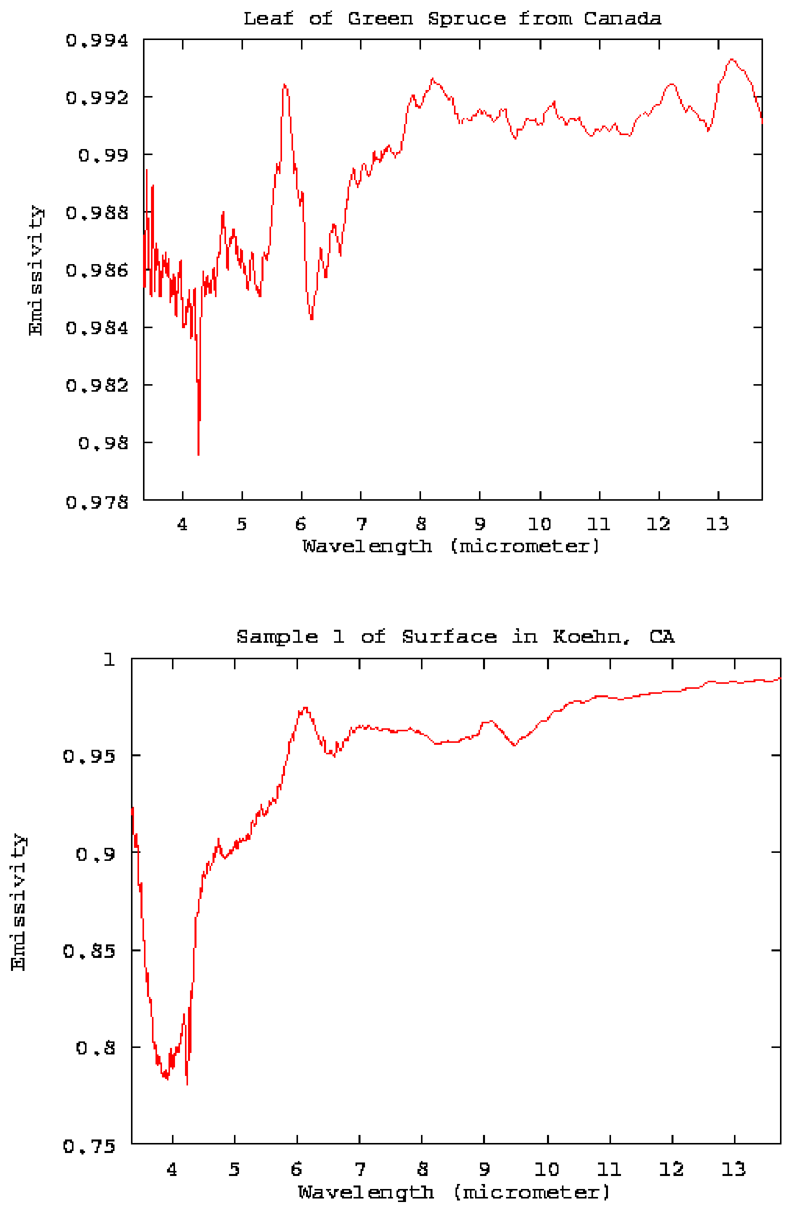


Figure 20.3 Examples of spectral emissivity (MODIS UCSB Emissivity Library)

<u>Inclusion of the surface emissivity into the radiative transfer equation:</u>

Recall the general solution of the upwelling radiance in the thermal IR:

$$I_\nu^{\uparrow}(\tau;\mu) = I_\nu^{\uparrow}(\tau^*;\mu)\exp(-\frac{\tau^*-\tau}{\mu}) + \frac{1}{\mu}\int_{\tau}^{\tau_*}\exp(-\frac{\tau'-\tau}{\mu})\ B_\nu(T(\tau'))d\tau'$$

where $I_\nu^{\uparrow}(\tau^*;\mu)$ is the contribution from the surface.

In general, contribution from the surface = emission + reflection

For *a specular surface*:

$$I_{\nu}^{\uparrow}(\tau^*;\mu) = \varepsilon_{\nu} B(T_s) + (1-\varepsilon_{\nu}) I_{\nu}^{\downarrow}(\tau^*,\mu) \qquad [20.14]$$

where ε_{ν} is the surface emissivity $r_{suf} = 1- \varepsilon_{\nu}$, and $I_{\nu}^{\downarrow}(\tau^*,\mu)$ is the downwelling radiances reaching the surface.

In the IR window, r_{suf} is negligibly small for the land and ocean surfaces => it is common in the radiative transfer modeling to keep only the first term in Eq.[20.14].

Table 20.1 Broadband emissivity of some surfaces in the IR window (10-to12 μm).

Surface	***Emissivity***
Water	**0.993-0.998**
Ice	**0.98**
Green grass	**0.975-0.986**
Sand	**0.949-0.962**
Frozen soil	**0.93**
Concrete	**0.94**
Snow	**0.969-0.997**
Granite	**0.898**

NOTE: There are several databases that have been developed to provide the spectral emission data of natural surfaces. Both surface emissivity and reflectance (BRDF and albedo) can be retrieved from satellite observations (sensors differ in the spatial footprint/coverage and spectral resolution)

Example: *ASTER spectral library* (http://speclib.jpl.nasa.gov/): includes data from three other spectral libraries: the Johns Hopkins University (JHU) Spectral Library the Jet Propulsion Laboratory (JPL) Spectral Library, and the United States Geological Survey (USGS - Reston) Spectral Library. Current Version 2.0 of the ASTER spectral library includes over 2400 spectra of natural and manmade materials.

3. Discrete-ordinate method for homogeneous and inhomogeneous atmosphere.

First, consider a homogeneous atmosphere.

