

Lecture 8.

Terrestrial infrared radiative processes.

Part 1: Line-by-line (LBL) method for solving IR radiative transfer.

1. Fundamentals of the thermal IR radiative transfer.
2. Line-by-line computations of radiative transfer in IR.

Required reading:

L02: 4.2.1-4.2.3

Additional reading:

LBLRTM code <http://rtweb.aer.com/>

Clough, S.A., et al. Atmospheric radiative transfer modeling: a summary of the AER codes. Journal of Quantitative Spectroscopy & Radiative Transfer 91 (2005) 233–244.

1. Fundamentals of the thermal IR radiative transfer.

Recall the **equation of radiative transfer** (see Lecture 3) for upward and downward intensities in *the plane-parallel atmosphere*

$$\mu \frac{dI_{\lambda}^{\uparrow}(\tau; \mu; \varphi)}{d\tau} = I_{\lambda}^{\uparrow}(\tau; \mu; \varphi) - J_{\lambda}^{\uparrow}(\tau; \mu; \varphi) \quad [3.25a]$$

$$-\mu \frac{dI_{\lambda}^{\downarrow}(\tau; -\mu; \varphi)}{d\tau} = I_{\lambda}^{\downarrow}(\tau; -\mu; \varphi) - J_{\lambda}^{\downarrow}(\tau; -\mu; \varphi) \quad [3.25b]$$

and its **solutions** (see Lecture 3):

$$I_{\lambda}^{\uparrow}(\tau; \mu; \varphi) = I_{\lambda}^{\uparrow}(\tau_{*}; \mu; \varphi) \exp\left(-\frac{\tau_{*} - \tau}{\mu}\right) + \frac{1}{\mu} \int_{\tau}^{\tau_{*}} \exp\left(-\frac{\tau' - \tau}{\mu}\right) J_{\lambda}^{\uparrow}(\tau'; \mu; \varphi) d\tau' \quad [3.26a]$$

$$I_{\lambda}^{\downarrow}(\tau; -\mu; \varphi) = I_{\lambda}^{\downarrow}(0; -\mu; \varphi) \exp\left(-\frac{\tau}{\mu}\right) + \frac{1}{\mu} \int_0^{\tau} \exp\left(-\frac{\tau - \tau'}{\mu}\right) J_{\lambda}^{\downarrow}(\tau'; -\mu; \varphi) d\tau' \quad [3.26b]$$

Infrared radiative transfer in the absorbing/emitting atmosphere:

For a non-scattering medium in the local thermodynamical equilibrium, the source function is given by the Plank's function $B_\lambda(T)$ (see Lecture 4) and

$$\beta_{e,\lambda} = \beta_{a,\lambda} = k_{a,\lambda}\rho ,$$

where $\beta_{e,\lambda}$ and $\beta_{a,\lambda}$ are the volume extinction and absorption coefficients, and $k_{a,\lambda}$ is the mass absorption coefficient.

Assuming that the thermal infrared radiation from the earth's atmosphere is independent on the azimuthal angle ϕ , **the equation of infrared radiative transfer** (in the wavenumber domain) **for the monochromatic upward and downward intensities** can be expressed as:

$$\mu \frac{dI_v^\uparrow(\tau; \mu)}{d\tau} = I_v^\uparrow(\tau; \mu) - B_v(T) \quad [8.1a]$$

$$-\mu \frac{dI_v^\downarrow(\tau; -\mu)}{d\tau} = I_v^\downarrow(\tau; -\mu) - B_v(T) \quad [8.1b]$$

and **the solutions** as

$$I_v^\uparrow(\tau; \mu) = I_v^\uparrow(\tau^*; \mu) \exp\left(-\frac{\tau^* - \tau}{\mu}\right) + \frac{1}{\mu} \int_{\tau}^{\tau^*} \exp\left(-\frac{\tau' - \tau}{\mu}\right) B_v(T(\tau')) d\tau' \quad [8.2a]$$

$$I_v^\downarrow(\tau; -\mu) = I_v^\downarrow(0; -\mu) \exp\left(-\frac{\tau}{\mu}\right) + \frac{1}{\mu} \int_0^{\tau} \exp\left(-\frac{\tau - \tau'}{\mu}\right) B_v(T(\tau')) d\tau' \quad [8.2b]$$

To solve Eqs.[8.2a, b] for the entire atmosphere with total optical depth τ_v^* , two boundary conditions are required:

