Lecture 11.

Principles of passive remote sensing using emission and applications: Remote sensing of atmospheric path-integrated quantities (cloud liquid content and precipitable water vapor).

1. Radiative transfer with emission.
2. Microwave radiative transfer.
3. Measurements of atmospheric path-integrated quantities (precipitable water, cloud liquid water).

Required reading:

S: 7.1, 7.3.1, 7.3.2, 7.2, Petty: 8

Additional reading:

Microwave remote sensing (various satellite data products, including SSM/I): http://www.remss.com/

NOAA AMSU-A retrieval algorithm for total precipitable water and cloud liquid water: http://www.star.nesdis.noaa.gov/corp/scsb/mspps/algorithms.html#ATPWCLW

1. Radiative transfer with emission.

Atmosphere and surfaces emit infrared and microwave radiation.

According to the Kirchhoff's law: **emission=absorption**

Recall the Beer-Bouguer-Lambert law (Eq.[6.26b], Lecture 6) for emission

$$dI_\lambda = k_{e,\lambda} J_\lambda ds$$

where $\boldsymbol{k_{e,\lambda}}$ is the volume extinction coefficient along path $\boldsymbol{ds}$.

- For a non-scattering medium in LTE, the Planck function gives the source function

$$\boxed{J_\lambda = B_\lambda} \quad [11.1]$$

Neglecting scattering => ***volume extinction coefficient = volume absorption coefficient***

Thus, the net change of radiation along path ***ds*** is due to the combination of emission and extinction

dI = dI (extinction) + dI(emission)

and thus the radiative transfer equation in the thermal region (ignoring scattering) is

$$dI_\lambda = -k_\lambda I_\lambda ds + k_\lambda B_\lambda ds \qquad [11.2]$$

or

$$\frac{dI_\lambda}{ds} = -k_\lambda [I_\lambda - B_\lambda] \qquad [11.3]$$

NOTE: Eqs.[11.2]-[11.3] are often called the differential forms of the radiative transfer equation.

Recall that by definition $d\tau_\lambda = -k_\lambda(s)ds$

Let's re-arrange terms in Eq.[11.3] and multiply both sides by ***exp(-τ_λ)***

$$-\frac{\exp(-\tau_\lambda)dI_\lambda}{d\tau_\lambda} + \exp(-\tau_\lambda)I_\lambda = \exp(-\tau_\lambda)B_\lambda \qquad [11.4]$$

and (using that *d[I(x)exp(-x)]=exp(-x)dI(x)* - *exp(-x)I(x)dx*) we have

$$-d\left[I_\lambda \exp(-\tau_\lambda)\right] = \exp(-\tau_\lambda)B_\lambda d\tau_\lambda \qquad [11.5]$$

Integrating the above equation along a path extending from some point s' to the end point s", it becomes

$$I_\lambda(s'')e^{-\tau_\lambda(s'')} - I_\lambda(s')e^{-\tau_\lambda(s')} = \int_{\tau(s'')}^{\tau(s')} B_\lambda(s)e^{-\tau_\lambda(s)}d\tau(s) \qquad [11.6]$$

and, re-arranging terms, we have **the solution** of the radiative transfer in IR

$$I_\lambda(s'') = I_\lambda(s')e^{-[\tau_\lambda(s')-\tau(s'')]} + \int_{\tau(s'')}^{\tau(s')} B_\lambda(s)e^{-[\tau_\lambda(s)-\tau(s'')]}d\tau(s) \qquad [11.7]$$

contribution from radiation incident at s' and transmitted to s''