Recall the radiative transfer equation (Lecture 19) for azimuthally independent diffuse intensity:

$$\mu \frac{dI(\tau,\mu)}{d\tau} = I(\tau,\mu) - \frac{\omega_0}{2}\int_{-1}^{1} I(\tau,\mu')P(\mu,\mu')d\mu' - \frac{\omega_0}{4\pi}F_0 P(\mu,-\mu_0)\exp(-\tau/\mu_0)$$

For isotropic scattering, the scattering phase function is 1. Hence we have

$$\mu \frac{dI(\tau,\mu)}{d\tau} = I(\tau,\mu) - \frac{\omega_0}{2}\int_{-1}^{1} I(\tau,\mu')d\mu' - \frac{\omega_0 F_0}{4\pi}\exp(-\tau/\mu_0) \qquad [20.15]$$

Let's apply the Gaussian quadratures to replace the integral in Eq.[20.15]

$$\mu_i \frac{dI(\tau,\mu_i)}{d\tau} = I(\tau,\mu_i) - \frac{\omega_0}{2}\sum_{j=-n}^{n} a_j I(\tau,\mu_j) - \boxed{\frac{\omega_0 F_0}{4\pi}\exp(-\tau/\mu_0)} \qquad [20.16]$$

where $i=-n, \ldots, n$ (2n terms) and $\boldsymbol{a_j}$ are the Gaussian weights (constants) and $\boldsymbol{\mu_j}$ are quadrature angles (or points).

Eq.[20.16] is a system of 2n inhomogeneous differential equations:

Solution of Eq.[20.16] = general solution + particular solution

where the general solution is a solution of the homogeneous part of the Eq.[20.16]

Denoting $\boldsymbol{I_i = I_i(\tau,\mu_i)}$, the general solution of Eq.[20.16] can be found as

$$I_i = g_i \exp(-k\tau) \qquad [20.17]$$

Inserting Eq.[20.17] into Eq.[20.16], we obtain

$$g_i(1+\mu_i k) = \frac{\omega_0}{2}\sum_{j=-n}^{n} a_j g_j \qquad [20.18]$$

We can find $\boldsymbol{g_i}$ in the form

$$g_i = L/(1+\mu_i k)$$

where L is a constant to be determined. Substituting this expression for $\boldsymbol{g_i}$ in Eq.[20.18], we have

$$1 = \frac{\omega_0}{2}\sum_{j=-n}^{n} \frac{a_j}{1+\mu_i k} = \omega_0 \sum_{j=1}^{n} \frac{a_j}{1-\mu_j^2 k^2} \qquad [20.19]$$

Eq.[20.19] gives 2n solutions for $+/-k_j$ (j=1,…,n).

Thus general solution is

$$I_i = \sum_j \frac{L_j}{1+\mu_i k_j} \exp(-k_j \tau) \qquad [20.20]$$

where L_j are constants.

The particular solution can be found as

$$I_i = \frac{\omega_0 F_0}{4\pi} h_i \exp(-\tau / \mu_0) \qquad [20.22]$$

where h_i are constants.

Inserting Eq.[20.22] into Eq.[20.16], we have

$$h_i (1+\mu_i / \mu_0) = \frac{\omega_0}{2} \sum_{j=-n}^{n} a_j h_j + 1 \qquad [20.23]$$

From Eq.[20.23], h_i is found as

$$h_i = \gamma / (1+\mu_i / \mu_0)$$

where γ is determined from

$$\gamma = 1/\{1 - \frac{\omega_0}{2} \sum_{j=1}^{n} a_j / (1-\mu_j^2 / \mu_0^2)\} \qquad [20.24]$$

Adding the general solution Eq.[20.20] and the particular solution Eq.[20.22], we have

$$I_i = \sum_j \frac{L_j}{1+\mu_i k_j} \exp(-k_j \tau) + \frac{\omega_0 F_0 \gamma}{4\pi(1+\mu_i / \mu_0)} \exp(-\tau / \mu_0) \qquad [20.25]$$

where L_j are constants to be determined from the boundary conditions.

H-function has been introduced by Chandrasekhar as

$$H(\mu) = \frac{1}{\mu_1 \ldots \mu_n} \frac{\prod_{j=1}^{n} (\mu + \mu_j)}{\prod_{j=1}^{1} (1+k_j \mu)} \qquad [20.26]$$

Expressing $\boldsymbol{\gamma}$ in the H-function, Eq.[20.25] becomes

$$I_i = \sum_j \frac{L_j}{1+\mu_i k_j} \exp(-k_j \tau) + \frac{\omega_0 F_0 H(\mu_0) H(-\mu_0)}{4\pi(1+\mu_i / \mu_0)} \exp(-\tau / \mu_0) \qquad [20.26]$$

Eq.[20.26] gives a simple solution for the semi-infinite isotropic atmosphere (see L02:6.2.2)

$$I^{\uparrow}(0,\mu) = \frac{1}{4\pi}\omega_0 F_0 \frac{\mu_0}{\mu+\mu_0} H(\mu_0) H(\mu) \qquad [20.27]$$

- **<u>Generalization of the discrete-ordinate method for an inhomogeneous atmosphere.</u>**

Let's consider the atmosphere with non-isotropic scattering. We can expand the diffuse intensity in the cosine series

$$I(\tau,\mu,\varphi) = \sum_{m=0}^{N} I^m(\tau,\mu)\cos(m(\varphi_0-\varphi))$$

So we need to solve

$$\mu \frac{dI^m(\tau,\mu)}{d\tau} = I^m(\tau,\mu) - (1+\delta_{0,m})\frac{\omega_0}{4}\sum_{l=m}^{N} \varpi_l^m P_l^m(\mu)\int_{-1}^{1} P_l^m(\mu') I^m(\tau,\mu')d\mu' -$$
$$-\frac{\omega_0}{4\pi}\sum_{l=m}^{N}\varpi_l^m P_l^m(\mu) P_l^m(-\mu_o) F_0 \exp(-\tau/\mu_0)$$

The **general solution** may be written as

$$I^m(\tau,\mu_i) = \sum_{j=-n}^{n} L_j^m \phi_j^m(\mu_j)\exp(-k_j^m \tau)$$

ϕ_j^m, k_j^m, L_j^m are coefficients to be determined.