Surface: it can be assumed to be a blackbody with the surface temperature T_s . Otherwise, the spectral surface emissivity (ϵ_v) is introduced.

$$I_v^\uparrow(\tau_v^*, \mu) = B_v(T_s) = B_v(T_s(\tau_v^*)) = B_v(\tau_v^*)$$

Top of the atmosphere (TOA), $\tau_v = 0$: no downward thermal emission

$$I_v^\downarrow(0, -\mu) = 0$$

Using the above boundary conditions, **the solutions (Eqs. 8.2a, b) for monochromatic upward and downward intensities are**

$I_v^\uparrow(\tau; \mu) = B_v(\tau^*) \exp\left(-\frac{\tau^* - \tau}{\mu}\right) + \frac{1}{\mu} \int_{\tau}^{\tau^*} \exp\left(-\frac{\tau' - \tau}{\mu}\right) B_v(\tau') d\tau'$	[8.3a]
$I_v^\downarrow(\tau; -\mu) = \frac{1}{\mu} \int_0^{\tau} \exp\left(-\frac{\tau - \tau'}{\mu}\right) B_v(\tau') d\tau'$	[8.3b]

- ✓ The solutions for monochromatic upward and downward intensities can be also expressed in terms of monochromatic transmittance.

By definition, the monochromatic transmittance is

$$T_v(\tau; \mu) = \exp\left(-\frac{\tau_v}{\mu}\right)$$

and the differential form is

$$\frac{dT_v(\tau; \mu)}{d\tau} = -\frac{1}{\mu} \exp\left(-\frac{\tau_v}{\mu}\right)$$

Thus **the formal solutions for monochromatic upward and downward intensities** given by Eq.[8.3a,b] in terms of transmittance are:

$$I_v^\uparrow(\tau; \mu) = B_v(\tau^*) T_v(\tau^* - \tau; \mu) - \int_\tau^{\tau^*} B_v(\tau') \frac{dT_v(\tau' - \tau; \mu)}{d\tau'} d\tau' \quad [8.4a]$$

$$I_v^\downarrow(\tau; -\mu) = \int_0^\tau B_v(\tau') \frac{dT_v(\tau - \tau'; \mu)}{d\tau'} d\tau' \quad [8.4b]$$

NOTE: Eq.(8.4 a, b) can be also written in terms of the weighing function which is defined as

$$W_v(\tau, \tau', \mu) = \left| \frac{dT_v(\tau - \tau', \mu)}{d\tau} \right| \quad [8.5]$$

Let's re-write the solutions of the radiative transfer equation for upward and downward radiances in the altitude coordinate z

$$\tau_v = \int_{z'}^z k_{a,v} \rho_{gas} dz'' \quad [8.6]$$

Thus transmission between z and z' along the path at μ is

$$T_v(z, z', \mu) = \exp\left(-\frac{1}{\mu} \int_{z'}^z k_{a,v} \rho_{gas} dz''\right) \quad [8.7]$$

and

$$\frac{dT_v(z, z', \mu)}{dz'} = -\frac{k_{a,v} \rho_{gas}}{\mu} \exp\left(-\frac{1}{\mu} \int_{z'}^z k_{a,v} \rho_{gas} dz''\right) \quad [8.8]$$

Thus

$$I_v^\uparrow(z, \mu) = I_v^\uparrow(0, \mu) \exp\left[-\frac{1}{\mu} \int_0^z k_{a,v} \rho_{gas} dz'\right] + \frac{1}{\mu} \int_0^z \exp\left[-\frac{1}{\mu} \int_{z'}^z k_{a,v} \rho_{gas} dz''\right] B_v(T(z')) k_{a,v} \rho_{gas} dz' \quad [8.9a]$$

$$I_v^\downarrow(z, -\mu) = \frac{1}{\mu} \int_z^\infty \exp\left[-\frac{1}{\mu} \int_z^z k_{a,v} \rho_{gas} dz''\right] B_v(T(z')) k_{a,v} \rho_{gas} dz' \quad [8.9b]$$

2. Line-by-line (LBL) computations of radiative transfer in IR.

The **LBL method** is considered to be an “exact” computation of radiative transfer in the gaseous absorbing/emitting inhomogeneous atmosphere and it accounts for all (known) gas absorption lines in the wavenumber range from 0 to about 23,000 cm^{-1} .