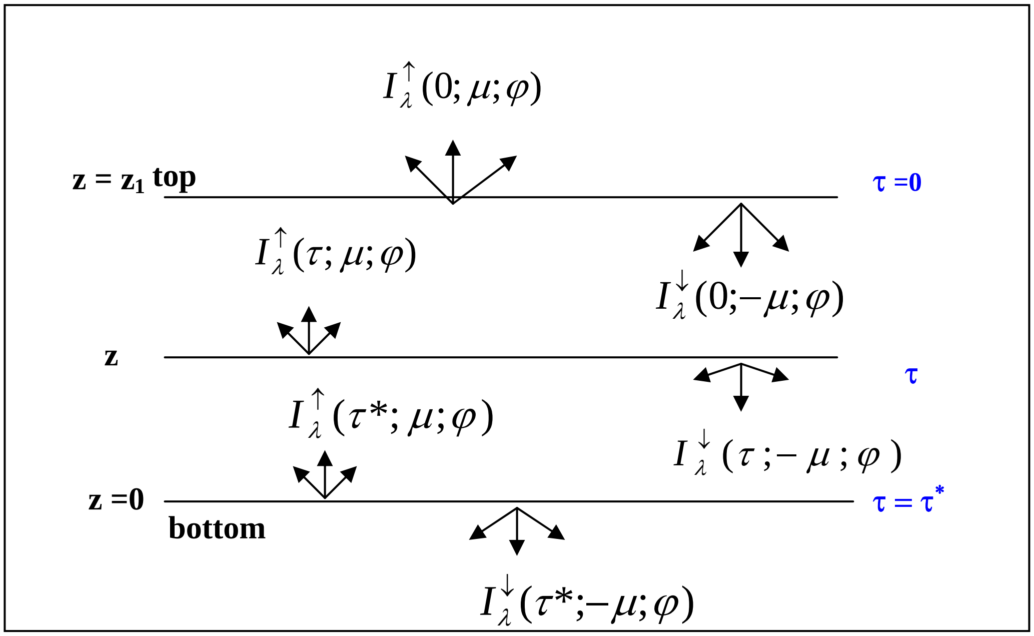
contribution from radiation emitted along the path and transmitted to s''

Let's consider a plane-parallel atmosphere ***(dz=*** μ ***ds*** and $\tau(z) = \mu\, \tau(s)$***)***

Upward intensity $I_\lambda^\uparrow$ is for $1 \ge \mu \ge 0$ (or $0 \le \theta \le \pi/2$);

Downward intensity $I_\lambda^\downarrow$ is for $-1 \le \mu \le 0$ (or $\pi/2 \le \theta \le \pi$)

(using that ***cos***(0)=1; ***cos***(π/2)=0 and ***cos***(π) =-1)



NOTE: For downward intensity, μ is replaced by **–**μ.

Eq.[11.7] gives both the upward intensity in the plane-parallel atmosphere

$$I_\lambda^\uparrow(\tau;\mu;\varphi) = I_\lambda^\uparrow(\tau^*;\mu;\varphi)\exp\left(-\frac{\tau^*-\tau}{\mu}\right) + \frac{1}{\mu}\int_\tau^{\tau^*}\exp\left(-\frac{\tau'-\tau}{\mu}\right) B_\lambda(\tau')d\tau' \quad [11.8]$$

and the downward intensity in the plane-parallel atmosphere:

$$I_\lambda^\downarrow(\tau;-\mu;\varphi) = I_\lambda^\downarrow(0;-\mu;\varphi)\exp\left(-\frac{\tau}{\mu}\right) + \frac{1}{\mu}\int_0^{\tau}\exp\left(-\frac{\tau-\tau'}{\mu}\right) B_\lambda(\tau')d\tau' \quad [11.9]$$

- In the atmospheric conditions for IR radiation, one can consider that at the surface

$$I_\lambda^\uparrow(\tau^*;\ \mu;\varphi) = B_\lambda(T_s) \text{ or } I_\lambda^\uparrow(\tau^*;\ \mu;\varphi) = \varepsilon_\lambda B_\lambda(T_s)$$

  no thermal incident radiation at the TOA

$$I_\lambda^\downarrow(0;\mu;\varphi) = 0$$

  no dependence on azimuthal angle φ.

