

- Tuesday, April 26: Last day of classes
- Wednesday, April 27 and part of Thursday, April 28: First reading period
- Thursday, April 28: Exams begin
- Tuesday, May 3 (part): Second reading period
- Thursday, May 5: Last regular day of exams
- Friday, May 6: Rescheduled exams (due to conflicts)

You may NOT schedule required class meetings, quizzes, tests, or any other assessments during the reading periods.

Lecture 25.

Problem solution examples.

Calculate the radiative forcing for a solar constant increase of 0.1% at sunspot maximum. Compare this with the current radiative forcing from the anthropogenic increase in trace gases.

For a change in the solar constant, ΔF_0 , it is the change in mean absorbed solar flux that matters, so

$$SWF = \frac{\Delta F_0}{4} (1 - \bar{r})$$

where \bar{r} is the spherical (or global) mean albedo.

Thus

$$SWF = \frac{\Delta F_0}{4} (1 - \bar{r}) = \frac{0.001 \frac{W}{m^2}}{4} (1 - 0.3) = 0.24 \frac{W}{m^2}$$

The radiative forcing is positive, so the Earth would warm.

The current radiative forcing from trace gases is about 2.64 W/m^2 .

The total solar irradiance changes over the sunspot cycle do not have significant climate effects.

Say a water cloud has an optical depth of 10 and asymmetry parameter of 0.85 at visible and near IR wavelengths. Use the Eddington solution to obtain the albedo of the cloud for a solar zenith angle of $\mu_0=2/3$.

Now suppose the cloud is above a grassy surface. Calculate the total albedo (cloud and surface, ignoring the small contribution from molecular scattering) at a wavelength of $\lambda=0.65 \mu\text{m}$ where the surface albedo is 0.10 and at $\lambda=0.85 \mu\text{m}$ where the surface albedo is 0.65. Use the Eddington approximation so the reflection matrix is treated as a scalar. What is the multiple reflection factor in each case.

At wavelengths of 0.65 μm and 0.85 μm water does not absorb, so this is conservative scattering. For a solar angle of $\mu_0=2/3$ the Eddington conservative scattering albedo is

$$R = \frac{(1-g)\tau}{\frac{4}{3}+(1-g)\tau} = \frac{(1-0.85)10}{\frac{4}{3}+(1-0.85)10} = 0.529$$

Since there is no absorption the transmission is $T = 1 - R = 0.471$. The cloud layer and surface may be combined radiatively using the adding formula:

$$R_T = R_a + T_a \Gamma R_s T_a \qquad \Gamma = (1 - R_s R_a)^{-1}$$

where R_a is the atmosphere reflection, T_a is the atmosphere transmission, R_s is the surface reflection, and Γ is the multiple reflection factor.

For $\lambda = 0.65 \mu\text{m}$ the total albedo is

$$R_{T,0.65} = 0.529 + \frac{0.471 * 0.1 * 0.471}{1 - 0.1 * 0.529} = 0.553$$

For $\lambda = 0.85 \mu\text{m}$ the total albedo is

$$R_{T,0.85} = 0.529 + \frac{0.471 * 0.65 * 0.471}{1 - 0.65 * 0.529} = 0.749$$

The multiple reflection factors are:

$$\Gamma_{0.65} = \frac{1}{1 - R_s R_a} = 1.056 \quad \Gamma_{0.85} = \frac{1}{1 - R_s R_a} = 1.524$$

Calculate the optical depth due to CO₂ absorption at $\nu = 692.000 \text{ cm}^{-1}$ in the 1 km thick layer at $P = 102 \text{ mb}$ and $T = 217 \text{ K}$. The volume mixing ratio of CO₂ in this layer is $q_{\text{CO}_2} = 3.7 \times 10^{-4}$. The CO₂ absorption the wavenumber of interest is mainly affected by the strong CO₂ line with the center $\nu_0 = 691.9725 \text{ cm}^{-1}$. Consider the Lorentz line. The parameters for this line (from HITRAN2004) are the following: Half-width $\alpha_0 = 0.0684 \text{ cm}^{-1}$ at $P_0 = 1013 \text{ mb}$ and $T_0 = 296 \text{ K}$; line intensity $S = 7.136 \times 10^{-20} \text{ (cm/molec)}$ at $T = 217 \text{ K}$; and $n = 0.78$.