The **particular solution** may be written as

$$I_p^m(\tau,\mu_i) = Z^m(\mu_i)\exp(-\tau/\mu_0)$$

Where $Z^m(\mu_i)$ is the following function

$$Z^m(\mu_i) = \frac{1}{4\pi}\omega_0 F_0 P_m^m(-\mu_0)\frac{H^m(\mu_0)H^m(-\mu_0)}{1+\mu_i/\mu_0}\sum_{l=0}^{N}\varpi_l^m \zeta_l^m \frac{1}{\mu_0} P_l^m(\mu_i)$$

The **complete solution** of the radiative transfer is

$$I^m(\tau,\mu_i) = \sum_{j=-n}^{n} L_j^m \phi_j^m(\mu_j)\exp(-k_j^m \tau) + Z^m(\mu_i)\exp(-\tau/\mu_0) \qquad [20.28]$$

i=-n,…,n

Let's generalize the **complete solution** Eq.[20.28] of the radiative transfer for the <u>inhomogeneous atmosphere</u>. The atmosphere can be divided into the N homogeneous layers, each is characterized by a single scattering albedo, phase function, and optical depth.

NOTE: If an atmospheric layer has gases, aerosols and/or clouds, one needs to calculate the effective optical properties of this layer.

For ***l***-th layer, we can write the solution using Eq.[20.28]. To simplify notations, let's consider the azimuthal independent case (i.e., ***m=0***), so we have

$$I^l(\tau,\mu_i) = \sum_{j=-n}^{n} L_j^l \phi_j^l(\mu_j)\exp(-k_j^l \tau) + Z^l(\mu_i)\exp(-\tau/\mu_0) \qquad [20.29]$$

Now, we need to match the boundary and continuity conditions between layers.

At the top of the atmosphere (TOA): no downward diffuse intensity

$$I^{l=1}(0,-\mu_i) = 0 \qquad [20.30]$$

At the layer's boundary: upward and downward intensities must be continuous

$$I^l(\tau_l,\mu_i) = I^{l+1}(\tau_l,\mu_i) \qquad [20.31]$$

At the bottom of the atmosphere (assuming the Lamdertian surface):

$$I^{l=N}(\tau_N,\mu_i) = \frac{r_{sur}}{\pi}[F^{\downarrow}(\tau_N) + \mu_0 F_0 \exp(-\tau_N/\mu_0)] \qquad [20.32]$$

Eqs.[20.30]-[20.32] provide necessary equations to find the unknown coefficients.

<u>Numerical implementation of the discrete-ordinate method: DISORT</u>

DISORT is a FORTRAN numerical code based on the discrete-ordinate method developed by Stamnes, Wiscombe et al.

DISORT is openly available and has a good user-guide.

Some features:

1) DISORT applies to the inhomogeneous nonithothermal plane-parallel atmosphere.

2) A user may set-up any numbers of the plane-parallel layers.

3) Each layer must be characterized by the effective optical depth, single scattering albedo and asymmetry parameter if the Henyey-Greenstein phase function is used.

4) A user may use any phase function by providing the Legendre polynomial expansion coefficients.

5) A user selects a number of streams (keeping in mind that the computation time varies as n^3).

6) A key problem is to obtain a solution for fluxes for strongly forward-peaked scattering.

7) DISORT allows predicting the intensity as a function of direction and position at any point in the atmosphere (i.e., not only at the boundaries of the layers).

DISORT is incorporated into the SBDART radiative transfer code.

4. Principles of invariance and Adding-doubling method.

Recall the definitions of reflection and transmission of a layer introduced in Lecture 19.

If the solar flux is incident on a layer of optical depth **τ***:

$$R(\mu, \varphi, \mu_0, \varphi_0) = \pi I^{\uparrow}(0, \mu, \varphi) / \mu_0 F_0$$

$$T(\mu, \varphi, \mu_0, \varphi_0) = \pi I^{\downarrow}(\tau^*, -\mu, \varphi) / \mu_0 F_0$$

Or in the general case:

$$I_r^{\uparrow}(0, \mu, \varphi) = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 R(\mu, \varphi, \mu', \varphi') I_{inc}(-\mu', \varphi')\mu' d\mu' d\varphi'$$

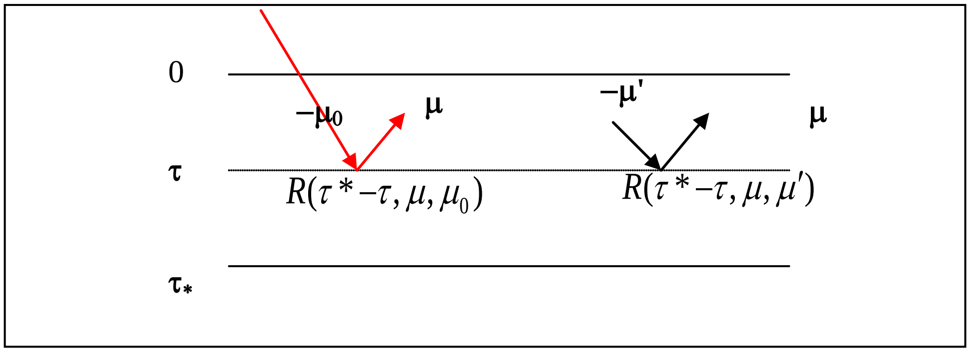
$$I_t^{\downarrow}(\tau^*, -\mu, \varphi) = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 T(\mu, \varphi, \mu', \varphi') I_{inc}(-\mu', \varphi')\mu' d\mu' d\varphi'$$

- **The principle of invariance for the <u>semi-infinite</u> atmosphere** (Ambartzumian, 1940): the diffuse reflected intensity cannot be changed if a layer of finite optical depth, having the same optical properties as those of the original layer, is added (see L02: 6.3.2).