Strategy to perform LBL calculations (i.e., to solve Eq.[8.3a,b]) for the plane-parallel atmosphere:

For a given wavenumber ν :

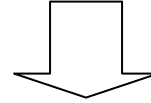
For the j -th atmospheric layer:(homogeneous; temperature T_j , pressure p_j , length ΔZ_j)

For n -th gas:

Absorption coefficient $k_{a\nu j,n}$ is

$$k_{a,\nu,j,n} = \sum_{l=1}^L k_{a,\nu,j,n,l} = \sum_{l=1}^L S_{n,l}(T_j) f_{\nu,n,l}(T_j, p_j)$$

where $l = 1, \dots, L$ is the number of absorbing lines of n -th gas at a selected ν ; $S_{\nu,n,l}$ and $f_{\nu,n,l}$ are the intensity and profile of the l -th line, respectively.



Optical depth $\tau_{\nu j,n}$, of the n -th gas of the j -th layer

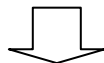
$$\tau_{\nu,j,n} = k_{a,\nu,n,j} u_{n,j}$$

where $u_{n,j}$ is the slant path for the n -gas in the j -th layer (i.e., the amount of the n -th gas in the j -th layer).

Repeating above calculations for all gases $n=1, \dots, N$, we find the optical depth of the j -th layer

$$\tau_{\nu,j} = \sum_{n=1}^N \tau_{\nu,j,n}$$

Repeating above calculations for all layers $j=1, \dots, J$, we find optical depth of each layer.



Using calculated optical depth of each layer, we find the **monochromatic upward and downward intensities** from Eq.[8.3a,b].

NOTE: Similar strategy is used to solve Eq.[8.4a,b] via monochromatic transmission function. For the n-th gas

$$T_{\nu,j,n} = \exp\left(-\frac{\tau_{\nu,j,n}}{\mu}\right)$$

and, using the **multiplication law** of transmittance, we have the transmittance of the j-th layer of N absorbing gases

$$T_{\nu,j} = T_{\nu,1}T_{\nu,2}\dots T_{\nu,N}$$

- **Multiplication law of transmittance** states that when several gases absorb, the monochromatic transmittance is a product of the monochromatic transmittances of individual gases:

$$T_{\nu,1,2\dots N} = T_{\nu,1}T_{\nu,2}\dots T_{\nu,N} \quad [8.10]$$

- ✓ **For many practical applications, one needs to know not monochromatic intensity (or flux) but intensity (or flux) averaged over a given wavenumber interval.**

Spectral intensity = intensity averaged over a very narrow interval (that B_ν is almost constant but the interval is large enough to consist of several absorption lines).

Narrow-band intensity= intensity averaged over a narrow band which includes a lot of lines;

Broad-band intensity= intensity averaged over a broad band (e.g., over a whole longwave region)

We can define the **spectral transmission function** for a band of a width $\Delta\nu$ as

$$T_{\bar{\nu}}(u) = \frac{1}{\Delta\nu} \int_{\Delta\nu} T_{\nu}(\tau) d\nu = \frac{1}{\Delta\nu} \int_{\Delta\nu} \exp(-\tau_{\nu}) d\nu = \frac{1}{\Delta\nu} \int_{\Delta\nu} \exp(-k_{a,\nu}u) d\nu \quad [8.11]$$

and **spectral absorptance** is defined as

$$A_{\bar{\nu}} = 1 - T_{\bar{\nu}}(u) = \frac{1}{\Delta\nu} \int_{\Delta\nu} (1 - \exp(-\tau_{\nu})) d\nu = \frac{1}{\Delta\nu} \int_{\Delta\nu} (1 - \exp(-k_{\nu}u)) d\nu \quad [8.12]$$

NOTE: Spectral intensify requires the calculations of **spectral transmission** which requires the calculations of **monochromatic optical depth** which are done with LBL computations.

LBL spectral resolution:

- Because LBL computes each line of absorbing gases in a non-homogeneous atmosphere, the adequate selection of an integration step (i.e., interval $d\nu$) is required to calculate the spectral transmittance in the interval $\Delta\nu$ ($\Delta\nu > d\nu$).
- Because P decreases exponentially with altitude, the line half-width and hence the integration step should be smaller at higher altitudes in the atmosphere.
- Because of these variable resolutions, the absorptions coefficients of two consequent layers must be merged – it is done by interpolating the coarser-resolution of lower layers into the finer-resolution of the higher-level => spectral absorptance for a given slant path is computed with the finest spectral resolution.

