Lecture 3

Blackbody radiation. Main Laws.

Objectives:
2. Main laws:
   - Planck function.
   - Stefan-Boltzmann law.
   - Wien’s displacement law.
   - Kirchhoff’s law.
3. Emissivity of the surfaces and the atmosphere.

Required reading:
L02: 1.2, 1.4.3

Additional/advanced reading:
L02: Appendix A;
T&S99: pp.96-97


Thermodynamical equilibrium describes the state of matter and radiation inside an isolated constant-temperature enclosure.

Blackbody radiation is the radiative field inside a cavity in thermodynamic equilibrium.
NOTE: A blackbody cavity is an important element in the design of radiometers. Cavities are used to provide a well-defined source for calibration of radiometers. Another use of a cavity is to measure the radiation that flows into the cavity (e.g., to measure the radiation of sun).

**Properties of blackbody radiation:**

- Radiation emitted by a blackbody is isotropic, homogeneous and unpolarized;
- Blackbody radiation at a given wavelength depends only on the temperature;
- Any two blackbodies at the same temperature emit precisely the same radiation;
- A blackbody emits more radiation than any other type of an object at the same temperature;

NOTE: The atmosphere is not strictly in the thermodynamic equilibrium because its temperature and pressure are functions of position. Therefore, it is usually subdivided into small subsystems each of which is effectively isothermal and isobaric referred to as Local Thermodynamical Equilibrium (LTE). (See Goody & Yung: 2.2.2)

- The concept of LTE plays a fundamental role in atmospheric studies: e.g., the main radiation laws discussed below, which are strictly speaking valid in thermodynamical equilibrium, can be applied to an atmospheric air parcel in LTE.

2. **Main laws.**

**Planck function** $B_\lambda(T)$ gives the *intensity* (or *radiance*) emitted by a blackbody having a given temperature.

NOTE: Planck derived $B_\lambda(T)$ by introducing photon energies, $E = n_h c / \lambda$

See Thomas & Stamnes or L02 (Appendix A)
• **Plank function** can be expressed in wavelength, frequency, or wavenumber domains as

\[
B_\lambda(T) = \frac{2hc^2}{\lambda^5 (\exp(hc / k_B T\lambda) - 1)} \tag{3.1}
\]

\[
B_\nu(T) = \frac{2h\tilde{\nu}^3}{c^2 (\exp(h\tilde{\nu} / k_B T) - 1)} \tag{3.2}
\]

\[
B_\nu(T) = \frac{2h\nu^3 c^2}{\exp(h\nu c / k_B T) - 1} \tag{3.3}
\]

where \(\lambda\) is the wavelength; \(\tilde{\nu}\) is the frequency; \(\nu\) is the wavenumber; \(h\) is the Plank’s constant; \(k_B\) is the Boltzmann’s constant (\(k_B = 1.3806x10^{-23}\) J K\(^{-1}\)); \(c\) is the velocity of light; and \(T\) is the absolute temperature of a blackbody.

**NOTE:** The relations between \(B_\nu(T);\) \(B_\nu(T)\) and \(B_\lambda(T)\) are derived using that

\[
B_\nu(T)d\tilde{\nu} = B_\nu(T)d\nu = B_\lambda(T)d\lambda , \text{ and that } \lambda = c / \tilde{\nu} = 1 / \nu \quad \Rightarrow
\]

\[
B_\nu(T) = \frac{\lambda^2}{c} B_\lambda(T) \quad \text{and} \quad B_\nu(T) = \lambda^2 B_\lambda(T)
\]

**Asymptotic behavior of Planck function:**

• If \(\lambda \to \infty\) (or \(\tilde{\nu} \to 0\)) (known as Rayleigh –Jeans distributions):

\[
B_\lambda(T) = \frac{2k_B Tc}{\lambda^4}
\]

\[
B_\nu = \frac{2k_B T\tilde{\nu}^2}{c^2}
\]

**NOTE:** This longwave limit has a direct application to passive **microwave** remote sensing.
If $\lambda \to 0$ (or $\nu \to \infty$):

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \exp\left(-\frac{hc}{\lambda k_B T}\right)$$

$$B_\nu = \frac{2h\nu^3}{c^2} \exp\left(-\frac{h\nu}{k_B T}\right)$$

Figure 3.1 Planck function on log-log plot for several temperatures.
NOTE: Because the sun and earth’s spectra have a very small overlap, the radiative transfer processes for solar and infrared regions are often considered as two independent problems.

**Stefan-Boltzmann law.**

The **Stefan-Boltzmann law** states that the total power (energy per unit time) emitted by a blackbody, per unit surface area of the blackbody, varies as the fourth power of the temperature.

\[
F = \pi B(T) = \sigma_b T^4
\]  

[3.4]

where \(\sigma_b\) is the *Stefan-Boltzmann constant* \((\sigma_b = 5.671 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4})\), \(F\) is radiant flux [W m\(^{-2}\)], and \(T\) is blackbody temperature [K];

\[
B(T) = \int_0^\infty B_\lambda(T) \, d\lambda
\]
Wien’s displacement law.