➢ ***The principles of invariance for a <u>finite</u> atmosphere*** (Chandrasekhar, 1950):

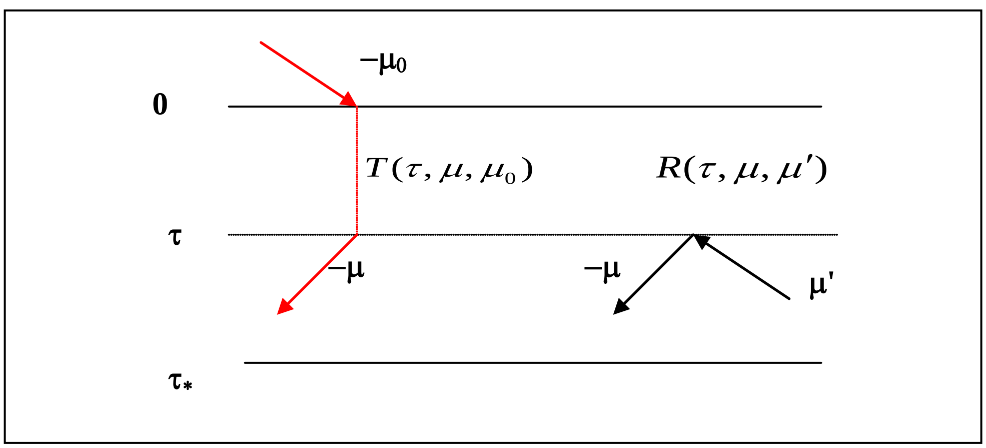
(1) The reflected (upward) intensity at any given optical depth **τ** results from the reflection of (a) the attenuated solar flux = $\mu_0 F_0 \exp(-\tau/\mu_0)$ and (b) the downward diffuse intensity at the level **τ:**

$$I^{\uparrow}(\tau,\mu) = \frac{\mu_0 F_0}{\pi}\exp(-\tau/\mu_0)R(\tau_1-\tau,\mu,\mu_0) + 2\int_0^1 R(\tau^*-\tau,\mu,\mu')I^{\downarrow}(\tau,-\mu')\mu' d\mu' \quad \textbf{[20.33]}$$



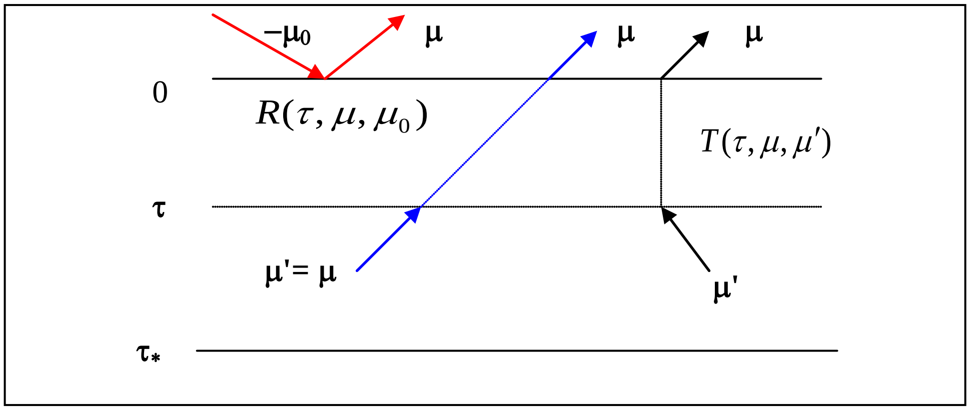
(2) The diffusely transmitted (downward) intensity at the level **τ** results from (a) the transmission of incident solar flux and (b) the reflection of the upward diffuse intensity above the level **τ**:

$$I^{\downarrow}(\tau,-\mu) = \frac{\mu_0 F_0}{\pi}T(\tau,\mu,\mu_0) + 2\int_0^1 R(\tau,\mu,\mu')I^{\uparrow}(\tau,\mu')\mu' d\mu' \quad [20.33]$$



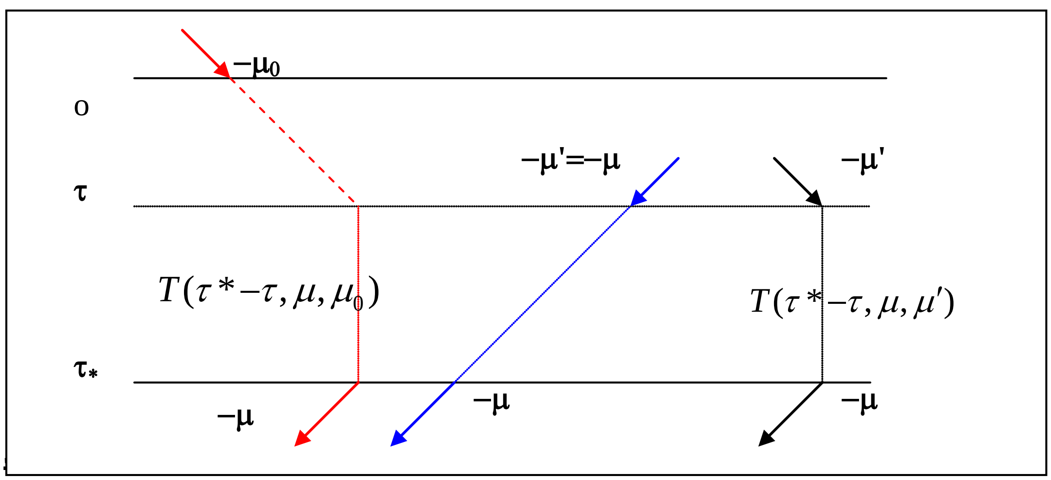
(3) The reflected (upward) intensity at the top of the finite atmosphere (**τ** = 0) is equivalent to (a) the reflection of solar flux plus (b) the direct and diffuse transmission of the upward diffuse intensity above the level **τ**:

$$I^{\uparrow}(0,\mu) = \frac{\mu_0 F_0}{\pi} R(\tau,\mu,\mu_0) + 2\int_0^1 T(\tau,\mu,\mu') I^{\uparrow}(\tau,\mu')\mu' d\mu' + I^{\uparrow}(\tau,\mu)\exp(-\tau/\mu) \qquad [20.34]$$