First, we need to calculate the half-width α at T and P of interest:

$$\alpha(P, T) = \alpha_0 \frac{P}{P_0} \left(\frac{T_0}{T} \right)^n = 0.00877 \text{ cm}^{-1}$$

The absorption coefficient is given by

$$k_\nu = S \frac{\alpha/\pi}{(\nu - \nu_0)^2 + \alpha^2}$$

For the desired wavenumber $\nu = 692.000 \text{ cm}^{-1}$, we have

$$\begin{aligned} k_{692} &= \left(1.736 * 10^{-20} \frac{\text{cm}}{\text{molec}} \right) \frac{0.00877 \text{ cm}/\pi}{(692 - 691.9725)^2 + (0.00877)^2} = \\ &= 2.39 * 10^{19} \text{ cm}^2 / \text{molec} \end{aligned}$$

The optical depth of the layer is the absorber amount times the absorption coefficient

$$\tau_{\nu} = k_{\nu} u_{CO_2}$$

To find u_{CO_2} , need to know the amount of air

$$u_{air} = \frac{(\Delta Z N_a P)}{(RT)} = 3.4 * 10^{23} \text{ molec/cm}^2$$

Thus

$$\tau_{692} = k_{692} u_{CO_2} = k_{692} q_{CO_2} u_{air} = 30.1$$

Derive an expression relating volume absorption coefficient to the integrated volume V (i.e., the volume of particles per volume of air). Consider the Rayleigh limit.

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In the Rayleigh limit, the absorption efficiency can be expressed as

$$Q_a = -4x \operatorname{Im} \left[\frac{m^2 - 1}{m^2 + 2} \right]$$

where x is the size parameter

To compute the volume absorption coefficient, we need to integrate over the particle size distribution

$$k_a = \int \pi r^2 Q_a N(r) dr$$

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$$= -\frac{6\pi}{\lambda} \operatorname{Im} \left[\frac{m^2 - 1}{m^2 + 2} \right] \int \frac{4\pi r^3}{3} N(r) dr =$$

$$= -\frac{6\pi}{\lambda} \operatorname{Im} \left[\frac{m^2 - 1}{m^2 + 2} \right] V$$

Assume that the Sun radiates as a blackbody at $T=5783$ K and is a uniform sphere with a radius of 6.96×10^5 km. Calculate total solar radiance and irradiance at the orbits of Venus and Earth. The distance between the Earth and Sun is 1.5×10^8 km and between the Sun and Venus is 1.08×10^8 km.

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Total flux:

$$F_{sun} = \sigma T^4 = (5.67 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4})(5783 \text{ K})^4 = 6.34 \cdot 10^7 \text{ W/m}^2$$

$$F_{Earth} = F_{sun} \left(\frac{R_{sun}}{R_{earth}} \right)^2 = 6.34 \cdot 10^7 \text{ W/m}^2 \left(\frac{6.96 \cdot 10^5 \text{ km}}{1.50 \cdot 10^8 \text{ km}} \right)^2 = 1365 \text{ W/m}^2$$

$$F_{Venus} = F_{sun} \left(\frac{R_{sun}}{R_{Venus}} \right)^2 = 6.34 \cdot 10^7 \text{ W/m}^2 \left(\frac{6.96 \cdot 10^5 \text{ km}}{1.08 \cdot 10^8 \text{ km}} \right)^2 = 2634 \text{ W/m}^2$$

Calculate the refractive index of aqueous ammonium sulfate particles with the 80% volume fraction of water. Take the refractive index of water $m_{\text{H}_2\text{O}} = 1.33 - i 10^{-8}$, and dry ammonium sulfate salt $m_{\text{sul}} = 1.55 - i 10^{-9}$

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The effective refractive index can be calculated by applying the volume weighted mixing rule:

Solution:

$$m_e = (1 - f)m_{sul} + fm_{H_2O}$$

where f is the volume fraction of water ($f = 0.8$).

$$\begin{aligned} m_e &= 0.2 m_{sul} + 0.8 m_{H_2O} \\ &= 0.2(1.55 - i10^{-9}) + 0.8(1.33 - i10^8) \\ &= 1.374 - i0.82 10^{-8} \end{aligned}$$