Thus Eqs.[11.8] and [11.9] can be re-written as (in the wavenumber domain)

$$I_\nu^\uparrow(\tau;\mu) = B_\nu(\tau^*)\exp(-\frac{\tau^* - \tau}{\mu}) + \frac{1}{\mu}\int_\tau^{\tau^*}\exp(-\frac{\tau' - \tau}{\mu})\ B_\nu(\tau')d\tau' \quad [11.10]$$

$$I_\nu^\downarrow(\tau;-\mu) = \frac{1}{\mu}\int_0^\tau \exp(-\frac{\tau - \tau'}{\mu})\ B_\nu(\tau')d\tau' \quad [11.11]$$

Eqs.[11.10] - [11.11] can be expressed in terns **of monochromatic transmittance.**
Recall that

$$T_\nu(\tau,\mu) = \exp(-\frac{\tau_\nu}{\mu}) \quad [11.12]$$

and the differential form is

$$\frac{dT_\nu(\tau,\mu)}{d\tau} = -\frac{1}{\mu}\exp(-\frac{\tau_\nu}{\mu}) \quad [11.13]$$

- **Multiplication law of transmittance** states that when several gases absorb, the **monochromatic transmittance** is a product of the monochromatic transmittances of individual gases:

$$T_{\nu,1,2...N} = T_{\nu,1}T_{\nu,2}...T_{\nu,N} \quad [11.14]$$

Thus the general solutions for monochromatic upward and downward radiances **in terms of transmittance are:**

$$I_\nu^{\uparrow}(\tau;\mu) = B_\nu(\tau^*)T_\nu(\tau^* - \tau, \mu) - \int_\tau^{\tau^*} B_\nu(\tau') \frac{dT_\nu(\tau' - \tau, \mu)}{d\tau'} d\tau' \quad [11.15]$$

$$I_\nu^{\downarrow}(\tau;-\mu) = \int_0^{\tau} B_\nu(\tau') \frac{dT_\nu((\tau - \tau'), \mu)}{d\tau'} d\tau' \quad [11.16]$$

The general solutions for monochromatic upward and downward radiances can be expressed in terms of the weighing function:

$$I_\nu^{\uparrow}(\tau;\mu) = B_\nu(\tau^*)T_\nu(\tau^* - \tau, \mu) + \int_{\tau^*}^{\tau} B_\nu(\tau')W(\tau, \tau', \mu) d\tau' \quad [11.17]$$

where the **weighting function** is defined as $W_\nu(\tau, \tau', \mu) = \left| \frac{dT_\nu(\tau, \tau', \mu)}{dz} \right|$ [11.18]

NOTE: The concept of the **weighting function** plays a central role in sounding techniques (i.e., retrievals of the vertical profile of T and gas (H2O, O3) concentrations from hyperspectral passive remote sensing.

Consider an isothermal atmosphere. Then B(τ) =B(z) = constant= $B(T_{atm})$

The radiance at the top of the isothermal atmosphere (from Eq.[11.15]) is

$$I_\nu^{\uparrow}(0;\mu) = B_\nu(\tau^*)T_\nu(\tau^*, \mu) + B_\nu(T_{atm})(1 - T_\nu(\tau^*, \mu)) \quad [11.19]$$

Where $T_\nu(\tau^*, \mu)$ is the transmission function of the entire atmosphere.

In no scattering, then

Emissivity = Absorptivity = 1- Transmission

Thus Eq.[11.19] can be re-written in terms of emissivity (or absorptivity).

2. Microwave radiative transfer.

According to the Rayleigh-Jeans distribution (see Lecture 3, Eqs.[3.13a,b]):

Brightness temperature is linear proportional to the radiance

- In the microwave, surface emissivities are low => need to account for reflection (i.e., the portion of microwave radiation emitted by the atmosphere toward the ocean is reflected back to the atmosphere and can be polarized depending on the viewing direction).