(4) The diffusely transmitted (downward) intensity at the bottom of the finite atmosphere (**τ=τ***) is equivalent to (a) the transmission of the attenuated solar flux at the level **τ** plus (b) the direct and diffuse transmission of the downward diffuse intensity at the level **τ** from above:

$$I^{\downarrow}(\tau^*,-\mu) = \frac{\mu_0 F_0}{\pi}\exp(-\tau/\mu_0) T(\tau^*-\tau,\mu,\mu_0) + 2\int_0^1 T(\tau^*-\tau,\mu,\mu') I^{\downarrow}(\tau,-\mu')\mu' d\mu' + \qquad [20.35]$$
$$+ I^{\downarrow}(\tau,-\mu)\exp(-(\tau^*-\tau)/\mu)$$

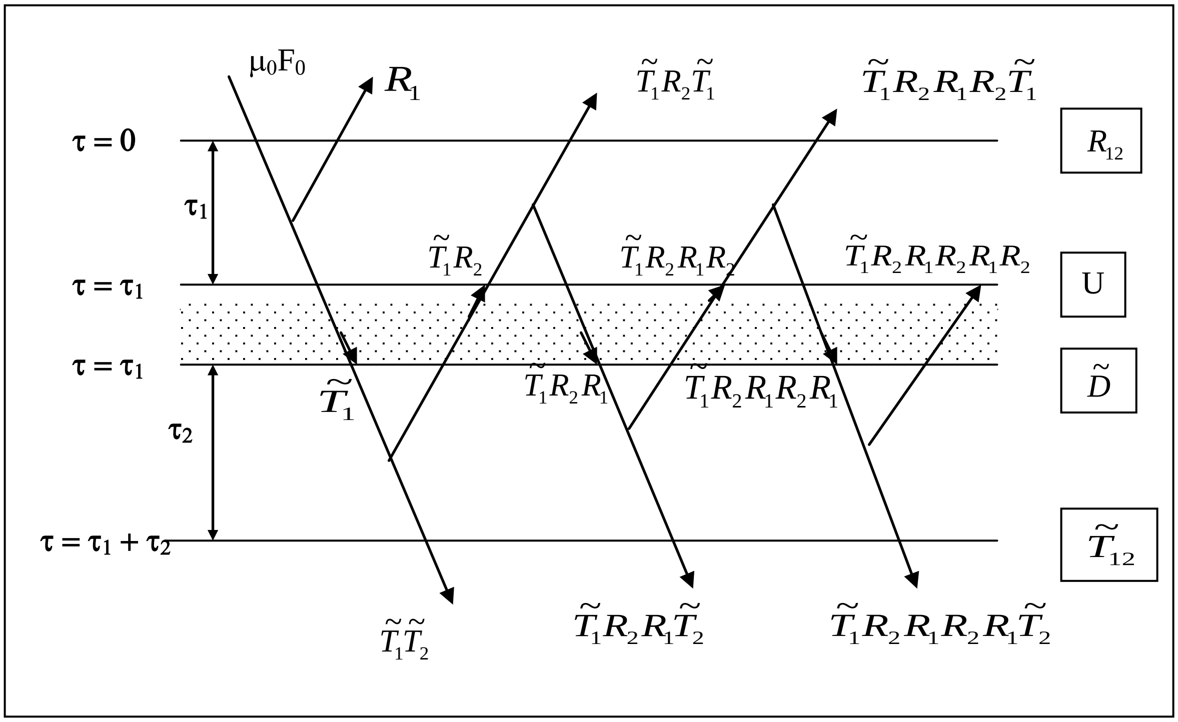


- ***Adding-doubling method*** is an "exact" method for solving the radiative transfer equation with multiple scattering. It uses geometrical ray-tracing approach and the reflection and transmission of each individual atmospheric layer.

Strategy: knowing the reflection and transmission of two individual layers, the reflection and transmission of the combined layer may be obtained by calculating the successive reflections and transmissions between these two layers.

NOTE: If optical depths of these two layers are equaled, this method is referred to as the doubling-adding method.

Consider two layers with reflection R_1 and R_2 and total (direct plus diffuse) transmission $\tilde{T}_1$ and $\tilde{T}_2$ functions, respectively. Let's denote the combined reflection and total transmission functions by R_{12} and $\tilde{T}_{12}$, and combined reflection and total transmission functions between layers 1 and 2 by U and $\tilde{D}$, respectively.



The combined reflection function R_{12} is

$$\begin{aligned}
&R_{12} = R_1 + \tilde{T}_1 R_2 \tilde{T}_1 + \tilde{T}_1 R_2 R_1 R_2 \tilde{T}_1 + \tilde{T}_1 R_2 R_1 R_2 R_1 R_2 \tilde{T}_1 + \ldots = \\
&= R_1 + \tilde{T}_1 R_2 \tilde{T}_1 [1 + R_1 R_2 + (R_1 R_2)^2 + \ldots] = \\
&= R_1 + R_2 \tilde{T}_1^{\,2} (1 - R_1 R_2)^{-1}
\end{aligned} \qquad [20.36]$$

NOTE: In Eq.[20.36] we use that $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$

The combined total transmission function $\tilde{T}_{12}$ is

$$\begin{aligned}
&\tilde{T}_{12} = \tilde{T}_1 + \tilde{T}_1 R_2 R_1 \tilde{T}_2 + \tilde{T}_1 R_2 R_1 R_2 R_1 \tilde{T}_2 + \ldots = \\
&= \tilde{T}_1 \tilde{T}_2 [1 + R_1 R_2 + (R_1 R_2)^2 + \ldots] = \\
&= \tilde{T}_1 \tilde{T}_2 (1 - R_1 R_2)^{-1}
\end{aligned} \qquad [20.37]$$

The combined reflection function U between layers 1 and 2:

$$\begin{aligned}
&U = \tilde{T}_1 R_2 + \tilde{T}_1 R_2 R_1 R_2 + \tilde{T}_1 R_2 R_1 R_2 R_1 R_2 + \ldots = \\
&= \tilde{T}_1 R_2 [1 + R_1 R_2 + (R_1 R_2)^2 + \ldots] = \\
&= \tilde{T}_1 R_2 (1 - R_1 R_2)^{-1}
\end{aligned} \qquad [20.38]$$