➤ **Continuum**

Absorption lines may have long wings (e.g., depending on a line half-width). To simplify calculations, the wings of a line are cut at a given distance from the line center Thus the absorption coefficient of the line may be expressed as

$$k_{a,\nu} = S f_{\nu} + k_{a,\nu}^c \quad [8.13]$$

where $k_{a,\nu}^c$ gives the absorption fraction in the wings (called **continuum absorption**).

CKD (Clough, Kneizys, and Davies) continuum model:

includes continuum absorption due to water vapor, nitrogen, oxygen, carbon dioxide, and ozone. The water vapor continuum is based upon a water vapor monomer line shape formalism applied to all spectral regions from the microwave to the shortwave.

The absorption coefficient for the water vapor continuum is the sum of the self- and foreign (air)-broadening components:

$$k_{a,v}^c = \sigma_{self} [p_w + \frac{\sigma_{air}}{\sigma_{self}} (p - p_w)] \quad [8.14]$$

where p_w and p denote the water vapor partial pressure and the air (ambient) pressure (in atm), respectively, and σ_{self} and σ_{air} are the self- and air-broadening coefficients for water vapor.

In the 8-12 μm region, the self-broadening coefficient is parameterized as (Roberts et al., 1976):

$$\sigma_{self}(\nu, T_r) = a + b \exp(-\beta\nu) \quad [8.15]$$

where $T_r = 296 \text{ K}$, $a=4.18$, $b=5578$, and $b=7.87 \times 10^{-3}$ (for σ_{self} in $\text{cm}^2 \text{ g}^{-1} \text{ atm}^{-1}$).

And $\sigma_{air} / \sigma_{self} = 0.002$ at $T_r = 296 \text{ K}$.

Moreover, σ_{self} depends on as:

$$\sigma_{self}(\nu, T) = \sigma_{self}(\nu, T_r) \exp(c(T_r / T - 1))$$

where $c=6.08$

NOTE: There are two competing effects on absorption of the water vapor continuum: the absorption coefficient increases as the temperature decreases, but colder atmospheric conditions have less water vapor.

➤ **Examples of LBL numerical codes: LBLRTM and FASTCODE**

LBLRTM was developed at the ATMOSPHERIC AND ENVIRONMENTAL RESEARCH INC. (available to the scientific community)

<http://rtweb.aer.com/>

General info:

A radiance algorithm has been used to treat the vertically inhomogeneous atmospheres resulting in substantially improved accuracy, and the model is directly applicable to longwave cooling rate calculations. A layered atmosphere is used with each layer assumed to be in local thermodynamic equilibrium with respect to absorption in the layer. The spectral lines are optimally sampled at each layer using an algorithm that effectively provides optimal sampling over the line. An accelerated approximation to the Voigt line

shape using a linear combination of precalculated functions is applied to all lines at all layers. The line shape calculation extends to 25 cm^{-1} from line center for all lines, and continua consistent with the adopted line shape definition are provided for self and foreign water vapor broadening and for carbon dioxide (CKD continua are used).

FASCODE (FAST Atmospheric Signature CODE)

<http://www.kirtland.af.mil/library/factsheets/factsheet.asp?id=7913>

H.J.P. Smith, D.J. Dube, M.E. Gardner, S.A. Clough, F.X. Kneizys, and L.S. Rothman, FASCODE- Fast Atmospheric Signature Code (Spectral Transmittance and Radiance), Air Force Geophysics Laboratory Technical Report AFGL-TR-78-0081, Hanscom AFB, MA, 1978.

General Info:

The Fast Atmospheric Signature Code is a first principles, line-by-line atmospheric radiance and transmittance code. FASCODE is the standard benchmark for atmospheric background codes based on band model approaches to radiation transport. It is applicable from the visible to long wavelength infrared. It is generally used to calculate atmospheric radiance and path transmission at low altitudes, but can be used for non-equilibrium high altitude calculations if supplied with the appropriate vibrational level temperatures.

Recent upgrades to FASCODE include vectorization of routines to increase the computational speed, parameterization of dimension statements to allow for easy changes in the size of arrays, configuration for use on a variety of computer platforms, and formulation of improvements to the atmospheric path geometry. New features (under development) include the capability to calculate spectra over a much wider spectral range, approximately a factor of four, and the option for output that is formatted to be compatible with inputs needed for multiple-scattering programs such as CHARTS and DISORT.