Eq.[11.8] can be modified to give the brightness temperature $T_{b,\nu}$ measured by a satellite passive microwave detector at a wavenumber ν

$$
\begin{aligned}
T_{b,\nu} &= \varepsilon_\nu^p T_{sur} \exp(-\tau^* / \mu) + \int_0^{\tau^*} T_{atm}(\tau') \exp(-\tau' / \mu) d\tau' / \mu \\
&+ R_\nu^p \exp(-\tau^* / \mu) \int_0^{\tau^*} T_{atm}(\tau') \exp(-(\tau^* - \tau') / \mu) d\tau' / \mu
\end{aligned} \qquad [11.20]
$$

where

T_{sur} is the surface **temperature**, T_{atm} is the atmospheric **temperatur**e,

ε_ν^p is the emissivity of the ocean surface with the given polarization state p, and

$R_\nu^p = (1 - \varepsilon_\nu^p)$ is reflectivity of the ocean surface with the given polarization state p.

Let's assume that **the absorption by water vapor only in the boundary layer**

$$\int_0^{\tau^*} T_{atm}(\tau') \exp(-\tau' / \mu) d\tau' / \mu \approx T_{sur} [1 - \exp(-\tau^* / \mu)] \qquad [11.21]$$

Thus we have from Eq.[11.20]

$$T_{b,\nu} = T_{sur} [1 - T_\nu^{\,2} (\tau^*, \mu)(1 - \varepsilon_\nu^p)] \qquad [11.22]$$

where $T_\nu(\tau^*, \mu) = \exp(-\frac{\tau_\nu^*}{\mu})$ is the transmission function.

3. Measurements of atmospheric path-integrated quantities: precipitable water vapor and cloud liquid water.

Let's consider brightness temperature measured at 19.35 GHz and 37 GHz for two measured polarization state (horizontal, H, and vertical V, polarization states)
Using Eq.[11.22], we have at each frequency

$$\Delta T_{b,\tilde{\nu}} = T_{sur}(R^V - R^H) T_{\tilde{\nu}}^{\,2}(\tau^*, \mu) \qquad [11.23]$$

where

$R_{\tilde{\nu}}^{H,V} = (1 - \varepsilon_{\tilde{\nu}}^{H,V})$

The atmospheric transmission can be represented as a combination of transmission for O_2, T_{O2}, cloud liquid water, T_W, and water vapor, T_ϖ, at each frequency

$$T^2 = T_{O_2}^2 T_W^2 T_\varpi^2 \qquad [11.24]$$

$T_W = \exp(-k_{a,W} LWP/\mu)$ where $k_{a,W}$ is the mass absorption coefficient of liquid water (cloud drops) and *LWP* is the liquid water path defined as liquid water content (*LWC*, Lecture 5, Eq.[5.5]) integrated over the path

$T_\varpi = \exp(-k_\varpi \varpi / \mu)$ where k_ϖ is the absorption coefficient of water vapor and ϖ is the amount of water vapor integrated over the part (called **precipitable water, often reported in mm**).

From Eq.[11.23] we have

$$k_{a,W} LWP + k_\varpi \varpi = -\frac{\mu}{2} \ln\left[\frac{\Delta T_b}{T_{sur}(R^V - R^H) T_{02}^2}\right] \qquad [11.25]$$

- Eq.[11.25] for two channels => we have two equations to solve for *LWP* and ϖ given the values of $T_{O2,19}$, $T_{O2,37}$, $k_{a,W}$, k_ϖ, and $R_{\tilde{\nu}}^{H,V}$