The combined total transmission function $\tilde{D}$ between layers 1 and 2:

$$\begin{aligned}
&\tilde{D} = \tilde{T}_1 + \tilde{T}_1 R_2 R_1 + \tilde{T}_1 R_2 R_1 R_2 R_1 + \ldots = \\
&= \tilde{T}_1 [1 + R_1 R_2 + (R_1 R_2)^2 + \ldots] = \\
&= \tilde{T}_1 (1 - R_1 R_2)^{-1}
\end{aligned} \qquad [20.39]$$

From Eqs.[20.36]-[20.39], we find that

$$R_{12} = R_1 + \tilde{T}_1 U; \;\; \tilde{T}_{12} = \tilde{T}_2 \tilde{D}; \;\; U = R_2 \tilde{D} \qquad [20.40]$$

Let's introduce $S = R_1 R_2 (1 - R_1 R_2)^{-1}$

Using that $\tilde{T} = T + \exp(-\tau / \mu')$, from Eqs.[20.39]-[20.40] we find

$$\begin{aligned}
&\tilde{D} = D + \exp(-\tau_1 / \mu_0) = \\
&= (1+S)(T_1 + \exp(-\tau_1 / \mu_0)) = (1+S)T_1 + S\exp(-\tau_1 / \mu_0) + \exp(-\tau_1 / \mu_0)
\end{aligned} \qquad [20.41]$$

$$\tilde{T}_{12} = (T_2 + \exp(-\tau_2/\mu_0))(D + \exp(-\tau_1/\mu_0))$$
$$= D\exp(-\tau_2/\mu_0) + T_2\exp(-\tau_1/\mu_0) + T_2 D + \exp\left(-\left[\frac{\tau_1}{\mu_0} + \frac{\tau_2}{\mu}\right]\right)\delta(\mu - \mu_0) \qquad [20.42]$$

Thus, we can write a system of iterative equations for the computation of diffuse transmission and reflection for the two layers in the form:

$$\boxed{\begin{aligned} &Q = R_1 R_2 \\ &S = Q(1-Q)^{-1} \\ &D = T_1 + ST_1 + S\exp(-\tau_1/\mu_0) \\ &U = R_2 D + R_2\exp(-\tau_1/\mu_0) \\ &R_{12} = R_1 + \exp(-\tau_1/\mu)U + T_1 U \\ &T_{12} = \exp(-\tau_2/\mu)D + T_2\exp(-\tau_1/\mu_0) + T_2 D \end{aligned}} \qquad [20.43]$$

NOTE: in Eq.[20.43], the product of two functions implies n integration over the appropriate angle so that all multiple-scattering contributions are included. For instance

$$R_1 R_2 = 2\int_0^1 R_1(\mu,\mu')R_2(\mu',\mu_0)\mu' d\mu'$$

Numerical procedure of the adding-doubling method:

1) As the starting point, one may calculate the reflection and transmission functions of an initial layer of very small optical depth (e.g., $\Delta\tau = 10^{-8}$) that the single scattering approximation is applicable.

2) Then, using Eq.[20.43], one computes the reflection and transmission functions of the layer of 2 $\Delta\tau$.

3) Using Eq.[20.43], one repeats the calculations adding the layers until a desirable optical depth is achieved.

5. Monte Carlo method.

- ✓ The absorption and scattering processes in the atmosphere can be considered as stochastic processes.

Recall that energy of one photon is hc/λ , where h = 6.626×10^{-34} J s

Solar flux at the top of the atmosphere at 550 nm = 2.55×10^{15} photons cm^{-2} s^{-1}

Thus, the radiative field can be predicted by statistical analysis of traveling photons.

The scattering phase function can be interpreted as a probability function for the redistribution of photons in different directions.

The single scattering albedo ω_0 can be interpreted as the probability that a photon will be scattered, given an extinction event.

NOTE: 1- ω_0 is called **co-albedo** and can be considered as the probability of absorption per extinction event.

The concept of the Monte Carlo method is to simulate photon propagation in an optically effective medium as a random process.

Generation of random numbers:

Using a random number generator (a numerical algorithm), the random numbers, ***rn***, between 0 and 1 with a probability distribution function ***PDF*** can be generated.

Using this ***rn***, we can generate another set of random numbers as

$$rx = -\ln(rn)$$

with ***PDF = exp(-rx)*** and ***rx*** between 0 and infinity.

Let's consider a homogeneous medium characterized by the extinction coefficient β_{ext}, single scattering albedo ω_0 and phase function $P(\mu, \mu')$.

Monte Carlo method simulates the trajectories of individual photons according to the following scheme:

(1) Determine starting position x_0 and direction (μ,φ) of a photon

(2) Generate a photon path length (using random numbers ***rx***)

$$x = rx$$

(3) Calculate a new photon position (x_0+x) called the event point (or collision point)

(4) Analyze what can happen with the photon at this event point by generating the random number ***rn*** and comparing it with ω_0

if ***rn*** **>** ω_0 => the photon is absorbed => go to (1) for a new photon.

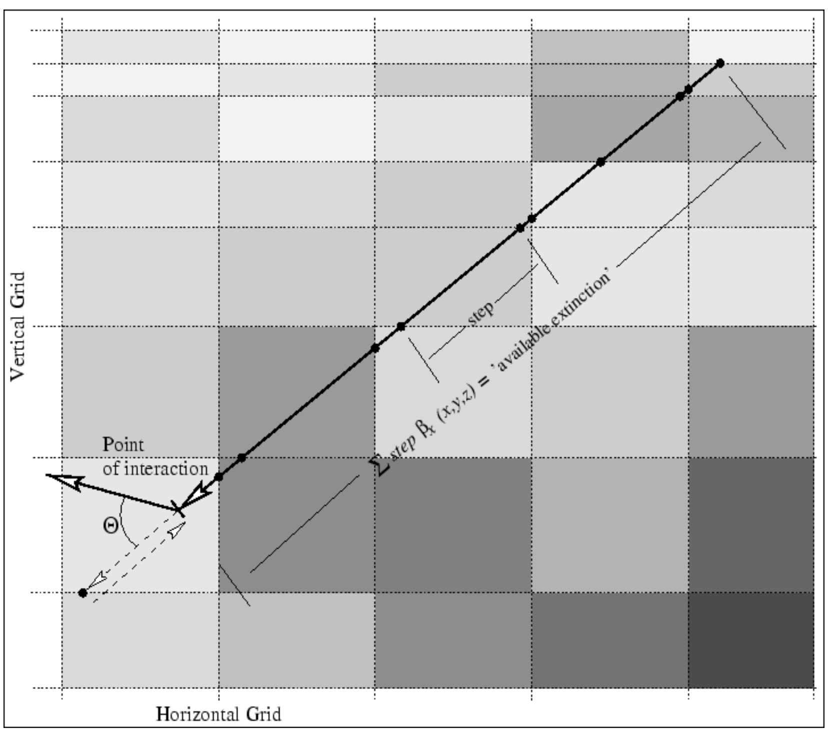
if ***rn*** **<** ω_0 => the photon is scattered => go to (5)