The **Wien’s displacement law** states that the wavelength at which the blackbody emission spectrum is most intense varies inversely with the blackbody’s temperature. The constant of proportionality is **Wien’s constant** (2897 K μm):

\[ \lambda_m = \frac{2897}{T} \quad [3.5] \]

where \( \lambda_m \) is the wavelength (in micrometers, μm) at which the peak emission intensity occurs, and \( T \) is the temperature of the blackbody (in degrees Kelvin, K).

**NOTE**: this law is simply derived from \( dB_\lambda/d\lambda = 0 \)

**NOTE**: Easy to remember statement of the Wien’s displacement law:

the hotter the object the shorter the wavelengths of the maximum intensity emitted

Kirchhoff’s law.

The **Kirchhoff’s law** states that the emissivity, \( \varepsilon_\lambda \), of a medium is equal to the absorptivity, \( A_\lambda \), of this medium under thermodynamic equilibrium:

\[ \varepsilon_\lambda = A_\lambda \quad [3.6] \]

where \( \varepsilon_\lambda \) is defined as the ratio of the emitting intensity to the Planck function; \( A_\lambda \) is defined as the ratio of the absorbed intensity to the Planck function.

For a **blackbody**: \( \varepsilon_\lambda = A_\lambda = 1 \)

For a **non-blackbody**: \( \varepsilon_\lambda = A_\lambda < 1 \)

For a **gray body**: \( \varepsilon = A < 1 \) (i.e., no dependency on the wavelength)

**NOTE**: **Kirchhoff’s law** applies to gases, liquids and solids if they in TE or LTE.
3. Emissivity of the surfaces and the atmosphere.

- For atmospheric radiation transfer applications, one needs to distinguish between the emissivity of the surface (e.g., various types of lands, ice, etc.) and the emissivity of an atmospheric volume (consisting of gases, aerosols, and/or clouds).

**Emissivity of the surfaces:**

- In general, emissivity depends on the direction of emission, surface temperature, wavelength and some physical properties of the surface (e.g., the refractive index).
- In thermal IR ($\lambda > 4 \mu m$), nearly all surfaces are efficient emitters with the emissivity $> 0.8$ and their emissivity does not depend on the direction. Therefore, the intensity emitted from a unit area surface at a given wavelength is $I_\lambda = \varepsilon_\lambda B_\lambda(T_s)$

![Figure 3.3 Examples of spectral emissivity of some surfaces.](image-url)
Table 3.1 Average emissivities of some surfaces in the IR region from 10-to12 µm.

<table>
<thead>
<tr>
<th>Surface</th>
<th>Emissivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>0.993-0.998</td>
</tr>
<tr>
<td>Ice</td>
<td>0.98</td>
</tr>
<tr>
<td>Green grass</td>
<td>0.975-0.986</td>
</tr>
<tr>
<td>Sand</td>
<td>0.949-0.962</td>
</tr>
<tr>
<td>Snow</td>
<td>0.969-0.997</td>
</tr>
<tr>
<td>Granite</td>
<td>0.898</td>
</tr>
</tbody>
</table>

**Emissivity of the atmospheric volume:**
Absorption and thermal emission of the atmosphere volume is *isotropic*. Kirchhoff’s law applied to volume thermal emission gives

\[
j_{\lambda,\text{thermal}} = \beta_{a,\lambda} B_{\lambda} (T)
\]  

[3.7]

where \(\beta_{a,\lambda}\) is the absorption coefficient of the atmospheric volume and \(j_{\lambda}\) is the thermal emission coefficient which relates to the source function \(J_{\lambda}\) (introduced in Lecture 2) as \(J_{\lambda} = (j_{\lambda,\text{thermal}} + j_{\lambda,\text{scattering}}) / \beta_{e,\lambda}\) and \(\beta_{e,\lambda}\) is the extinction coefficient of the atmospheric volume.

Recall the elementary solution of radiative transfer equation (Eq.[2.16] derived in Lecture 2):

\[
I_{\lambda}(s_1) = I_{\lambda}(0) \exp(-\tau_{\lambda}(s_1;0)) + \int_{0}^{s_1} \exp(-\tau_{\lambda}(s_1;s)) J_{\lambda} \beta_{e,\lambda} ds
\]

- For a **non-scattering medium in the thermodynamical equilibrium:**
  \[J_{\lambda} = B_{\lambda}(T), \text{ where } B_{\lambda}(T) \text{ is Plank’s function.}\]
Also, for non-scattering media, we have $\beta_{e,\lambda} = \beta_{a,\lambda} = k_{\lambda} \rho$, where $k_{\lambda}$ is the mass absorption coefficient and $\rho$ is the density (see Lecture 2).

Thus, the solution of the equation radiative transfer for this case is

\[
I_{\lambda}(s_1) = I_{\lambda}(0) \exp(-\tau_{\lambda}(s_1;0)) + \int_0^{s_1} \exp(-\tau_{\lambda}(s_1;s)) B_{\lambda}(T(s)) k_{\lambda} \rho ds
\]

\[\text{[3.8]}\]

**NOTE:** The optical depth in Eq.[3.8] is due to absorption only, so

\[
\tau_{\lambda}(s_1; s) = \int_s^{s_1} \beta_{e,\lambda}(s) ds = \int_s^{s_1} k_{\lambda} \rho ds
\]