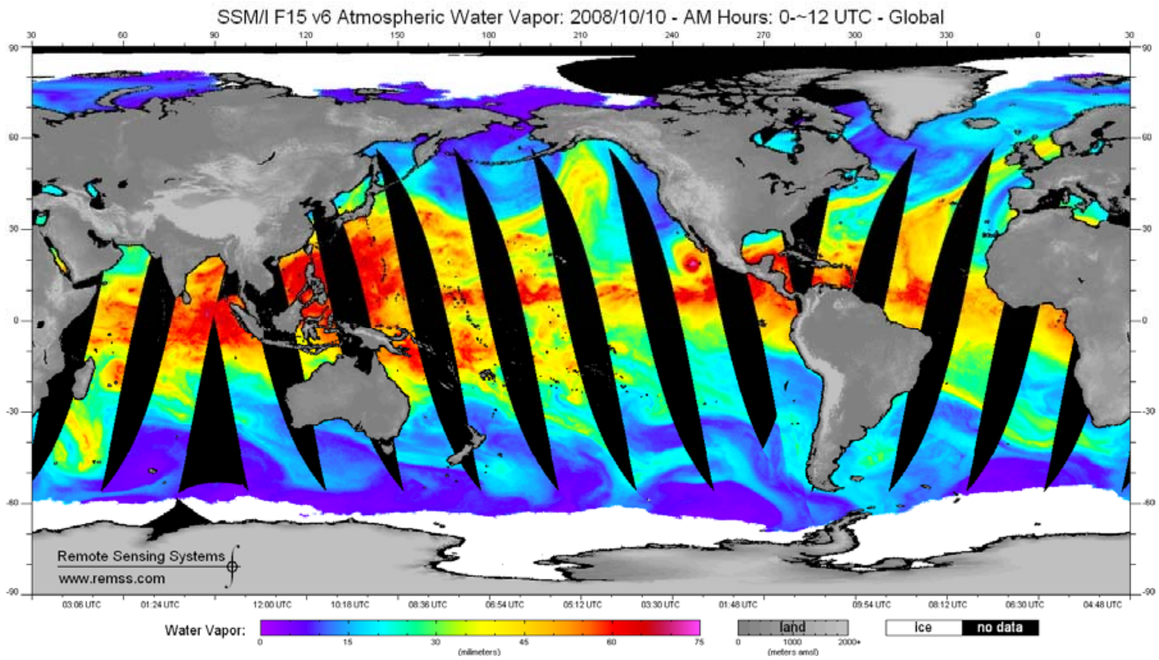
Problems:

1) Need to know absorption coefficients
2) $R_{\tilde{\nu}}^{H,V}$ are functions of wind speed

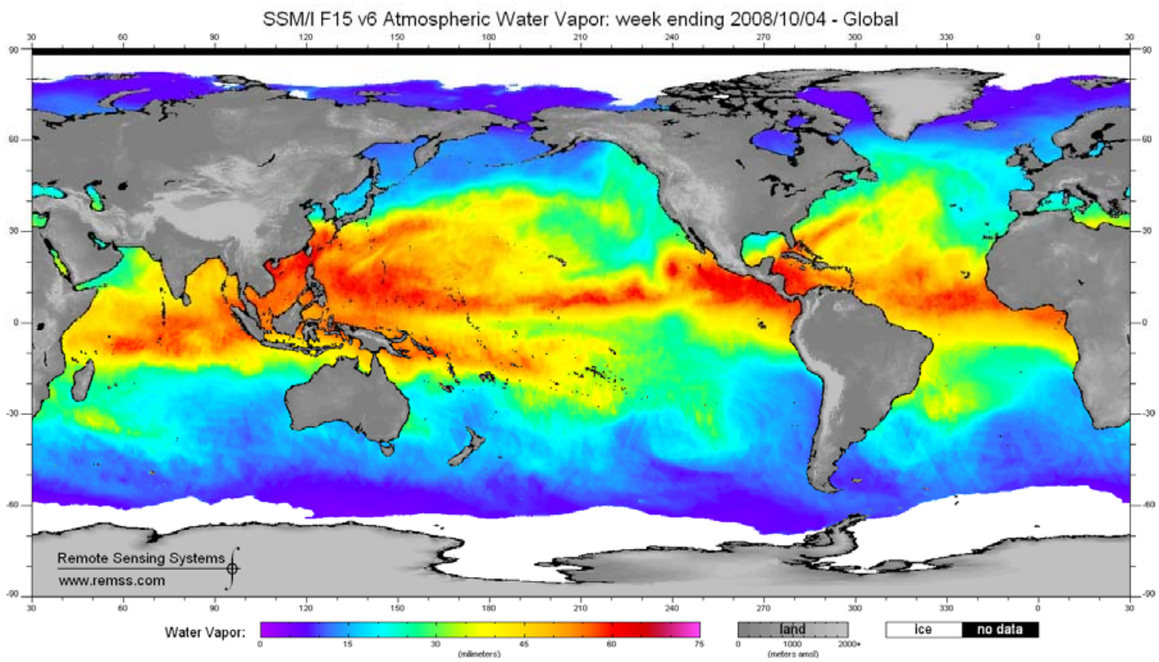
NOTE: The above principle is used in the retrieval algorithm of Special Sensor Microwave/Imager (SSM/I). SSM/I is a passive microwave sensor aboard the DMSP satellite series http://www.remss.com/