(5) Find a new direction for the scattered photon using the phase function to calculate the cumulative probability function to relate the scattering angle to a random number.

(6) Then repeat starting with (3) until the all photons are analyzed.

- ✓ Monte Carlo requires about 10^6 –10^9 photons to produce statistically reliable results.
- ✓ **Backward Monte Carlo method**: starts with the photon at the point of interest and traces back to the source.

Let's consider the inhomogeneous atmosphere. We can split it into the homogeneous grids.



Point of interaction (or event point)

$$rx = \sum_{step} step \;\; \beta_{ext}(x, y, z)$$

where ***step*** = step-size in each grid

***l* =1/β$_{ext}$** is called **free path-length**.

$$\frac{I}{I_0} = \exp(-\tau) = \exp(-\beta_{ext} x) = \exp(-x/l) \qquad [20.44]$$

➢ **Radiative transfer techniques for inhomogeneous clouds.**

Clouds exhibit high variability in space (x, y, z) => one-dimensional radiative transfer has limited applications

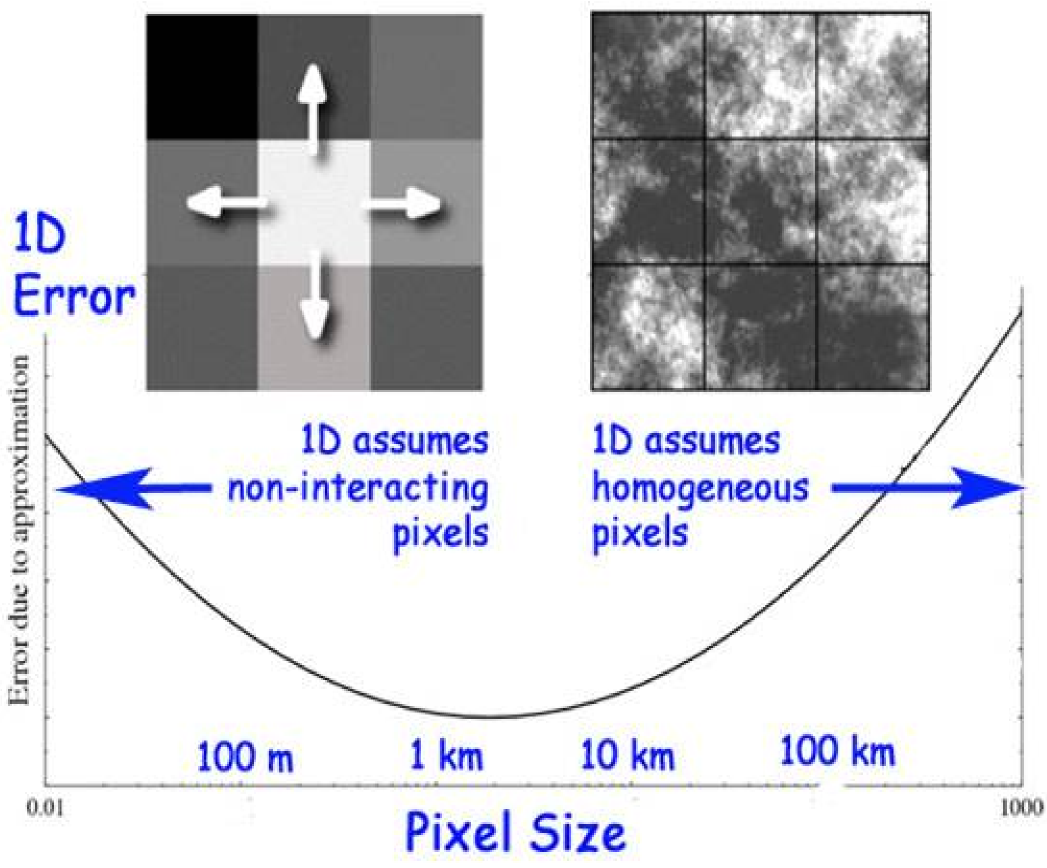


Figure 20.4 Illustration of the sources of errors in one-dimensional radiative transfer in cloudy conditions.

<u>How to treat the inhomogeneity of clouds:</u>

The simplest method: **introduce a cloud fraction f_{cl}**

(f_{cl} is commonly reported from meteorological observations)

$$I = f_{cl} I_{cl} + (1 - f_{cl}) I_{clear}$$

where I_{cl} is the intensity calculated with one-dimensional cloud, and I_{clear} is the intensity of clear sky.

But the problem is that

$$f_{cl}(observed) \neq f_{cl}(radiative)$$

Another problem is cloud overlap.

<u>Independent Column Approximation (ICA)</u>

- ✓ ICA is computational efficient technique to calculate the radiative transfer accounting for the cloud inhomogeneity. A cloud is subdivided into columns, plane-parallel radiative transfer is applied to each column, and the overall radiative transfer effect is the summation from the individual columns. Thus, ICA calculates the domain-averaged radiative properties.
- ✓ ICA requires the probability distribution of optical depth PDF (τ) in the cloudy part of the scene, instead of just the mean optical depth.
- ✓ ICA concept is well suitable for GCM models, but does not work well in the remote sensing of cloud properties.
- ✓ Modification of ICA, NICA (nonlocal independent column approximation) has been proposed to account for 'radiative smoothing' effect (i.e., the tendency of horizontal photon transport to smooth the radiative field predicted by ICA.

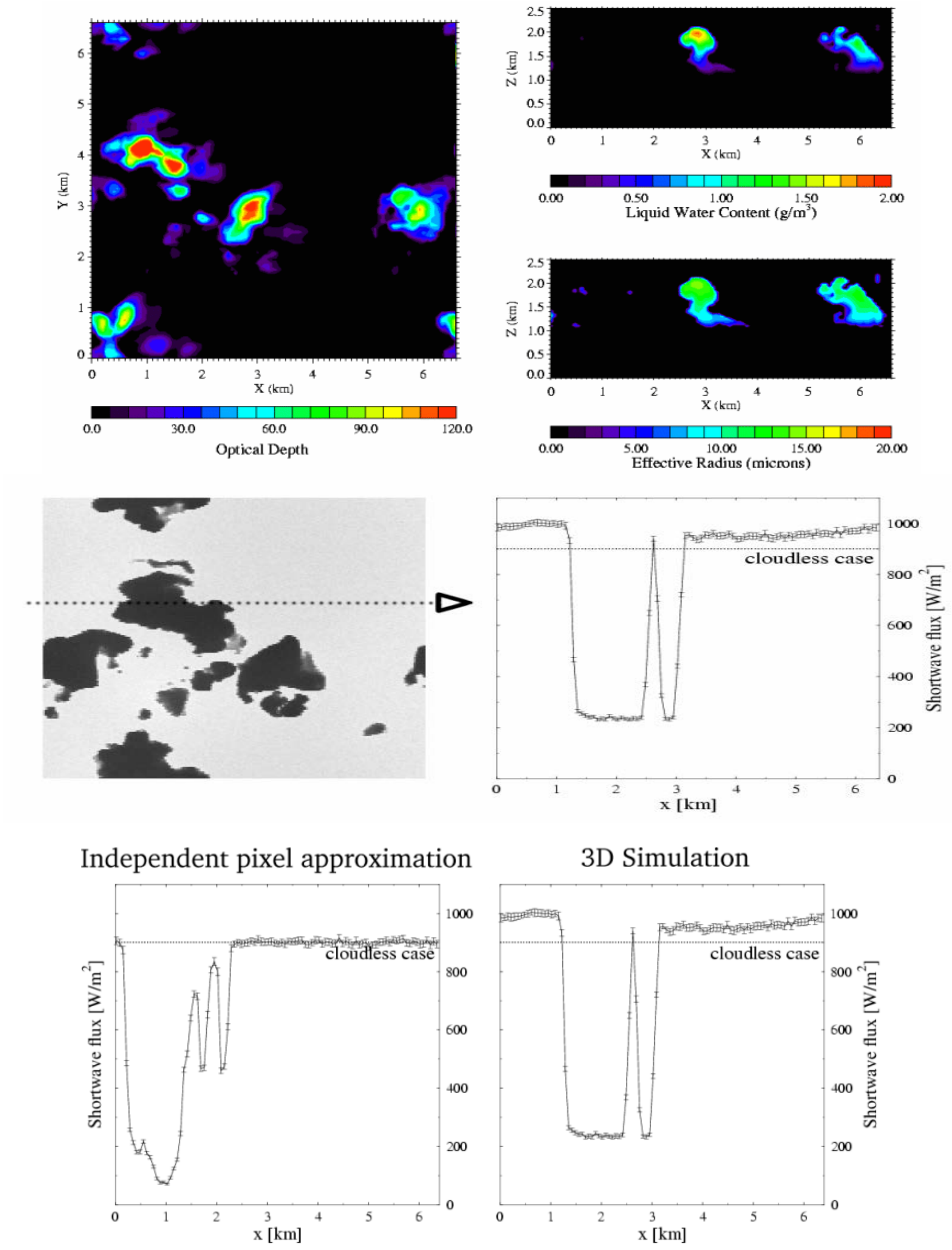


Figure 20.5 Solar flux (0.2µm - 4µm) at the Earth's surface calculated with ICA and Monte Carlo for the cloud field shown on the upper panels. The sun is shining from the left, solar zenith angle is 30°. The right image shows a cross section along the dotted line (Mayer et al).

NOTE in Fig. 20.5:

- ✓ The 3D shadows do not appear below the clouds, but of course offset into the direction of the direct solar beam.
- ✓ The 3D radiation field outside the cloud shadows is enhanced compared to the one-dimensional approximation.
- ✓ The 3D flux inside the cloud shadows is considerably enhanced compared to the independent pixel approximation by more than a factor of 2. This is caused by sidewards scattering of radiation under the cloud.

Barker, et al., Assessing 1D Atmospheric Solar Radiative Transfer Models: Interpretation and Handling of Unresolved Clouds. Journal of Climate, vol. 16, Issue 16, pp.2676-2699, 2003.

The Monte Carlo Independent Column Approximation (McICA):

combines IAC and Monte Carlo (incorporated into the ECMWF forecasting model)

Spherical Harmonic Discrete Ordinate Method (SHDOM):

(developed by F. Evans, http://nit.colorado.edu/~evans/shdom.html)

- ✓ SHDOM is a highly efficient and flexible 3D atmospheric radiative transfer model.
- ✓ SHDOM uses an iterative process to compute the source function (including the scattering integral) on a grid of points in space. The angular part of the source function is represented with a spherical harmonic expansion.
- ✓ SHDOM can compute unpolarized, monochromatic and broadband (with a k-distribution), shortwave and longwave radiative fluxes. The medium properties (extinction, phase function, etc.) are specified at each grid point, and the surface albedo may vary as well.

Pincus, R., and K. F. Evans, 2009: Computational cost and accuracy in calculating three-dimensional radiative transfer: Results for new implementations of Monte Carlo and SHDOM. J. Atmos. Sci., 66, 3131-3146.