Example of SSM/I products

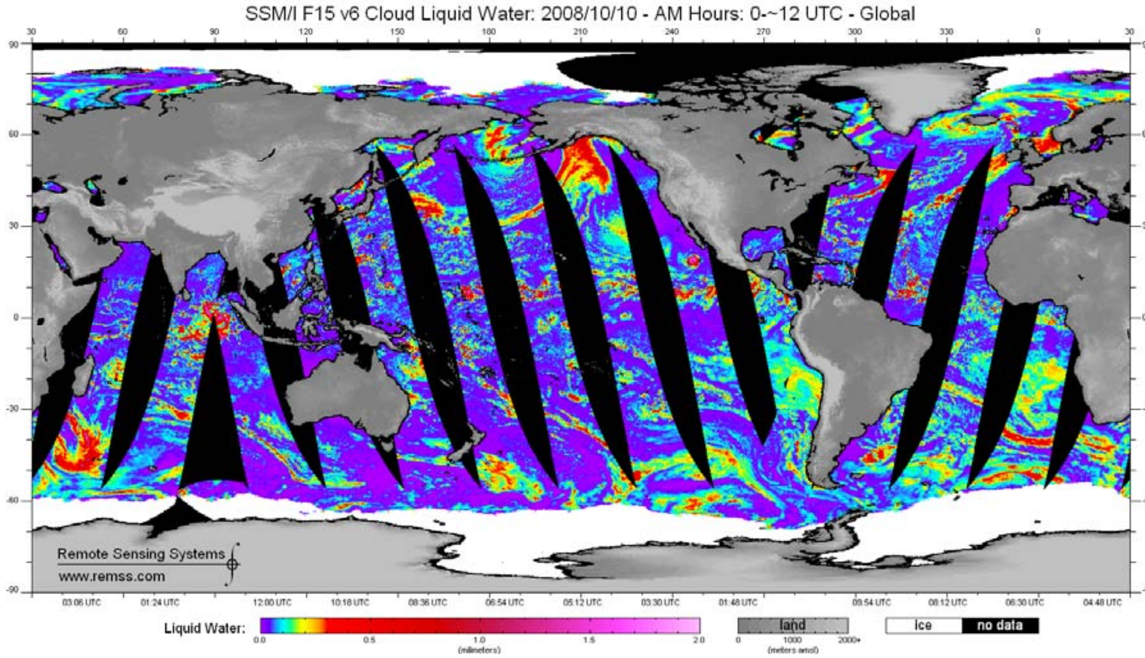
Daily (UTC AM) retrievals of precipitable water vapor



Weekly retrievals of precipitable water vapor



Daily (UTC AM) retrievals of cloud liquid water - compare to the above image for precipitable water



Weekly retrievals of cloud liquid water - compare to the above image for precipitable water